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Optimal solution for a cargo revenue management problem with allotment and spot arrivals

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ABSTRACT

We consider a single-leg cargo revenue management problem, in which a two-dimensional cargo capacity is sold through allotment contracts and in the spot market. Capacity sold on an allotment basis is guaranteed. We optimally solve the problem of determining how much of the total weight and volume capacity to sell on an allotment basis, by deriving a closed-form expression of the objective function. We provide numerical examples of industry-size problems and perform sensitivity analysis by changing some problem parameters. The sensitivity analysis illustrates the dependency of the optimal decisions on the spot and allotment booking types. The remaining capacity is then sold over a booking horizon in the spot market. Allotment bookings and spot requests can arrive any time over the booking horizon. Since some of the allotment bookings might not show up at departure, cargo carriers tend to overbook the remaining capacity allocated to spot requests. For these requests, we formulate a discrete-time dynamic capacity control model, to decide which of the spot requests to accept, based on the total weight and volume of the allotment show-ups and spot bookings accepted at the time of an arrival. We solve the exact dynamic programming model for medium-size industry problems. Since the booking policy based on critical booking levels or time periods is not optimal, we propose several heuristics to solve large industry problems and derive an upper bound on the value function. We test their performance via simulation against the optimal solution, the upper bound, and the first-come first-served policy, and recommend a heuristic that performs well in a wide variety of numerical cases. Finally, we show via simulation, that our model outperforms the one existing in the literature, for small and medium-size industry problems.

1. Introduction

Cargo management is gaining importance as it is generating substantial revenues for airlines in the saturated passenger markets. According to Boeing (World air cargo forecast, 2011), cargo revenue, which represents about 15% of the total traffic revenue, is expected to grow at an average annual rate of 5.9% for the next 20 years. According to the BTS (Bureau of Transportation Statistics, 2011), cargo transportation is one of the fastest growing segments of the U.S. economy. The bureau also notes that air freight has experienced one of the fastest growth rates in the cargo industry. Such expectations of growth are not new (Kasilingam, 1996). As a growing market, air cargo business in general deserves more attention. In particular, it is important to efficiently manage the cargo bookings.

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Cargo revenue management (RM) is different than that of passengers (Kasilingam, 1996; Amaruchkul et al., 2007; Slager and Kapteijns, 2003). Among the main differences is the two-dimensional capacity, characterized by weight and volume, versus a one-dimensional capacity in passenger RM (i.e. number of seats). The cargo capacity is perishable, but unlike passenger airlines where the capacity is fixed, cargo capacity is affected by factors such as the number of passengers and baggage carriage, unless the carrier has dedicated freighters. Another distinctive feature for cargo RM is demand. Passenger demand follows a seasonality pattern and is itinerary-bound, whereas cargo demand patterns are more difficult to anticipate and are service bound, i.e. shippers can employ varying service levels at different prices as long as the shipments are delivered on time. Furthermore, cargo business serves a few key customers which are granted space by the carrier. Finally, cargo carriers charge shippers based on the chargeable weight of a shipment, which is the maximum of the shipment weight and the weight equivalent of the volume.

Cargo is either sold on a contracted basis or on a request-reply basis (Slager and Kapteijns, 2003; Billings et al., 2003). In the former, available capacity is sold as allotments to intermediaries, called forwarders (Gupta, 2008; Hellermann, 2006; Slager and Kapteijns, 2003), for a predetermined commitment period (months in advance) on a specific flight. These preallocations translate into granted space for the duration of the commitment period. Allotment contracts are long-term signed agreements between the airline and the forwarders and usually have a lifespan of six or twelve months. An important objective of these contracts is the implied shift of capacity utilization risk. In the absence of advance-sale agreements, the air carrier would bear the entire capacity utilization risk (Hellermann, 2006). In return for the risk shift, airlines grant lower prices for contract capacity. If, close to departure, capacity is returned, a cancelation fee is charged, but this is seldom practiced as forwarders tend to be big businesses with market power: "Long-term customer relations take priority [in cargo RM]" where "a few important customers ...ship large volumes" as stated in Talluri and van Ryzin (2005). Thus, contract enforcement is hindered by market power of forwarders. One distinctive feature of these contracts is that a forwarder usually does not pay a cargo carrier if the forwarder books space for the cargo but changes his opinion and does not ship the cargo. This results in no-shows. As a result, the committed granted space for allotments may not be fully utilized.

Allotment contracts are renewed or canceled twice a year, at the beginning of the IATA winter and summer schedule. During those periods, new requests for contracted capacity arrive per customer per flight. When contracts are submitted, the airline makes the decision of which contracted requests to accept. The remaining capacity is a "free sale" and sold at standard rates to either direct shippers or forwarders in the spot market, with no capacity guaranteed. One main challenge that cargo revenue management practitioners face is that the actual utilization of the granted space for allotments, which realizes shortly before the departure when the shipments actually show up, is different than the space initially reserved by these allotments. This is aggravated by the fact that forwarders can cancel their contracted capacity anytime before the departure without paying any contractual penalties. To avoid the situation of flying with spoiled capacity in case of low allotment show-up, cargo carriers tend to overbook the remaining capacity sold to spot arrivals. In the case when the total allotment and spot show-ups exceed either the weight or the volume capacities, some of the spot show-ups are offloaded, resulting in an offload cost that depends on the weight and volume being offloaded.

In this paper, we consider two problems that are closely inter-related, with their dynamics motivated by the real practice in air cargo business, as described in Hellermann (2006). The first problem is to determine which contracts should be signed with allotment customers. This specifies the portion of the total capacity sold as allotments. The second problem is to dynamically decide which spot requests should be accepted or rejected, once the contracted capacity has been allocated. We first develop a static model to be solved at the beginning of the booking horizon, that manages allotment bookings by deciding which contracts among available bids to accept. Our model takes into account the possibility of allotments no shows. For this model, a non-linear program is presented, with the objective of maximizing the expected revenue from allotment and spot sales, less the offload cost when either the weight or volume capacity is exceeded. We optimally solve this program for industry-size problems and illustrate the dependency of the optimal decisions on the spot and allotment types. Second, we develop a dynamic model for managing spot bookings, with the state being the weight and volume of the showing up allotments and spot bookings. In our model, both allotment and spot bookings arrive dynamically over the booking horizon. Since a portion of the allotment bookings may not show-up, the cargo capacity is overbooked. We

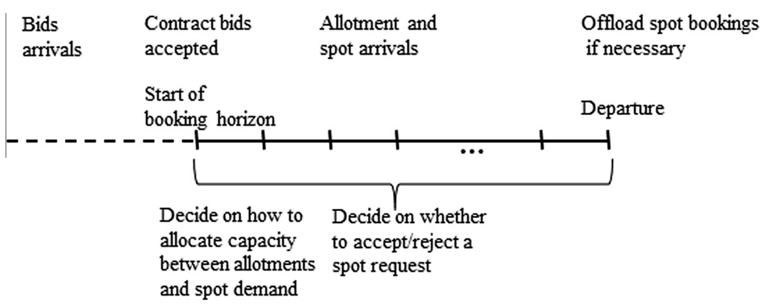

Fig. 1. Timing of events over the booking horizon.

assume that only spot market show-ups can be offloaded, while allotment show-ups have to be accommodated. The timing of events is depicted in Fig. 1.

Both models capture the variability of allotment and spot demand by assuming that packages arrive according to a Poisson Process. The dynamic model takes as inputs the bid decisions obtained from the static model and is linked to the static model through the time-dependent arrival probabilities. We solve the exact dynamic control model for medium-size problems. For very large problems, and in the absence of monotonicity properties, our dynamic capacity control model cannot be solved optimally via the classical dynamic program (DP) recursive scheme due to excessive storage requirement, so we propose heuristics that give near optimal solutions. We construct an upper bound on the DP value function and numerically test these heuristics against the optimal solution, upper bound, and first-come first-served policy via simulation. We recommend a well-performing heuristic based on decoupling the original DP model. Finally, we compare our model to a model for allocating capacity between allotments and spot market bookings studied by Levin et al. (2012), and using simulation, we show that our model performs better for small and medium size industry problems, which is the main contribution of our work.

The DP formulation we present, in the second part of our paper, for the dynamic spot booking control model is close to that in Amaruchkul et al. (2007) in the sense that the state of our DP is based on the weight and volume of the showing up bookings. However, three features distinguish our work from theirs: First, this paper studies the allotment and spot booking allocation problems which are inter-related. So our DP model accounts not only for the spot arrivals, but also for the arrival of allotments. Once the allotment bids show up over the booking horizon before the departure, they have to be accommodated onto the flight, and the remaining capacity is sold in the spot market. Thus, our DP model tracks how much of the total weight and volume accepted is due to allotment show-ups and how much is due to spot show-ups. Amaruchkul et al. (2007) only consider the spot booking allocation, without the consideration of show-ups, thus, the state of their Markov Decision Process (MDP) is based on the weight and volume of the spot bookings only. Second, the time-dependent arrival probabilities of the spot booking requests of our DP model are derived using the static demands modeled by Poisson random variables of the static capacity allocation problem that occurs at the beginning of the booking horizon. Third, the heuristics we present are based on the decomposition of the multi-dimensional DP into single-dimensional ones, a method that has been used in the literature (Amaruchkul et al., 2007; Maddah et al., 2010; Hoffmann, 2013). Distinctively, our proposed heuristics are designed to account for the allotment as well as the spot show-ups, with the arrival probabilities also linked to the static demand models of the capacity allocation problem.

Our study of allotment and spot market demand is the closest to that of Levin et al. (2012), who consider allotment and spot market cargo booking allocations for parallel flights. The approach we adopt in this paper allows us to obtain the exact DP solution which is shown to be superior over Levin et al. (2012) heuristics for medium-size problems. Levin et al. (2012) provide an approximation that depends on the choice of the Lagrange multipliers, which are a function of the allotment selection decision when the number of accepted spot bookings is zero. In their spot market booking control formulation, the state of the DP is the total accepted number of spot bookings of each type, while our DP tracks the total weight and volume accepted. The most important difference from our work is that the approximation of the optimality equation in their work no longer depends on the numbers of accepted spot bookings. This is stated in the last paragraph, on Page 360 of their paper: "The chief drawback of the booking control policy ··· is that the optimal solution to this problem does not depend on the numbers of accepted spot market bookings. In particular, irrespective of whether we reach time period t with too many accepted spot market bookings or with too few, we always open the same set of flight- and cargo-type combinations for sale." In our paper, the optimal solution to the DP and the heuristics proposed depend on the total weight and volume accepted, updated in each time period upon an allotment or a spot arrival.

Moreover, they show that the spot booking problem decomposes by cargo type. In practice, (as also established in a wide array of revenue management problems), there is a clear dependency of the spot booking request decisions on the spot types. In the dynamic model, our results are driven by the various spot cargo types and the number of packages of each type available in each bid. We also illustrate the dependency among bid and spot cargo types in the static capacity allocation model. In fact, the objective value of the spot booking problem has a "very special structure that makes it solvable in a tractable fashion". This special structure reduces to a simple problem with optimal solution studied by Talluri and van Ryzin (2005). The over-simplification of the problem has resulted in a solution that does not capture the dynamics of the multi-dimensional cargo problem. Thus, it remains important to investigate the quality of their proposed solution compared to the optimal solution. In addition, Levin et al. (2012) assume that allotments arrive only at departure. In our paper, allotments arrive randomly over the entire booking horizon. Also, allotment arrivals are automatically accepted, unlike Levin et al. (2012) in which spot and allotment arrivals might be offloaded. We make this assumption, motivated by Hellermann (2006), since bookings on allotment basis are made through contracts with air carriers that guarantee committed capacity for the allotments on each flight. In our model, only spot can be offloaded.

The contributions of our paper are the following: First, we consider a capacity allocation problem with allotments and spot bookings commonly used in practice, as presented in Hellermann (2006), with the allotment bookings arriving any time throughout the booking horizon. The capacity sold on allotment basis is committed, so the allotment arrivals have to be accommodated on the aircraft, and only the spot market arrivals can be offloaded. Second, we optimally solve the allotment problem for industry-size problems by deriving a closed-form expression for the objective function, and provide sensitivity analysis to some of the problem parameters. The sensitivity analysis illustrates the dependency among bid and spot types and their effect on the optimal decisions. Third, we solve the exact DP spot booking problem for medium-size industry problems, a major contribution to the literature, with the decision in each time period depending on the total weight and total volume of the allotment and spot show-ups. This model is linked to the capacity allocation model via the arrival probabilities, and takes as input the accepted bids at the beginning of the booking horizon. Our numerical results illustrate the dependency of the decisions on the spot cargo types chosen and the number of packages of each type in each bid accepted. For large industry problems, we propose heuristics that also utilize the showing up weight and volume, and test their performance numerically to recommend a well-performing heuristic. Thus, our proposed solution methodology has preserved the essence of the two-dimensional aspect of the problem studied. Finally, we numerically show, via simulation, that the exact DP solution that we obtain for medium size problems is superior to that obtained using Levin et al. (2012) approach, so our model outperforms that proposed in Levin et al. (2012) for small and medium size problems. For large problems, the heuristic we propose does not perform as well when compared to Levin et al. (2012). It is noteworthy that general purpose software were used in the numerical analysis, thus, with the increasing computational capability that is becoming available, our approach could be used to solve much larger problems to optimality.

The rest of the paper is organized as follows. In Section 2, we review the related literature. Section 3 discusses the pricing structure adopted in practice, to be used in the model formulation. In Section 4, we provide the formulation of our problem: In SubSection 4.1, we present the static capacity allocation model and numerically solve the resulting non-linear integer program for small and industry-size problems. In SubSection 4.2, we formulate the dynamic model for controlling the spot booking requests, taking into consideration the possibility of no-shows of allotments. In Section 5, we propose several heuristics that give near-optimal solution and develop an upper bound on the value function. Numerical testing of the heuristics against the optimal solution, upper bound, and first-come first-served policy is performed in Section 6. In 6.3, we compare our model to the literature, highlighting the contribution of our paper. A conclusion is provided in Section 7.

2. Literature review

Unlike cargo revenue management (RM), extensive research has been done on passenger revenue management (e.g. Rothstein, 1985; McGill and van Ryzin, 1999; Talluri and van Ryzin, 2005). There are only a few research works that deal with single- and multi-leg cargo RM, which we survey below.

Several papers survey the practices of cargo RM. For example, Kasilingam (1996), Billings et al. (2003) compare passenger and cargo RM, and conclude that air cargo carriers must adopt RM or they would face the consequences of losing the opportunities to make (more) revenue. Kasilingam (1996) also notes that the cargo capacity available for sale is not known in advance on aircrafts carrying passengers. Talluri and van Ryzin (2005) go further by saying that "the decisions for both passenger and cargo are interrelated and ideally should be coordinated by a single RM system". The practice of cargo RM is far from such coordination so we aim our models for aircrafts reserved only for cargo transportation. Slager and Kapteijns (2003) describe cargo RM specifically at KLM by focusing on development and implementation of RM processes, and on critical success factors during implementation.

A few recent studies on cargo RM provide a static formulation of the cargo RM problem. Kasilingam (1997) develops cargo overbooking models for a given distribution of the capacity and the show-up rate. However, his models are one-dimensional. Karaesman (2001) develops a bid pricing method in air cargo revenue management for accepting or rejecting a shipment request. Based on the weight, volume, and fare of the shipment, the bid prices are determined by computing the shadow prices of the capacity constraints in a linear program. Zhang et al. (2004) investigate the effect of an air cargo alliance on competition in passenger markets. Popescu et al. (2006) statistically justify the use of a discrete estimator of the show-up rate in cargo RM and show that a discrete estimator can significantly impact the profits and service levels. Yan et al. (2006) develop an integrated scheduling model, combining airport selection, air cargo fleet routing, and timetable setting by formulating a mixed integer program. Tang et al. (2008) develop a scheduling model that combines passenger, cargo, and combination flights. Luo et al. (2009) present the first two dimensional model for overbooking. They have a cost objective which is additive over volume and weight dimensions. Moussawi-Haidar and Çakanyildirim (2012) consider a two-dimensional profit maximization model with non-additive revenue and offload cost, i.e. the revenue and cost of a package are computed based on the maximum of the cargo weight and the weight equivalent of its volume. Zou et al. (2013) find the optimal overbooking decisions for a two-segment air cargo flight network, using the inventory transshipment modeling approach. They compare the profitability outcomes between local and global optimization models and conclude that profit improvement through global optimization is significant when local shipments have a higher freight yield compared to flow-through shipments. In a recent paper, Amaruchkul et al. (2011) study capacity contracts between a carrier and a forwarder when certain parameters such as the forwarder's demand, operating cost to the carrier, margin, and reservation profit are its private information. They propose contracts in which the forwarder pays a lump sum in exchange for a guaranteed capacity allotment and receives a refund for each unit of unused capacity according to a pre-announced refund rate.

Other studies consider a dynamic formulation. Amaruchkul et al. (2007) is the first to formulate a two-dimensional Markov Decision Process (MDP) for managing the spot booking requests for a single-leg cargo RM problem. The volume of each accepted cargo at the time of the booking is random and known just before the departure. In their multi-period model, the number of booked cargos of a certain type is known in each period. But because of random volumes, there may be spoilages and offloads. They use a two-dimensional revenue function but a linear offload cost function as in Luo et al. (2009). They develop a heuristic that decomposes the two-dimensional MDP into single-dimensional ones and perform numerical analysis. Zhuang et al. (2012) study a single-resource revenue management problem with multiple classes and random resource consumptions. They analyze how the variability in resource consumption impacts the optimal booking decisions, by developing a dynamic programming model with random resource consumptions. They also suggest heuristics that solve the DP, using the information regarding the degree of randomness in resource consumption. Xiao and Yang (2010) formulate the revenue management problem with two capacity dimensions as a continuous-time stochastic control model, derive the optimal solution analytically and explore the structural properties of the optimal solution. They show that the optimal solution does not preserve the nested price structure as in one-dimensional single-resource problems. For the capacity allocation of booking requests, Huang and Chang (2010) propose a heuristic based on approximating the expected revenue function of the dynamic program and compare it to the decoupling heuristic. Han et al. (2010) formulate the problem as a discrete-time Markov chain and derive an optimal bid-price booking control policy. Amaruchkul and Lorchirachoonkul (2011) study the problem of accepting/rejecting allotment requests of multiple forwarders of a single air-cargo carrier. They develop a DP to choose the allotments that maximize the expected total profit, and propose heuristics for solving the problem. Azadian et al. (2012) consider a freight forwarder's routing of a time-sensitive air-cargo in the presence of real-time information, and show that dynamic routing with real-time information can reduce cost.

This paper studies the capacity allocation problem between allotments and spot demand, and the booking problem of the spot requests. To the best of our knowledge, it is the first work in cargo RM that solves the exact DP model for medium-size problems,which is shown to be superior to the existing work in the literature. We note that we use general purpose software to solve the DP. While the exact solution approach may not be feasible for industry scale problems, this may not be a very important limitation given the rate at which computational capabilities are increasing.

3. Two-dimensional pricing structure

In this section, we present the pricing structure commonly adopted in the air cargo management practice, and which will be used in the problem formulation. Cargo carriers charge the shippers depending on both the volume and the weight of the cargo. They specify a standard inverse density d_s, which is currently 6 m^3/ton, (Levin et al., 2012; Moussawi-Haidar and Çakanyildirim, 2012; Amaruchkul et al., 2007), and which is the same for all aircraft types. It represents the ideal ratio of the volume to weight for cargos so the aircraft capacity is utilized efficiently. Airlines combine weight and volume of a cargo to compute a "chargeable weight", which is used to compute the revenue from the cargo. The chargeable weight is $\max\{\text{Volume}/d_s, \text{Weight}\}$. Then, the revenue obtained from a cargo is given as

$$\text{Revenue}(\text{Volume}, \text{Weight}) = a \max\{\text{Volume}/d_s, \text{Weight}\} \tag{1}$$

where a is the price per chargeable weight. Both the static and dynamic models developed in this paper use the non-linear revenue structure in (1). However, for the same chargeable weight, allotment and spot bookings generate different revenues, due to different revenue multipliers. In general, the price per ton of chargeable weight is smaller for an allotment booking than for a spot booking. However, spot prices are highly unpredictable. If the spot price for space falls below the forward price agreed upon between the carrier and the forwarder, the forwarder can simply rebook space on the spot market, without paying penalty to the carrier (Hellermann, 2006).

When a cargo shipment cannot be accommodated due to lack of either volume or weight capacity, the cargo is offloaded to be carried by alternative flights. This leads to additional costs for airlines. Similarly to the revenue, offload costs vary depending on the chargeable weight, but with a different multiplier b. In reality, the problem of deciding which shipments to offload is complex, as the carrier would need to solve an NP-hard integer program to determine which shipments to offload (Amaruchkul et al., 2007). However, for simplification, we assume, as in Amaruchkul et al. (2007), that the offload cost function is a separable function of the weight and volume of the bookings. Specifically, if the total weight and volume offloaded are w and v, then the offload cost is $h_w(w) + h_v(v)$, where $h_w(.)$ and $h_v(.)$ are non-negative increasing convex functions. We also assume that cargo is divisible, i.e. can be divided into smaller cargos. This implies that the total loaded weight (volume) capacity can be made equal to the weight (volume) capacity by dividing one of the cargos into smaller boxes.

4. Problem formulation

We consider a single flight with respective volume and weight capacities k_v and k_w. The airline sells capacity through long-term contracts based on bids submitted from forwarders, and on the spot market. Each bid specifies, among other parameters, the size of the allotment and the corresponding rate. Once a bid is accepted, a long-term contract is signed with the carrier at the beginning of the booking horizon. We assume that the number of bids awarded to each forwarder is at most one. Packages of an accepted bid arrive randomly over the booking horizon, and may not all show-up, as forwarders are not penalized for the booked but unutilized capacity in the case of no-shows. The physical cargo capacity utilized by an allotment booking becomes known only at the departure. To maximize its revenues, a cargo carrier needs to optimize the distribution of the cargo capacity between allocating part of it to allotments and selling the remaining capacity in the spot market.

Spot booking requests occur continuously over a short period of time $[1, .., T]$ before the departure, with T being the start of the booking horizon and 0 the departure time of the flight. Each time period in the booking horizon corresponds to a small interval of time in which at most one arrival occurs, either of spot or allotment type, or no arrival. According to Hellermann

(2006), spot booking requests start arriving thirty days before departure at a standard rate which is higher than the contract rate. Some spot bookings are made few days or even hours before the departure and are considered high margin products booked on short notice.

Since a large portion of the spot arrivals may come from forwarders (Hellermann, 2006), we assume in this paper that an allotment or spot booking request belongs to a type k, where $k = 1, \cdots, m$, m being the maximum number of cargo types. As in Amaruchkul et al. (2007), we assume that cargo types differ according to their capacity utilizations and profit margins. At time T, the air carrier receives a finite set $A = \{1, .., n\}$ of bids. We now describe the arrival process of allotment and spot packages. Let X_k^i be the number of packages of type k available in bid i, that will arrive over the time horizon $[1, .., T]$. Each bid $i \in A$ specifies the parameters of the contract, including the number of packages of each type available in a bid, and the revenue associated with a particular type k, r_k^c. The superscripts c and s are used to refer to allotment and spot types respectively. Similarly to Levin et al. (2012), we assume that the revenue is bid-independent, and is related to the capacity utilization of accepted bids through the show-up probability of each accepted bid. We represent a decision to grant a contract on the terms of bid i by a binary decision variable x_i. Thus, for bid i, $x_i = 1$ if bid i is accepted and 0 otherwise. The randomness in X_k^i captures the uncertainty in the actual utilization of the capacity reserved by the allotment bookings. We assume that $X_k^i \sim \text{Poiss}(\lambda_k^i)$, where λ_k^i is the average arrival rate of allotment bookings of type k coming from bid i.

A spot booking of type k has weight w_k and volume v_k, and generates revenue r_k^s which is based on its chargeable weight, as discussed in Section 3, and is paid when a spot booking shows up. In general, $r_k^s > r_k^c$. In case of low demand, air carriers lower the spot rate to attract bookings, in which case forwarders may cancel their contract-based bookings and re-purchase capacity at a lower rate. The models proposed in this paper account for the effect of no show-ups of the allotment bookings, but not for the effect of cancellations. This will be discussed in more detail in Section 4.2, when the dynamic model for spot arrivals is developed. Similarly to allotment arrivals, we let X_k^s be the number of spot packages of type k that arrive dynamically over the time horizon, then $X_k^s \sim \text{Poiss}(\lambda_k^s)$, where λ_k^s is the average arrival rate of spot requests of type k. We assume that the spot market and allotment demands are independent. This assumption is a simplification that several revenue management models use (Weatherford and Pfeifer, 1994; Hellermann, 2006). Hellermann (2006) states that "...it can be justified to regard these markets [allotment and spot] as sufficiently separated because the market structure differs and the asset provider aims at serving different market segments with different price sensitivities" (Hellermann, 2006, Page 56). Weatherford and Pfeifer (1994) study whether or not advance bookings should be offered by quantifying the benefits from demand stimulation, and using advance booking as a leading indicator of total demand. They show that even if booked demand gives the decision maker no information concerning unbooked demand (i.e. no correlation), benefits will still result. Hellermann (2006) develops a two-period model with an intermediary and an asset-provider who interact with each other in the contract market and with other buyers and sellers in the spot market. He examines what impact demand correlation has on the expected profits of the asset provider, by considering several levels of correlation between allotment and spot market demands. He shows that the expected profits of the asset provider are not affected by the correlation of demands, thus, concludes that no indication can be found that an interdependence of demands in the contract and spot market affect the expected profits of the asset provider. Following Hellermann (2006), we assume that the spot market demand and spot price are independent. Our model does not take into account the correlation between spot demand and spot price, which can be done as an extension of the current paper.

Also, let n_k^s be the maximum number of spot packages of type k to accept over the entire booking horizon. Thus, n_k^s is the booking level of spot requests of type k up to which spot requests of type k are accepted, and beyond which they are rejected. As a result, the total number of spot requests of type k that are accepted is the minimum of the spot arrivals and booking level for that type, i.e.

$$\text{Accepted number of spot requests of type } k = \min(X_k^s, n_k^s). \tag{2}$$

Since spot arrivals occur shortly before the departure, we assume in what follows that all the accepted spot bookings will show up. Thus, the expression in (2) is also the number of spot bookings that will show-up.

We let the offload cost function be $\psi(.,.)$. Following the discussion of the offload cost computation in Section 3, $\psi(.,.)$ takes as parameters the weight and volume offloaded, and is expressed as the summation of the total offloaded weight and total offloaded volume, with the weight and volume offload cost parameters being $h_w(.)$ and $h_v(.)$ respectively.

4.1. Allocation of capacity between allotment purchases and spot market sales

In this section, we formulate a non-linear integer program with two decisions, x_i, the decision of whether or not to accept bid i for all $i = 1, \cdots, n$, and n_k^s, the booking level of spot requests of type k, with $k = 1, \cdots, m$. This program is to be solved at time T (beginning of the booking horizon). The objective is to maximize the expected profit function, expressed as the expected revenue obtained from allotment show-ups, plus the expected revenue from the spot arrivals, less the expected offload cost when the total weight or volume from allotment and spot arrivals exceeds the weight or volume capacity. To model allotment no shows, we let ξ^i be a random variable to denote the show-up rate corresponding to bid i, which is the proportion of show-ups to the total bookings from bid i. Thus, we obtain the following nonlinear integer program (P):

$$\mathrm{Max}_{x_i,n_k^s}\left(\sum_{i=1}^{n}\mathrm{E}(\xi^i)\sum_{k=1}^{m}r_k^c\, x_i\, \lambda_k^i\right)+\mathrm{E}\left(\sum_{k=1}^{m}r_k^s\min(X_k^s,n_k^s)\right)$$

$$-\,\mathrm{E}\psi\left[\left(\sum_{i=1}^{n}\sum_{k=1}^{m}x_iX_k^i\, v_k+\sum_{k=1}^{m}\min(X_k^s,n_k^s)v_k-k_v\right)^+,\left(\sum_{i=1}^{n}\sum_{k=1}^{m}x_iX_k^i\, w_k+\sum_{k=1}^{m}\min(X_k^s,n_k^s)w_k-k_w\right)^+\right] \quad (3)$$

subject to $\quad x_i$ binary $\quad$ for all $i=1,..,n,$ (4)

$\qquad n_k^s \geqslant 0$ and integer $\quad$ for all $k=1,..,m.$ (5)

A closed-form expression for the expected profit function (3) is presented and derived in Appendix A. To validate our model, we first solve numerically a small-size problem with two bids and four cargo types. Then, we solve (P) for an industry-size problem with 10 bids and 24 cargo types. The following solution procedure is used in the numerical examples: The non-linear integer program in (P) in (3)–(5) is solved optimally by implementing it into an optimization software (Excel), with the objective being to maximize the closed-form approximation of the expected profit (derived in Appendix A), subject to x_i binary, and n_k^s integer and non-negative. The optimal solution is obtained for several numerical cases, for small and large problems. For the larger problem presented in Section 4.1.2, we implement (P) into an optimization software (Excel) and write a programming code by creating an Excel Visual Basic macro, that enumerates all the combinations of the possible values of bids, i.e. $2^{10}=1024$ cases in our example; For each combination, we find the optimal decisions, x_i and n_k^s, and the corresponding expected profit, using the Excel optimization software. Our code then searches for the largest expected profit over all the 1024 cases, which is the optimal expected profit. The computational time for solving each numerical case is 3 min on average.

4.1.1. Results for small-size problems

Consider a small-size problem with 2 bids and 4 cargo types, i.e. $i=1,2$ and $k=1,\ldots,4$. The weight and volume capacities are respectively $k_w=10$ tons and $k_v=58\ \mathrm{m}^3$. The weight (kg) and volume ($\times 10^4\ \mathrm{cm}^3$) of the four types are $(w_k,v_k)=\{(50,30),(100,55),(100,59),(250,147)\}, k=1,\cdots,4$. Bid 1 consists of only types 1 and 2, with 10 packages of type 1 and 15 packages of type 2, i.e. $\lambda_k^1=(10,15,0,0)$. Bid 2 consists of only types 3 and 4, with 20 packages of type 3 and 35 packages of type 4, i.e. $\lambda_k^2=(0,0,20,35)$. We consider an expected show-up rate of 80% for each bid. For our base case, we assume that the spot demand is two-third of the contract demand, i.e. $\lambda_k^s=(6.66,10,13.33,23.33)$. The revenue per unit chargeable weight is $r_k^c=0.8$ if capacity is sold on an allotment basis, and $r_k^s=1.12$ if sold in the spot market. To specify the offload cost rates, we define the benchmark volume and weight offload cost, η_v and η_w as the total expected revenue per expected unit volume and unit weight demand. Thus, η_v is mathematically defined as $\eta_v=\left(\sum_{i=1}^2\sum_{k=1}^4 r_k^c\lambda_k^i+\sum_{k=1}^4 r_k^s\lambda_k^s\right)/d_v$, where $d_v=\sum_{i=1}^2\sum_{k=1}^4 v_k\lambda_k^i+\sum_{k=1}^4 v_k\lambda_k^s$. Similarly, we define η_w. Then, the offload cost rates (h_v,h_w) for the base case are chosen so that $(h_v/\eta_v,h_w/\eta_w)=(1.25,1.25)$, which implies $h_v=\$2.02$ per unit volume offloaded and \$1.18 per unit weight offloaded.

Table 1 presents a summary of our numerical study. It gives the expected profit, the optimal decision of whether to accept or reject each bid, represented by x_1 and x_2, and the optimal booking level $n_1,..,n_4$ of each spot type. The first row in Table 1 gives the results of the base case. The results in the other rows are based on changing the parameters as indicated in the Change column. We consider changes in the offload cost, spot demand, allotment demand, and spot revenue. Table 1 indicates the following insights:

- In Case 1, we accept only bid 1, which has types 1 and 2 in it, and only a small portion of type 4 spot requests. Note that due to the high offload cost, $(h_v,h_w)=(2.02,1.18)$, it is optimal to reject most of type 4 spot requests.
- In Case 2, we decrease the offload cost, $(h_v,h_w)=(1.62,0.95)$, which results in higher profits and accepting more of type 1, 2, and 4 spot requests. Here, we accept less of type 3 and more type 4 spot requests, compared to the base case, as there is a trade off between the bulky types 3 and 4 spot requests.

Table 1
Optimal solution with two bids and four spot types, $i=1,2,$ and $k=1,\ldots,4$. Base Case: (k_v,k_w)=(58 m^3,10 tons), $r_k^c=0.8, r_k^s=1.12, (h_v,h_w)=(2.02,1.18), (w_k,v_k)$ = {(50, 30), (100,55), (100,59), (250,147)}, k = 1,..,4 $\lambda_k^1=(10,15,0,0), \lambda_k^2=(0,0,20,35), \lambda_k^s=(6.66,10,13.33,23.33)$), and $\mathrm{E}(\xi^i)=0.8$.

Case	Change	Expected Profit	x_1	x_2	n_1	n_2	n_3	n_4
1	Base	\$9626.1	1	0	17	31	60	10
2	$(h_v,h_w)=(1.62,0.95)$	\$9860.9	1	0	128	53	50	54
3	$(h_v,h_w)=(0.8,0.5)$	\$10,330.5	1	0	18	34	265	238
4	$\lambda_k^s=(13.33,20,26.66,46.66)$	\$11,192.2	0	0	18	134	25	25
5	$\lambda_k^1=(20,30,0,0)$	\$8841.9	1	0	33	48	9	10
6	$\lambda_k^2=(0,0,25,35)$	\$9893.3	0	0	21	56	43	55
7	$r_k^s=\$2.24$/ton	\$19,040	0	0	62	31	30	60

Table 2
Weight and volume characteristics of each type.

Category	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Weight(kg)	50	50	50	50	100	100	100	100	200	200	200	250	250	300	400	500	1000	1500	2500	3500	70	70	210	210
Volume($\times 10^4$ cm^3)	30	29	27	25	59	58	55	52	125	119	100	147	138	179	235	277	598	898	1488	2083	233	17	700	52

Table 3
Number of packages of each type arriving in each bid.

Bid i	λ_1^c	λ_2^c	λ_3^c	λ_4^c	λ_5^c	λ_6^c	λ_7^c	λ_8^c	λ_9^c	λ_{10}^c	λ_{11}^c	λ_{12}^c	λ_{13}^c	λ_{14}^c	λ_{15}^c	λ_{16}^c	λ_{17}^c	λ_{18}^c	λ_{19}^c	λ_{20}^c	λ_{21}^c	λ_{22}^c	λ_{23}^c	λ_{24}^c
1	4	4	4	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	2	2	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	2	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2

- As the offload cost decreases further(Case 3), $(h_v, h_w) = (0.8, 0.5)$, we reject bid 2 as it contains type 4 bulky requests in order to free up capacity, and we accept more of types 3 and 4, as spot demand is more profitable. The expected profit increases.
- As the spot demand doubles compared to the base case, we reject all allotment requests and accept only from the spot demand which is more profitable. The expected profit increases to $11,192.2.
- As bid 1 demand increases (Case 5), we accept very little of types 3 and 4 spot requests, in order to free up capacity for bid 1 request.
- If we increase the demand for bid 2 (Case 6), the weight demand increases to 19.4 tons and the volume demand increases to 113 m^3, both exceeding the weight and volume capacities. This is the case since bid 2 consists of types 3 and 4, which have bulky items. Thus, we reject bid 2 to free up capacity sold at a higher rate on the spot market. So we accept more of type 4 spot demand. The expected profit increases.
- As the spot rate doubles (Case 7), we free up capacity by rejecting both bids 1 and 2, and we accept more spot. Because of available capacity, more of type 4 is accepted, and the expected profit increases due to a higher spot price.

The results above validate our model and demonstrate the dependency of the optimal profit and optimal decisions (the allotment purchases and booking level of the spot requests) on the spot types and the types of the bid packages. Our results illustrate the effect of the volume and weight offload costs, spot demand, and allotment demand, on the optimal decisions and expected profit.

4.1.2. Results for industry-size problems

We now solve (P) in (3)–(5) for a problem of realistic size, with weight and volume capacities $k_v = 250\,\text{m}^3$ and $k_w = 40$ tons. We can convert the volume capacity into dimensional weight by dividing the volume by the standard density. The volume capacity level of 250 m^3 corresponds to dimensional weight of 41.67 tons. We consider 10 bids and 24 cargo types, with the weight and volume of each cargo type adopted from Amaruchkul et al. (2007) and presented in Table 2. The weights range from 50 to 3500 kilograms. Types 1–20 have densities that vary from 160 kg/m^3 to 200 kg/m^3 and represent normal cargo (e.g apparel, computer equipment, electronics, plastic products, sports equipment and drugs and medicine, (van de Reyd and Wouters, 2005)). Extreme types are 21–24, with densities as low as 30 kg/m^3 and as high as 400 kg/m^3 (e.g. metal products and machine parts (van de Reyd and Wouters, 2005)). The allotment demand is presented in Table 3, which gives the number of packages of each type arriving in each bid, $\lambda_k^i, k = 1, \cdots, 24$. Following Hellermann (2006), the spot demand is assumed to be two thirds of the total demand. The revenue and offload cost computations follow Section 3. The allotment revenue rate is $0.8/ton and the expected allotment show up rate is 80%. The revenue rate for the spot demand is the same as in the numerical tests of Amaruchkul et al. (2007), that is, a function of the chargeable weight, as follows: Letting $\hat{w}$ denote the chargeable weight of a particular package, the spot revenue multipliers are 1.12 for $0 \leqslant \hat{w} \leqslant 90$; 1.11 for $90 < \hat{w} \leqslant 990$; 1.09 for $990 < \hat{w} \leqslant 1990$; 1.08 for $\hat{w} > 1990$ ($\hat{w}$ in kg). The offload cost parameters are chosen so that $(h_v/\eta_v, h_w/\eta_w) = (1.25, 1.25)$, as in SubSection 4.1.1, which implies that the weight and volume offload cost are $h_w = \$1.5$/unit weight offloaded and $h_v = \$2.2$/unit volume offloaded.

Table 4 presents the optimal allotment decisions for each of the 10 bids. Note that bids 6, 7, and 8 consist only of bulky packages, and bids 9 and 10 consist of packages with extreme densities. Thus, we first solve (P) for the base case, then we vary the demand to capacity ratio in Cases 1–6, the allotment rate in Case 7 and the spot revenue rates in Case 8. The base

Table 4
Optimal allotment decisions when varying the weight and volume demand to capacity ratios, allotment and spot rates.

Cases	$(d_w/k_w, d_v/k_v)$	1	2	3	4	5	6	7	8	9	10
1	(1,1)	1	1	1	1	1	1	1	0	1	1
2	(1.1,1)	1	1	1	1	1	1	0	0	1	1
3	(1.2,1)	1	1	1	1	1	0	0	0	1	1
4	(1,1.1)	1	1	1	1	1	1	1	0	1	1
5	(1,1.2)	1	1	1	1	1	1	0	0	1	1
6	(0.9,1)	1	1	1	1	1	1	1	1	1	1
7	$r_k^c = 0.4$	1	1	1	1	1	1	0	0	1	1
8	$r_k^s < r_k^c = 0.8$	1	1	1	1	1	1	1	1	1	1

case corresponds to Case 4, as it has demand to capacity ratio of $(d_w/k_w, d_v/k_v) = (1.0, 1.1)$, where $d_w = 40.9$ tons is the weight of the total showing up demand, and $d_v = 277.5\ \text{m}^3$ is for the showing up volume.

From Table 4, the optimal solution for Case 1 ($k_w = 41$ tons and $k_v = 280\ \text{m}^3$ with demand to capacity ratio of (1,1)) is to accept all bids except bid 8 which contains the bulkiest package in terms of both weight and volume. Next, we compare the optimal decisions in the remaining cases to Case 1 with the ratio of $(1, 1)$. In Case 2, the weight capacity decreases by 10%, so the ratio becomes 1.1, (i.e. $k_w = 37$ tons) compared to Case 1, while the volume capacity is the same, so the decision is to reject bids 7 and 8, which have bulky packages with respective weight and volume equal to 25 tons and 14.88 m^3 for bid 7, and 35 tons and 20.83 m^3 for bid 8. In Case 3, we decrease the weight capacity even further, (by 20% compared to Case 1), which leads to rejecting more of the bids with bulky packages, i.e. bids 6, 7 and 8. Similarly, in Case 4, we keep the ratio d_w/k_w at 1, and we decrease the volume capacity by 10% compared to Case 1, so $k_v = 250\ \text{m}^3$. This leads to rejecting bid 8, with volume demand equal to 14.88 m^3. Reducing the volume capacity further, (Case 5), $k_v = 230\ \text{m}^3$, the optimal decision becomes to reject both bids 7 and 8. In Case 6, we increase the weight capacity by 10 %, the optimal decision is to accept all bid requests. In Case 7, we reduce the allotment rate by half, $r_k^c = 0.4$, the optimal decision becomes to reject bids 7 and 8 with the bulky packages, so we free-up capacity for higher paying spot requests. In Case 8, we decrease the spot revenue beyond the allotment rate, i.e. $r_k^s < r_k^c = 0.8$, as follows: $r_k^s = 0.55$ for $0 \leqslant \hat{w} \leqslant 90; 0.54$ for $90 < \hat{w} \leqslant 990; 0.52$ for $990 < \hat{w} \leqslant 1990; 0.51$ for $\hat{w} > 1990$ ($\hat{w}$ in kg). Then, all bids are accepted as they generate higher revenue than selling the capacity at the spot rate.

The above results indicate that the optimal allotment and spot booking decisions depend on the types of allotment and spot packages, and illustrate the effect of changing the weight and volume capacities (using the demand to capacity ratio), the spot revenue, and allotment rate.

4.2. Capacity control for spot requests

In this section, we formulate a DP model for the spot requests. The booking horizon is divided into small decision periods such that no more than one request of either type arrives in each period. That is, in each time period, we can have either a spot arrival, or an allotment arrival, or neither. The aircraft type is known, i.e. the total weight and volume capacities are known in advance. Allotment decisions are known at the beginning of the booking horizon by solving the static problem (P), in (3)–(5), where arrivals occur randomly over the booking horizon. Allotment arrivals have to be accommodated. Due to the possibility that a package from bid i may not show up, we allow overbooking. When one of the capacities is exceeded, only spot packages can be offloaded. Our goal is to obtain an optimal control policy for the spot arrival process, based on the total weight and volume of the allotment and spot show-ups.

As in Section 4.1, there are m different types of cargo booking requests, for spot and allotment types, with type k having weight w_k and volume v_k. We assume that, at time t, the probabilities of a spot arrival of type k and an allotment arrival of type k from bid i, denoted respectively by p_{kt}^s and p_{kt}^i, are known and time-dependent. The probability of no-arrival in a period, p_{0t}, is also assumed to be known and time-dependent. As we illustrate in Section 6, these probabilities are derived using the static demands of the capacity allocation problem of SubSection 4.1, modeled by Poisson random variables. Note that with the assumption that there is at most one arrival per period, we have

$$\sum_{i=1}^{n}\sum_{k=1}^{m} p_{kt}^i + \sum_{k=1}^{m} p_{kt}^s + p_{0t} = 1.$$

We formulate a finite-horizon Markov decision process with the state variable being the total weight and volume of the cargo conveyed for shipment, denoted by (x_w, x_v). Because the spot bookings arrive over a short horizon before the departure, we assume that all the accepted spot requests will materialize. Accepted allotment bookings arrive dynamically over the booking horizon, and have to be accommodated whenever they arrive. However, a portion of the allotment bookings might not materialize. We model the allotment show-ups by using a show-up rate ξ^i for bid i, which is a random variable indicating the proportion of bid i bookings that will show-up over the booking horizon. The total weight conveyed for shipment x_w is the summation of the weight from allotment and spot show-ups. Similarly, x_v is the total showing up volume, and is equal to the volume of the allotment and spot show-ups. Due to the allotment no-shows, we allow overbooking the remaining capacity reserved to spot requests. At each stage, the booking agent decides whether or not to accept a spot request, based on the type of spot arrival and the current state of the DP, (x_w, x_v). If the booking agent accepts a request of type k, the air carrier earns r_k^s, and whenever an allotment booking of type k from accepted bid i shows up, the air carrier earns r_k^c. Our objective is to maximize the expected total profit over the booking horizon, starting from the state $(x_w, x_v) = (0, 0)$, at the beginning of the booking horizon. We formulate the DP such that we do not reserve actual capacity for allotments. Instead, we start the booking horizon with the entire empty capacity, and whenever there is an allotment show-up, it is accommodated, which might lead to offloading some of the spot show-ups at the departure. Since we are filling up the capacity from the allotment show-ups, our DP model does not account for cancellations of the allotment bookings.

Denote by $V_t(x_w, x_v)$ the maximum expected controllable profit of operating the system over periods t to 0. The optimal value functions, $V_t(.)$, are determined recursively as we explain next. Let $A(\mathbf{x})$ be the set of accepted bids, which is known at time T, with $\mathbf{x}$ being the set of optimal allotment decisions obtained from solving problem (P) in SubSection 4.1, i.e. $A(\mathbf{x}) = \{i \in A \mid x_i = 1\}$. Then, the $V_t(.)$ value functions are evaluated as follows.

$$\begin{aligned} V_t(x_w, x_v) = & \sum_{i \in A(\mathbf{x})} \mathrm{E}(\xi^i) \sum_{k=1}^{m} p_{kt}^i \{r_k^c + V_{t-1}(x_w + w_k, x_v + v_k)\} + \sum_{k=1}^{m} p_{kt}^s \max\{r_k^s + V_{t-1}(x_w + w_k, x_v + v_k), V_{t-1}(x_w, x_v)\} \\ & + p_{0t} V_{t-1}(x_w, x_v), \\ V_0(x_w, x_v) = & -h_v(x_v - k_v)^+ - h_w(x_w - k_w)^+. \end{aligned} \tag{6}$$

At departure, the objective is to minimize the offload cost due to overbooking the weight and/or volume capacities. Moreover, no arrivals are expected at time 0, thus we have $p_{00} = 1$.

Whenever an allotment booking of type k arrives at time t from accepted bid i when the system state is (x_w, x_v), it generates an additional revenue r_k^c and the state becomes $(x_w + w_k, x_v + v_k)$. An accepted bid i will show up with an expected show-up rate $\mathrm{E}(\xi^i)$, which is multiplied by the expected revenue obtained from a package arrival of an accepted bid, to give the expected revenue from an arrival in a showing up bid. If a spot booking request of type k is accepted in period t when the system state is (x_w, x_v), the maximum profit is $r_k^s + V_{t-1}(x_w + w_k, x_v + v_k)$. If the spot request is rejected, the maximum profit is $V_{t-1}(x_w, x_v)$. Thus, a spot request is accepted in period t when the system state is (x_w, x_v) if and only if

$$r_k^s > V_{t-1}(x_w, x_v) - V_{t-1}(x_w + w_k, x_v + v_k). \tag{7}$$

The right hand side of (7) can be interpreted as the opportunity cost of a spot booking request of type k, given that the reservation level is (x_w, x_v). We formally define this opportunity cost as

$$u_t^k(x_w, x_v) \equiv V_{t-1}(x_w, x_v) - V_{t-1}(x_w + w_k, x_v + v_k), \quad 1 \leqslant t \leqslant T. \tag{8}$$

Then, a spot booking request of type k is accepted in period t when the reservation state is (x_w, x_v) if and only if $r_k^s > u_t^k(x_w, x_v)$. Once the optimal value functions, $V_t(.)$, are determined, the booking agent decides which spot requests to accept. However, when the number of remaining time periods is large, and the number of accessible states is large, storing and retrieving the $V_t(.)$'s is not possible in practice.

When at least one of the weight or volume capacities is exceeded, the value function becomes:

$$\begin{aligned} V_t(x_w, x_v) = & \sum_{i \in A(\mathbf{x})} \mathrm{E}(\xi^i) \sum_{k=1}^{m} p_{kt}^i \{r_k^c + V_{t-1}(x_w + w_k, x_v + v_k)\} + \left(1 - \sum_{i \in A(\mathbf{x})} \sum_{k=1}^{m} p_{kt}^i\right) V_{t-1}(x_w, x_v) - h_v(x_v - k_v)^+ \\ & - h_w(x_w - k_w)^+, \end{aligned} \tag{9}$$

$$\text{for } x_v > k_v \text{ or } x_w > k_w, \text{ and } t = 1, \cdots, T. \tag{10}$$

Condition (10) indicates that at any time t, whenever one of the capacity limits is reached, we pay a penalty for offloading the excess spot bookings, but we still accommodate any arrival of the allotment type.

Similarly to Amaruchkul et al. (2007), the value function of the DP in (6) is based on the total volume and weight accepted. However, unlike prior literature, our value function accounts not only for the spot arrivals, but also for the arrival of allotment bookings. Our DP tracks both the allotment and spot arrivals and their contribution to the total weight and volume accepted. Moreover, the time-dependent arrival probabilities in our DP are based on the spot and allotment static demand models of the static capacity allocation problem, as detailed in Section 6.1.

The non-monotone behavior of a booking-limit policy of a multi-dimensional capacity has been discussed in the literature (see, for example, Lee and Hersh, 1993; Maddah et al., 2010; Xiao and Yang, 2010). Specifically, Xiao and Yang (2010) show that if two products consume the same amount of capacity in one dimension and different amount in the other, and if revenue rate is concave in the capacity usage, the optimal control follows a threshold policy which differs from the nested-fare policy of the single-dimension single-resource problems.

5. Heuristics and upper bound

Optimal policies are too complex to store and compute for large problems. In this section, we describe heuristics which are based on reducing the two-dimensional state space of the original Markov Decision Process (MDP) in (6) into one- dimensional space, thus reducing the storage and computational requirements. This method has been used in the literature in different problem settings (Amaruchkul et al., 2007; Maddah et al., 2010; Hoffmann, 2013). We also derive an upper bound on the value function which is useful in assessing the heuristics performance as well as quantifying an upper limit on the profitability of cargo RM. In what follows, we describe the heuristics then we derive an upper bound on the value function. The performance of these heuristics is tested in Section 6.

5.1. Decouple heuristic (DC)

The idea behind this heuristic is to reduce the two-dimensional MDP by decoupling the original problem into two subproblems, a weight subproblem and a volume subproblem, each being one-dimensional. We allocate the allotment and spot revenues from type k among the two subproblems as follows. The revenues from a request of type k allocated to the volume and weight subproblems are respectively $f_k^{c,v}$ and $f_k^{c,w}$ for the allotment arrivals, and $f_k^{s,v}$ and $f_k^{s,w}$ for the spot requests, such that

$$r_k^c = f_k^{c,v} + f_k^{c,w}, \text{ with } f_k^{c,w} = \gamma^c w_k \text{ and } f_k^{c,v} = (\gamma^c v_k/d_s - \gamma^c w_k)1_{v_k > w_k d_s},$$
$$r_k^s = f_k^{s,v} + f_k^{s,w}, \text{ with } f_k^{s,w} = \gamma^s w_k \text{ and } f_k^{s,v} = (\gamma^s v_k/d_s - \gamma^s w_k)1_{v_k > w_k d_s},$$

where γ^c and γ^s are respectively the allotment and spot revenue per unit of chargeable weight, and $1_{v_k > w_k d_s}$ is an indicator function that takes the value 1 if $v_k > w_k d_s$, and 0 otherwise. In the above revenue allocations, if the chargeable weight of the request of either type, v_k/d_s, is greater than its weight, then the revenue is based on the cargo volume only, otherwise it is based on its weight.

The volume subproblem is a one-dimensional problem, with the state being the total volume already reserved from allotment arrivals and accepted spot requests. Then, the value function of the volume subproblem is defined as

$$V_t^v(x_v) = \sum_{i \in A(\mathbf{x})} \mathrm{E}(\xi^i) \sum_{k=1}^{m} p_{kt}^i \{f_k^{c,v} + V_{t-1}^v(x_v + v_k)\} + \sum_{k=1}^{m} p_{kt}^s \max\{f_k^{s,v} + V_{t-1}^v(x_v + v_k), V_{t-1}^v(x_v)\}$$
$$+ \left(1 - \sum_{i \in A(\mathbf{x})} \sum_{k=1}^{m} p_{kt}^i - \sum_{k=1}^{m} p_{kt}^s\right) V_{t-1}^v(x_v),$$
$$V_0^v(x_v) = -h_v(x_v - k_v)^+.$$

When the volume capacity is reached, the value function $V_t^v(x_v)$ becomes:

$$V_t^v(x_v) = \sum_{i \in A(\mathbf{x})} \mathrm{E}(\xi^i) \sum_{k=1}^{m} p_{kt}^i \{f_k^{c,v} + V_{t-1}^v(x_v + v_k)\} + \left(1 - \sum_{i \in A(\mathbf{x})} \sum_{k=1}^{m} p_{kt}^i\right) V_{t-1}^v(x_v) - h_v(x_v - k_v)^+, \text{for } x_v > k_v, \text{ and } t = 1, \cdots, T.$$

Similarly, the value function of the weight subproblem is written as

$$V_t^w(x_w) = \sum_{i \in A(\mathbf{x})} \mathrm{E}(\xi^i) \sum_{k=1}^{m} p_{kt}^i \{f_k^{c,w} + V_{t-1}^w(x_w + w_k)\} + \sum_{k=1}^{m} p_{kt}^s \max\{f_k^{s,w} + V_{t-1}^w(x_w + w_k), V_{t-1}^w(x_w)\}$$
$$+ \left(1 - \sum_{i \in A(\mathbf{x})} \sum_{k=1}^{m} p_{kt}^i - \sum_{k=1}^{m} p_{kt}^s\right) V_{t-1}^w(x_w),$$
$$V_0^w(x_w) = -h_w(x_w - k_w)^+.$$

When the weight capacity is reached, we get

$$V_t^w(x_w) = \sum_{i \in A(\mathbf{x})} \mathrm{E}(\xi^i) \sum_{k=1}^{m} p_{kt}^i \{f_k^{c,w} + V_{t-1}^w(x_w + v_k)\} + \left(1 - \sum_{i \in A(\mathbf{x})} \sum_{k=1}^{m} p_{kt}^i\right) V_{t-1}^w(x_w) - h_w(x_w - k_w)^+, \text{ for } x_w > k_w, \text{ and } t = 1, \cdots, T.$$

Define the opportunity cost for the volume and weight subproblems as

$$u_t^{k,v}(x_v) \equiv V_{t-1}^v(x_v) - V_{t-1}^v(x_v + v_k), \quad 1 \leqslant t \leqslant T,$$
$$u_t^{k,w}(x_w) \equiv V_{t-1}^w(x_w) - V_{t-1}^w(x_w + w_k), \quad 1 \leqslant t \leqslant T. \tag{11}$$

Then the decision rule under the decouple heuristic becomes: In period t, accept a spot request of type k if and only if $r_k^s > u_t^{k,v}(x_v) + u_t^{k,w}(x_w)$.

5.2. Infinite weight/volume heuristic (W^∞/V^∞)

We also consider two other heuristics, the first corresponds to assuming that the weight capacity is infinite, while the second corresponds to assuming infinite volume capacity. For the problem with infinite weight, we define the value function as follows:

$$V_t^{v,\infty}(x_v) = \sum_{i\in A_(\mathbf{x})} \mathrm{E}(\xi^i)\sum_{k=1}^m p_{kt}^i\{r_k^c + V_{t-1}^{v,\infty}(x_v+v_k)\} + \sum_{k=1}^m p_{kt}^s \max\{r_k^s + V_{t-1}^{v,\infty}(x_v+v_k), V_{t-1}^{v,\infty}(x_v)\} + p_{0t}V_{t-1}^{v,\infty}(x_v).$$

$$V_0^{v,\infty}(x_v) = -h_v(x_v-k_v)^+.$$

When the volume capacity is reached, the value function becomes:

$$V_t^{v,\infty}(x_v) = \sum_{i\in A_i} \mathrm{E}(\xi^i)\sum_{k=1}^m p_{kt}^i\{r_k^c + V_{t-1}^{v,\infty}(x_v+v_k)\} + \left(1-\sum_{i\in A_i}\sum_{k=1}^m p_{kt}^i\right)V_{t-1}^{v,\infty}(x_v) - h_v(x_v-k_v)^+, \text{ for } x_v > k_v,\ t=1,\cdots,T.$$

Define the opportunity cost $u_t^{k,v,\infty}(x_v) \equiv V_{t-1}^{v,\infty}(x_v) - V_{t-1}^{v,\infty}(x_v+v_k),\ 1\leqslant t\leqslant T$, then the decision rule under the Infinite Volume heuristic becomes: In period t, accept a spot request of type k if and only if $r_k^s > u_t^{k,v,\infty}(x_v)$. The Infinite Weight heuristic is defined similarly, with the following decision rule: Accept a spot request of type k if and only if $r_k^s > u_t^{k,w,\infty}(x_w)$, with $u_t^{k,w,\infty}(x_w) \equiv V_{t-1}^{w,\infty}(x_w) - V_{t-1}^{w,\infty}(x_w+w_k),\ 1\leqslant t\leqslant T.$

5.3. Upper bound (UB)

The value function of the MDP in (6) is bounded above by the sum of the value functions of the weight and volume subproblems of the decouple heuristic (DC). This is shown in the following theorem.

Theorem 1. Let x_w, x_v be the total weight and volume of the showing up allotment and spot bookings. Then, $V_t(x_w,x_v) \leqslant V_t^v(x_v) + V_t^w(x_w)$, for $t = 0,1,\cdots,T$.

Proof. See Appendix B. □

As a result of Theorem 1, the total expected profit has the following upper bound

$$\overline{V} = V_T^v(0) + V_T^w(0).$$

This upper bound will help us evaluate the performance of the heuristics in Section 6 for large-size problems by comparing their estimated expected profit obtained from simulation to the upper bound. Note that this upper bound requires solving two subproblems, each being one-dimensional.

6. Numerical results

In this section, we present numerical results to test the performance of the proposed heuristics. Only the medium-size problem could be solved optimally due to storage and computational requirements. To use (6), one should store all the values of $V_t(x_w,x_v)$ and all decisions specified by this function, which specify what spot types are acceptable. This requires too much computer memory when the number of decision periods is large and the number of types is large. Thus, SubSection 6.1 presents numerical results for the optimal solution, which we compare to the proposed heuristics and upper bound. We also compare the performance of the optimal solution and heuristics to the first-come first-served policy (FCFS). The large-size problems are solved only by the proposed heuristics and are analyzed in SubSection 6.2. Finally, we compare our model to Levin et al. (2012) in SubSection 6.3 to emphasize the contribution of the paper.

All simulations (each simulation has 1000 runs) in the numerical experiments for medium and large problems are implemented in Matlab version 2013, on an Intel (R) Core (TM) i5–2540 M CPU2.60 GHz. For the medium-size problem, the computational times were as follows: 3.5 min for the optimal DP solution, 1.09 min for FCFS, 1.44 min for the decouple heuristic, 1.36 and 1.38 min for the infinite volume and weight respectively. For the large problem, the computational times become 2.75, 7.43, 5.34, and 4.54 min for the FCFS, decouple, infinite volume and infinite weight respectively.

6.1. Results for medium-size problems

In this subsection, we consider an aircraft with respective weight and volume capacities $k_w = 15$ tons and $k_v = 135$ m^3. To make the scale of capacity dimensions easier to compare, we can convert the volume capacity value into dimensional weight by dividing the volume capacity by the standard density. This implies a volume capacity of 22.5 tons of dimensional weight. The distribution of weight and cargo is the same as in the numerical tests of Amaruchkul et al. (2007), and is presented in Table 2, resulting in 24 cargo types. In our tests, we consider 10 bids, with the number of arrivals of each cargo type for each bid, λ_k^i, given in Table 3. Following Amaruchkul et al. (2007), we assume that a shipment belongs to type k with probability 0.072 for $k = 1, \cdots, 10$; 0.04 for $k = 11, \cdots, 16$; 0.009 for $k = 17, \cdots, 20$; and 0.001 for $k = 21, \cdots, 24$. To find the probability of an arrival in time t, we divide the booking horizon T into six time intervals, and we use the request probabilities in Amaruchkul et al. (2007) (Table 3, p. 465). Thus, the probability of an arrival in time t is the summation of the probabilities over all classes of the corresponding time interval, with a class-l shipment defined in Amaruchkul et al. (2007) as generating revenue according to a function that maps the class and the shipment chargeable weight to revenue. As a result, we compute the probability that there is an arrival of type k at time t, p_{kt}, as the product of the probability of an arrival in time t and the probability that this arrival is of type k. Next, we decompose this probability into probability of an allotment arrival, p^i_{kt}, and spot arrival, p^s_{kt}, as follows: We assume that one third of the total demand is of the allotment type, and two third is of the spot type. Furthermore, the probability that an allotment arrival of type k belongs to bid i is $\lambda^i_k/\sum_i \lambda^i_k$, where λ^i_k is the average number of arrivals of type k from bid i, and is given in Table 3. As a result, the arrival probabilities can be found as follows: $p^i_{kt} = (1/3)\left(\lambda^i_k/\sum_i \lambda^i_k\right) p_{kt}$ and $p^s_{kt} = (2/3)\, p_{kt}$. We consider a booking period length $T = 300$ for the results in Table 5. The show-up rate for a bid is assumed to follow the beta distribution (following Moussawi-Haidar and Çakanyildirim, 2012), with the parameters chosen so that we have an 80% show-ups on average. The base case corresponds to demand to capacity ratio $(d_w/k_w, d_v/k_v) = (1.3, 1.4)$, and the following spot revenue multipliers: 1.12 for $0 \leqslant \hat{w} \leqslant 90$; 1.11 for $90 < \hat{w} \leqslant 990$; 1.09 for $990 < \hat{w} \leqslant 1990$; 1.08 for $\hat{w} > 1990$. The allotment revenue rate is \$0.8/ton. The offload cost parameters are such that the offload cost to spot revenue ratio is $(h_v/r^{s,v}_k, h_w/r^{s,w}_k) = (1.67, 1.67)$, where $r^{s,v}_k$ and $r^{s,w}_k$ are the spot revenue of type k per unit volume and weight, i.e. $r^{s,v}_k = \left(\sum_{k=1}^m r^s_k \lambda^s_k\right)/\left(\sum_{k=1}^m v_k \lambda^s_k\right)$. $r^{s,w}_k$ is defined similarly. As a result, the weight and volume offload cost are $h_w = \$2.35$/unit weight and $h_v = \$2.47$/unit volume.

Table 5 presents a summary of the results. It gives the expected profit for each of the heuristics and the upper bound $\Pi^J, J \in \{UB, FCFS, DC, V^\infty, W^\infty\}$, as a percentage of the optimal expected profit. The first row in Table 5 gives the results for our base case. The results for the other cases are based on changing the parameters from the base case. We consider changes in the capacities of the volume and weight to test the effect of the demand to capacity ratio (since demand given by the arrival probabilities and Table 3 is unchanged). This is shown in Cases 1 through 10. We also change the weight and volume offload costs (h_w, h_v) in Cases 11–14. In Case 15–16, we vary the allotment rate. The last row in Table 5 shows the average performance of all the 16 cases. Comparing the base case in Table 5 with that obtained in Table 1 for the static problem, we observe the following: The total expected profit for the base case in Table 1, is \$9626. For the base case in Table 5, it is \$14,323, which indicates the opportunity of increasing profits when dynamically managing the spot arrivals.

Table 5 indicates the following insights: First, heuristic DC performs the best among all heuristics, with an average optimality gap of 2.91%. On average, the expected profit of the DC heuristic is 4.84% better than the trivial FCFS benchmark. The DC heuristic does not perform well in Case 9 with demand to capacity ratio equal to (1.5, 1), i.e. when the weight capacity is tight, and also in Case 15, when the allotment rate is very low, half of its value in the base case. In the other 14 cases, the DC heuristic is very close to the optimal, and is in the range of [96.27–99.9%] of the optimal. The infinite volume heuristic is 4.86% less than the optimal, and 1.95% less than the decouple heuristic. The infinite weight heuristic is very close to the FCFS, and is the worst performing heuristic. The profit from the upper bound is very close to the optimal except in Case 9 in which it is 7.2% higher, and Case 15 in which it is 17.9% higher. On average, the upper bound we derive is very close to the optimal with an optimality gap of 2.52%. It is noteworthy that the DC heuristic takes the longest to run. Given that general purpose solver was used, one can overcome this limitation by using better software. Even large problems could be solved to optimality, which can be studied as an extension to the current work.

Table 5
Expected profit as a percentage of the optimal profit. Base case $(k_w, k_v) = (15 \text{ tons}, 135 \text{ m}^3), T = 300$.

Case	$(d_w/k_w, d_v/k_v)$	Π^{UB}	Π^{FCFS}	Π^{DC}	V^∞	W^∞
1	(1.3,1.4)	101.16	90.03	98.82	93.97	90.02
2	(1,1)	100.88	97.34	98.91	98.15	97.34
3	(1.1,1)	101.63	96.03	97.72	97.05	96.03
4	(1,1.1)	100.63	98.16	99.37	98.56	98.16
5	(1.1,1.1)	100.82	98.01	99.11	98.46	98.01
6	(0.9,1)	100.23	99.10	99.76	99.30	99.10
7	(1,0.9)	101.29	96.96	98.07	97.76	96.96
8	(0.9,0.9)	100.81	97.96	99.07	98.49	97.96
9	(1.5,1)	107.20	75.91	85.20	84.51	75.91
10	(0.5,1)	102.60	96.27	96.27	96.07	96.27
11	(h_w, h_v)=(2.35, 1.24)	101.17	90.63	98.65	94.43	90.63
12	(h_w, h_v)=(2.35, 4.95)	101.17	88.66	98.83	92.89	88.73
13	(h_w, h_v)=(1.17, 2.47)	100.45	96.74	99.54	97.70	96.74
14	(h_w, h_v)=(4.7, 2.47)	101.81	77.69	99.9	92.91	77.69
15	r^c_k=0.4	117.93	83.82	84.79	83.87	83.82
16	r^c_k=1.2	100.53	92.71	99.42	98.12	92.71
	Mean	102.52	92.25	97.09	95.14	92.26

Table 6
Number of packages of each type arriving from an allotment booking used in the large-size problem; data below is such that $(d_w/k_w, d_v/k_v) = (1.2, 1.3)$.

Bid i	λ_1^c	λ_2^c	λ_3^c	λ_4^c	λ_5^c	λ_6^c	λ_7^c	λ_8^c	λ_9^c	λ_{10}^c	λ_{11}^c	λ_{12}^c	λ_{13}^c	λ_{14}^c	λ_{15}^c	λ_{16}^c	λ_{17}^c	λ_{18}^c	λ_{19}^c	λ_{20}^c	λ_{21}^c	λ_{22}^c	λ_{23}^c	λ_{24}^c
1	10	10	10	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	5	5	5	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	5	5	5	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	5	5	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	3	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	7	7	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	5
10	5	5	5	5	2	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 7
Expected profit as a percentage of the upper bound. Base case $(\mathbf{k_w}, \mathbf{k_v}) = (40 \text{ tons}, 250 \text{ m}^3), T = 700$.

Case	$(d_w/k_w, d_v/k_v)$	Π^{FCFS}	Π^{DC}	V^∞	W^∞
1	(1.06, 1.2)	74.58	99.15	99.13	74.58
2	(1,1)	71.08	98.67	98.17	71.08
3	(1.1,1)	68.83	98.66	98.46	68.83
4	(1, 1.1)	80.79	97.48	97.26	80.79
5	(1.1,1.1)	69.99	99.05	98.89	69.99
6	(0.9, 1)	81.96	98.87	98.86	81.96
7	(1, 0.9)	71.08	98.67	98.65	71.08
8	(0.9, 0.9)	67.48	96.76	96.67	67.48
9	(0.8,1.5)	51.57	51.62	51.59	51.57
10	(1.5, 0.5)	84.87	99.55	99.35	84.87
11	(h_w, h_v)=(1.94, 1.37)	85.92	99.13	99.10	85.92
12	(h_w, h_v)=(1.94, 5.48)	74.54	99.15	99.03	74.54
13	(h_w, h_v)=(0.97, 2.74)	92.04	99.72	99.62	92.04
14	(h_w, h_v)=(3.88, 2.74)	39.37	98.70	98.41	39.16
15	r_k^c=0.4	57.80	61.36	57.91	37.77
16	r_k^c=1.2	81.48	99.59	99.20	81.48
	Mean	72.09	93.51	93.14	70.82

6.2. Results for large-size problems

In this Section, we solve a large problem with our heuristics with the following weight and volume capacities: $k_w = 40$ tons and $k_v = 250$ m^3, as considered in §4.1.2, with $T = 700$. We increase the number of arrivals in each bid, λ_k^c as in Table 6 below. We obtain total weight demand of 42.5 tons and total volume demand of 301.01 m^3. The arrival probabilities and parameters used are the same as in §6.1. Since we can not find the optimal solution of the DP for the large problem, we compare the expected profit for each heuristic to the upper bound. The results are presented in Table 7. The leading heuristic is DC, achieving an improvement of approximately 22.7% over the worst performing heuristic, W^∞. It has a gap of 6.5% from the upper bound. The decouple heuristic does not perform well in Case 9, when the weight capacity is too high and the volume capacity too low, with $k_w = 54$ tons and $k_v = 20$ m^3, and in Case 15 when the allotment rate is too low. For the other cases, the DC heuristic is in the range [96.7–99.7%] of the upper bound. The infinite volume heuristic also performs well and ranks second after the DC heuristic.

Clearly, our solution depends on the types of all allotment and spot arrivals. In summary, we solve the dynamic spot capacity control model for medium size problems and compare the optimal solution to the other heuristics and the upper bound, to conclude that the DC heuristic is the closest to the optimal solution. For large problems, in which the optimal solution is difficult to obtain, the DC heuristic can be used as it is a well-performing heuristic. Moreover, we study the effect of changing the capacities (by changing the demand to capacity ratios), the offload costs, and the allotment rate. Our results indicate in all the cases that the DC heuristic is leading. Thus, when the problem is medium size, one can solve for the optimal solution. When the problem at hand is large, the decouple heuristics can be applied. The performance of the DC heuristic can be further improved with the use of better software with more powerful computational capability.

6.3. Comparison with Levin et al. (2012)

In this section, we numerically compare our model to Levin et al. (2012). In Section 7.2 of their paper, Levin et al. (2012) simulate the performance of their allotment selection method and spot market booking control, compared to the theoretical upper bound for the optimal expected profit. They consider 16 experimental settings and present their results in Table 1 in their paper (P. 362), which shows the performance in each setting. Their findings indicate that the simulated profit is in the range $[87.1\%, 94.8\%]$ of the upper bound.

We consider the same experimental setup as in Levin et al. (2012), which specifies the distributions used in the random sampling of data, the volume and weight distributions of allotments and spot market cargo, and the revenue and cost models. First, we describe the distributions of the weight, volume, revenue and cost of the spot cargo types. The weight of each spot type is assumed to be constant in each experiment. The value of this constant is sampled from an exponential distribution with mean 0.25 truncated to the interval $[0.05, 3.5]$ (in tons). The reference density is sampled from the normal distribution with mean 0.167 and standard deviation 0.04 truncated to the interval $[0.03, 3.4]$ (in ton/m^3). The volume of a type k follows the lognormal distribution with mean equal to the weight of type k divided by the sampled reference density for type k, and with coefficient of variation θ^s. We use a shipping rate ρ_k for spot, sampled from the interval $[1500, 3000]$ in \$/ton, and an offload cost for type k proportional to the weight and volume of that type. The revenue computation is based on the chargeable weight, with the revenue multiplier ρ_k, similar to the one we use in our model. The offload cost for weight is $1.5\rho_k$ and for volume $0.15\rho_k$.

Next, we consider the numerical setting of the allotments. The maximum allotted weight for bid i is sampled from the normal distribution with mean 5 and standard deviation 2 tons. The utilized fraction of the allotted bid i is sampled from the normal distribution with mean 0.9 and standard deviation $1/9.7$ truncated to the interval $[0, 1]$, and the utilized volume is lognormal with mean equal to the weight divided by the standard density and coefficient of variation θ^c. The allotment revenues are also based on the chargeable weight pricing structure we adopt in our model.

As in Levin et al. (2012), we consider four combinations of volume and weight capacities, $k_v = \{75, 150\}$ m^3 and $k_w = \{10, 20\}$ tons, two levels of the coefficient of variation of the spot demand, $\theta^s = \{0.2, 0.8\}$, and two levels of the coefficient of variation of the allotments $\theta^c = \{0.2, 0.4\}$. This results in a total of 16 experimental scenarios. When $k_v = 75$ m^3, the problem is of medium size, so we obtain the average simulated profits by optimally solving the DP in (6), and report the results in the first two rows of Table 8. When $k_v = 150$ m^3, the problem becomes large, and cannot be solved optimally, so we obtain the average simulated profits by using our best-performing heuristic, the decouple heuristic, and summarize the result in the last two rows of Table 8. We compare these results to those obtained in Levin et al. (2012), and which are shown in Table 9.

Comparing the average simulated profits as a percentage of the upper bound obtained by using our model (Table 8) to those obtained in the literature (Table 9), it is clear that the optimal solution of our DP outperforms the solution obtained in Levin et al. (2012) when the capacities are not too large, since in the eight scenarios when $k_v = 75$ (the first two rows), our ratios are better. For the other eight scenarios when $k_v = 150$ (last two rows), we use the Decouple heuristic, since solving optimally the DP is not possible due to the excessive storage requirements, as we discuss in Section 4.1.2. Comparing the results obtained using the decouple heuristic to those in Table 9, we observe that in three out of eight scenarios, our results are better. This indicates that when the capacities are not too large, using the DP solution we propose in this paper results in more profits. For large problems, Levin et al. (2012) can be applied.

The optimal solution we obtain for medium-size problems by solving the exact DP is superior to Levin et al. (2012). For large problems, Levin et al. (2012) approximate solution performs better than our heuristics. An explanation of these results is that the revenue allocations to the weight and volume subproblems, $f_k^{c,v}$ and $f_k^{c,w}$, are static inputs to the decouple heuristic that we use to solve large problems, which presents a limitation to that heuristic. Levin et al. (2012) approach of approximating the value functions in each time period is based on Lagrange multipliers that correspond to the tighter upper bound of the approximate value function and are updated dynamically. The underlying concept of using revenue allocations is very similar to the Lagrangian relaxation approach for network revenue management; there, one optimizes over these multipliers to find the best upper bound, while here, these allocations are static. Our paper can be extended to consider dynamic updates of the weight and volume revenue allocations, based on the remaining weight and volume capacities, the type, and the time

Table 8
Expected profit as a percentage of the upper bound.

	k_v	k_w	$\theta^s = 0.4$ (%)		$\theta^s = 0.8$ (%)	
			$\theta^c = 0.2$	$\theta^c = 0.4$	$\theta^c = 0.2$	$\theta^c = 0.4$
Using optimal Solution	75	10	98.9	94.3	92.6	96.5
	75	20	95.4	96.1	95.7	96.6
Using decouple Heuristic	150	10	87.6	92.5	97.6	92.8
	150	20	99.7	93.8	91.8	91.4

Table 9
Levin et al. (2012) results.

		$\theta^s = 0.4$ (%)		$\theta^s = 0.8$ (%)	
k_v	k_w	$\theta^c = 0.2$	$\theta^c = 0.4$	$\theta^c = 0.2$	$\theta^c = 0.4$
75	10	92.5	90.0	89.8	88.9
75	20	89.9	87.2	89.2	87.1
150	10	93.6	94.9	93.5	93.9
150	20	94.8	93.7	94.7	93.4

of an arrival. Thus, our model can be used by air cargo practitioners when the weight and volume capacities are of medium size, as it outperforms the one proposed in the literature. For large problems, the optimal solution proposed by Levin et al. (2012), and obtained through approximating the value function can be used.

7. Conclusions

In this paper, we consider two cargo revenue management problems that are closely inter-related, as they are used in practice. First, at the beginning of the booking horizon, the air carrier decides how much of the total cargo capacity to sell on a contracted basis. Thus, the decision is to determine which contracts from available bids should be accepted. Once this decision has been made, the second decision is to devise a capacity control policy for the spot booking requests arriving dynamically over the planning horizon until departure. In our model, we assume that whenever an allotment booking from an accepted bid shows up, it has to be accommodated on the aircraft, and only the spot bookings can be offloaded at departure. Also, the allotment bookings arrive dynamically along with the spot requests over the booking horizon. We formulate and solve two problems: In the first, a static formulation is presented to determine the optimal distribution of the cargo capacity between advanced capacity purchases and spot market sales. For this model, an integer non-linear program is formulated. We optimally solve this program for small and industry-size problems by deriving a closed-form expression of the objective function and perform sensitivity analysis by varying the demand to capacity ratio, the allotment rate, and the spot revenue rate. The sensitivity analysis illustrates the dependency among bid and spot types and their effect on the optimal decisions. Also, we illustrate the effect on the optimal profit and decisions of increasing and decreasing the volume and weight offload costs, increasing the spot demand, and increasing the demand of each bid.

For the spot booking control problem, we develop a dynamic programming model, with the spot and allotment bookings arriving dynamically over the booking horizon, and the possibility of offloading some of the spot arrivals at departure, if the allotment show ups and spot arrivals exceed the total weight or volume capacity. In a given time period, we can have an allotment arrival, or a spot arrival, or none. At each stage, the booking agent decides whether or not to accept a spot request, based on the type of spot request and the current state, which is the total weight and total volume of both the allotment bookings accepted and spot bookings that show-up. We solve this problem optimally for a wide array of medium-size problems and illustrate the dependency of the decisions on the spot cargo types chosen and the number of packages of each type in every bid accepted. Our solution preserves the two-dimensional aspect of the cargo problem. For very large problems, our model cannot be solved optimally via the classical DP recursive scheme, so we propose a well-performing heuristic that gives near optimal solution. We numerically test the proposed heuristics against the optimal solution, upper bound, and first-come first-served policy via simulation, and consider several cases by varying the demand to capacity ratio, offload cost to spot revenue ratio, and allotment rate. Our results indicate that the leading heuristic is the decouple heuristic, which is based on collapsing the two-dimensional state space into one-dimensional space. This heuristic performs the best among all heuristics for medium and large size problems. We compare our model with the literature, and show via simulation that our approach of solving the exact DP is superior to the one in the literature for medium size problems. It is noteworthy that while the decouple heuristic is not the best performing one when compared with the literature, better technology with increased computational capability to solve large problems to optimality is an area of future research.

Appendix A

We show how a closed form expression of the expected profit in (3)–(5),which is used in the numerical analysis of Sections 4.1.1 and 4.1.2, is obtained.

First, note that the allotment and spot arrivals follow the Poisson distribution (refer to Section 4), i.e. $X_k^i \sim$ Poiss (λ_k^i) and $X_k^s \sim$ Poiss (λ_k^s) respectively. Thus, for a large number of arrivals, we can use the Normal distribution as an approximation of the allotment and spot arrivals, with mean and variance being $(\mu^i, \sigma_i^2) = (\lambda_k^i, \lambda_k^i)$ and $(\mu^s, \sigma_s^2) = (\lambda_k^s, \lambda_k^s)$ respectively. In what follows, we let $X \sim N(\mu, \sigma^2)$, and denote the pdf and cdf of the standard normal function by $f_S(.)$ and $F_S(.)$ respectively. Then, we can compute a closed form expression for $\mathrm{E}(X-a)^+$ as follows:

$$\begin{aligned}
\mathrm{E}(X-a)^+ &= \int_{X=a}^{\infty}(x-a)f(x)dx = \int_{x=a}^{\infty}(x-a)\frac{1}{\sigma\sqrt{2\pi}}e^{\frac{-(x-\mu)^2}{2\sigma^2}}dx \\
&= \int_{z=(a-\mu)/\sigma}^{\infty}(z\sigma+\mu-a)\frac{1}{\sigma\sqrt{2\pi}}e^{\frac{-z^2}{2}}\sigma dz \\
&= \int_{z=(a-\mu)/\sigma}^{\infty} z\frac{\sigma}{\sqrt{2\pi}}e^{\frac{-z^2}{2}}dz + \int_{z=(a-\mu)/\sigma}^{\infty}\frac{\mu-a}{\sqrt{2\pi}}e^{\frac{-z^2}{2}}dz \\
&= \int_{w=(\frac{(a-\mu)}{\sigma})^2/2}^{\infty}\frac{\sigma}{\sqrt{2\pi}}e^{-w}dw + (\mu-a)\Big(1-F_S\Big(\frac{a-\mu}{\sigma}\Big)\Big) \\
&= \sigma f_S\Big(\frac{a-\mu}{\sigma}\Big) + (\mu-a)\Big(1-F_S\Big(\frac{a-\mu}{\sigma}\Big)\Big),
\end{aligned} \tag{12}$$

where the second and before last equalities are obtained by the change of variables $z=(x-\mu)/\sigma$ and $w=z^2/2$. Also, we can find a closed form expression for E min (a, X) as follows:

$$\begin{aligned}\mathrm{E}\ \min(a,X) &= \int_0^a xf(x)dx + \int_a^\infty af(x)dx = \int_0^a x\frac{1}{\sigma\sqrt{2\pi}}e^{\frac{-(x-\mu)^2}{2\sigma^2}}dx + a(1-F(a))\\ &= \int_{z=-\mu/\sigma}^{(a-\mu)/\sigma}\mu\frac{1}{\sqrt{2\pi}}e^{\frac{-z^2}{2}}dz + \int_{z=-\mu/\sigma}^{(a-\mu)/\sigma}\frac{z\sigma}{\sqrt{2\pi}}e^{\frac{-z^2}{2}}dz + a(1-F(a))\\ &= \mu\Big[F_S\Big(\frac{a-\mu}{\sigma}\Big) - F_S\Big(-\frac{\mu}{\sigma}\Big)\Big] - \frac{\sigma}{\sqrt{2\pi}}\Big(e^{-[(a-\mu)/\sigma]^2/2} - e^{-(\mu/\sigma)^2/2}\Big)\\ &\quad + a(1-F(a)).\end{aligned}$$

Finally, we find a closed form expression for $\mathrm{E}(X+Y-a)^+$, where $X \sim N(\mu,\sigma^2)$ and $Y = \min(b,X')$, with $X' \sim N(\mu',\sigma'^2)$. Approximate Y by its expected value, $\mathrm{E}\ \min(b,X')$, we get

$$\mathrm{E}(X+Y-a)^+ \simeq \mathrm{E}(X-(a-E(Y)))^+ = \sigma f_S\Big(\frac{(a-E(Y))-\mu}{\sigma}\Big) + (\mu-(a-E(Y)))\Big(1-F_S\Big(\frac{(a-E(Y))-\mu}{\sigma}\Big)\Big). \tag{13}$$

Using the closed form approximations obtained above, and the fact that the sum of two normal variables is a normal variable with mean and variance equal to the sum of the means and variances, a closed form expression of the expected profit in (3) is obtained for the numerical analysis in Sections 4.1.1 and 4.1.2.

Closed-form expression:

Note that $\mathrm{E}\ \min(a,X) = \mathrm{E}(X) - \mathrm{E}(X-a)^+$.

The second term in the profit expression is

$$\mathrm{E}\Big[\sum_{k=1}^m r_k^s\ \min(X_k^s,n_k^s)\Big] = \sum_{k=1}^m r_k^s\lambda_k^s - \Big[\sqrt{\lambda_k^s}f_S\Big(\frac{n_k^s-\lambda_k^s}{\sqrt{\lambda_k^s}}\Big) + (\lambda_k^s-n_k^s)\Big(1-F_S\Big(\frac{n_k^s-\lambda_k^s}{\sqrt{\lambda_k^s}}\Big)\Big)\Big].$$

Similarly, we have

$$k_v - \mathrm{E}\Big(\sum_{k=1}^m v_k\ \min(X_k^s,n_k^s)\Big) = k_v - \Big\{\sum_{k=1}^m v_k\lambda_k^s - \Big[\sqrt{\lambda_k^s}f_S\Big(\frac{n_k^s-\lambda_k^s}{\sqrt{\lambda_k^s}}\Big) + (\lambda_k^s-n_k^s)\Big(1-F_S\Big(\frac{n_k^s-\lambda_k^s}{\sqrt{\lambda_k^s}}\Big)\Big)\Big]\Big\} \tag{14}$$

$$k_w - \mathrm{E}\Big(\sum_{k=1}^m w_k\ \min(X_k^s,n_k^s)\Big) = k_w - \Big\{\sum_{k=1}^m w_k\lambda_k^s - \Big[\sqrt{\lambda_k^s}f_S\Big(\frac{n_k^s-\lambda_k^s}{\sqrt{\lambda_k^s}}\Big) + (\lambda_k^s-n_k^s)\Big(1-F_S\Big(\frac{n_k^s-\lambda_k^s}{\sqrt{\lambda_k^s}}\Big)\Big)\Big]\Big\}. \tag{15}$$

Next, we consider the last term in the profit expression. Following the approximation in (13), we get

$$\mathrm{E}\Big[\Big(\sum_{i=1}^n\sum_{k=1}^m x_iX_k^i\ v_k + \sum_{k=1}^m \min(X_k^s,n_k^s)v_k - k_v\Big)\Big]^+ = \mathrm{E}\Big[\Big(\sum_{i=1}^n\sum_{k=1}^m x_iX_k^i\ v_k - \Big(k_v - \mathrm{E}\Big(\sum_{k=1}^m \min(X_k^s,n_k^s)v_k\Big)\Big)\Big)\Big]^+,$$

where the expression $k_v - \mathrm{E}(\sum_{k=1}^m \min(X_k^s,n_k^s)v_k)$ is computed in (14).

Now define the constant $\overline{K} := k_v - \mathrm{E}(\sum_{k=1}^m \min(X_k^s,n_k^s)v_k)$. Then, using the closed-form expression of $\mathrm{E}(X-a)^+$ in (12), we get

$$\begin{aligned}\mathrm{E}\Big[\Big(\sum_{i=1}^n\sum_{k=1}^m x_iX_k^i\ v_k + \sum_{k=1}^m \min(X_k^s,n_k^s)v_k - k_v\Big)\Big]^+ &= \mathrm{E}\Big[\Big(\sum_{i=1}^n\sum_{k=1}^m x_iX_k^i\ v_k - \overline{K}\Big)\Big]^+\\ &= \sqrt{\sum_{i=1}\sum_{k=1}^m x_i^2 v_k^2\lambda_k^i}\ f_S\left(\frac{\overline{K} - \sum_{i=1}\sum_{k=1}^m x_i\lambda_k^i\ v_k}{\sqrt{\sum_{i=1}\sum_{k=1}^m x_i^2 v_k^2\lambda_k^i}}\right)\\ &\quad + \Big(\sum_{i=1}\sum_{k=1}^m x_i\lambda_k^i\ v_k - \overline{K}\Big)\left(1 - F_S\left(\frac{\overline{K} - \sum_{i=1}\sum_{k=1}^m x_i\lambda_k^i\ v_k}{\sqrt{\sum_{i=1}\sum_{k=1}^m x_i^2 v_k^2\lambda_k^i}}\right)\right).\end{aligned}$$

where the last expression was obtained since $\sum_{k=1}^m x_i\lambda_k^i\ v_k$ is normally distributed with mean $\sum_{k=1}^m x_i\lambda_k^i\ v_k$ and variance $\sum_{k=1}^m x_i^2 v_k^2\lambda_k^i$.

Similarly, we define $\overline{\overline{K}} := k_w - \mathrm{E}(\sum_{k=1}^m \min(X_k^s,n_k^s)w_k)$, we get

$$\begin{aligned}\mathrm{E}\Big[\Big(\sum_{i=1}^n\sum_{k=1}^m x_iX_k^i\ w_k + \sum_{k=1}^m \min(X_k^s,n_k^s)w_k - k_w\Big)\Big]^+ &= \sqrt{\sum_{i=1}\sum_{k=1}^m x_i^2 w_k^2\lambda_k^i}\ f_S\left(\frac{\overline{\overline{K}} - \sum_{i=1}\sum_{k=1}^m x_i\lambda_k^i\ v_k}{\sqrt{\sum_{i=1}\sum_{k=1}^m x_i^2 w_k^2\lambda_k^i}}\right)\\ &\quad + \Big(\sum_{i=1}\sum_{k=1}^m x_i\lambda_k^i\ w_k - \overline{\overline{K}}\Big)\left(1 - F_S\left(\frac{\overline{\overline{K}} - \sum_{i=1}\sum_{k=1}^m x_i\lambda_k^i\ v_k}{\sqrt{\sum_{i=1}\sum_{k=1}^m x_i^2 w_k^2\lambda_k^i}}\right)\right).\end{aligned}$$

Finally, the expected profit expression can be approximated by the following closed-form expression:

$$\text{Expected Profit} = \sum_{i=1}^{n} \mathrm{E}(\xi^i) \sum_{k=1}^{m} r_k^c x_i \lambda_k^i + \sum_{k=1}^{m} r_k^s \lambda_k^s - \left[\sqrt{\lambda_k^s} f_S\left(\frac{n_k^s - \lambda_k^s}{\sqrt{\lambda_k^s}}\right) + (\lambda_k^s - n_k^s)\left(1 - F_S\left(\frac{n_k^s - \lambda_k^s}{\sqrt{\lambda_k^s}}\right)\right)\right]$$
$$- h_v \left\{ \sqrt{\sum_{i=1}^{n} \mathrm{E}(\xi^i) \sum_{k=1}^{m} x_i^2 \upsilon_k^2 \lambda_k^i} \; f_S\left(\frac{\overline{K} - \sum_{i=1} \mathrm{E}(\xi^i) \sum_{k=1}^{m} x_i \lambda_k^i \; \upsilon_k}{\sqrt{\sum_{i=1} \mathrm{E}(\xi^i) \sum_{k=1}^{m} x_i^2 \upsilon_k^2 \lambda_k^i}}\right)\right.$$
$$\left. + \left(\sum_{i=1} \mathrm{E}(\xi^i) \sum_{k=1}^{m} x_i \lambda_k^i \; \upsilon_k - \overline{K}\right)\left(1 - F_S\left(\frac{\overline{K} - \sum_{i=1} \mathrm{E}(\xi^i) \sum_{k=1}^{m} x_i \lambda_k^i \; \upsilon_k}{\sqrt{\sum_{i=1} \mathrm{E}(\xi^i) \sum_{k=1}^{m} x_i^2 \upsilon_k^2 \lambda_k^i}}\right)\right)\right\}$$
$$- h_w \left\{ \sqrt{\sum_{i=1}^{n} \mathrm{E}(\xi^i) \sum_{k=1}^{m} x_i^2 w_k^2 \lambda_k^i} \; f_S\left(\frac{\overline{\overline{K}} - \sum_{i=1} \mathrm{E}(\xi^i) \sum_{k=1}^{m} x_i \lambda_k^i \; \upsilon_k}{\sqrt{\sum_{i=1} \mathrm{E}(\xi^i) \sum_{k=1}^{m} x_i^2 w_k^2 \lambda_k^i}}\right)\right.$$
$$\left. + \left(\sum_{i=1} \mathrm{E}(\xi^i) \sum_{k=1}^{m} x_i \lambda_k^i \; w_k - \overline{\overline{K}}\right)\left(1 - F_S\left(\frac{\overline{\overline{K}} - \sum_{i=1} \mathrm{E}(\xi^i) \sum_{k=1}^{m} x_i \lambda_k^i \; \upsilon_k}{\sqrt{\sum_{i=1} \mathrm{E}(\xi^i) \sum_{k=1}^{m} x_i^2 w_k^2 \lambda_k^i}}\right)\right)\right\},$$

where the expressions of $\overline{K}$ and $\overline{\overline{K}}$ are in (14) and (15).

Appendix B

Proof of Theorem 1. We show by induction that $V_t(x_w, x_v) \leqslant V_t^v(x_v) + V_t^w(x_w)$, for $t = 0, 1, \cdots, T$.

First, for $t = 0$, we have

$$V_0(x_w, x_v) = -h_v(x_v - K_v)^+ - h_w(x_w - K_w)^+ \leqslant V_0^v(x_v) + V_0^w(x_w).$$

Suppose the theorem is true for $t-1$, then $V_{t-1}(x_w, x_v) \leqslant V_{t-1}^v(x_v) + V_{t-1}^w(x_w)$. Then

$$\begin{aligned}
V_t(x_w, x_v) = &\sum_{i \in A(\mathbf{x})} \sum_{k=1}^{m} p_{kt}^i \{ f_k^{c,v} + f_k^{c,w} + V_{t-1}(x_w + w_k, x_v + \upsilon_k)\} \\
&+ \sum_{k=1}^{m} p_{kt}^s \max\{ f_k^{s,v} + f_k^{s,w} + V_{t-1}(x_w + w_k, x_v + \upsilon_k), V_{t-1}(x_w, x_v)\} \\
&+ \left(1 - \sum_{i \in A(\mathbf{x})} \sum_{k=1}^{m} p_{kt}^i - \sum_{k=1}^{m} p_{kt}^s\right) V_{t-1}(x_w, x_v) \\
\leqslant &\sum_{i \in A(\mathbf{x})} \sum_{k=1}^{m} p_{kt}^i \{ f_k^{c,v} + f_k^{c,w} + V_{t-1}^w(x_w + w_k) + V_{t-1}^v(x_v + \upsilon_k)\} \\
&+ \sum_{k=1}^{m} p_{kt}^s \max\{ f_k^{s,v} + f_k^{s,w} + V_{t-1}^w(x_w + w_k) + V_{t-1}^v(x_v + \upsilon_k), V_{t-1}^w(x_w) + V_{t-1}^v(x_v)\} \\
&+ \left(1 - \sum_{i \in A(\mathbf{x})} \sum_{k=1}^{m} p_{kt}^i - \sum_{k=1}^{m} p_{kt}^s\right) \{V_{t-1}^v(x_v) + V_{t-1}^w(x_w)\} \\
\leqslant &\sum_{i \in A(\mathbf{x})} \sum_{k=1}^{m} p_{kt}^i \{ f_k^{c,v} + V_{t-1}^v(x_v + \upsilon_k)\} \\
&+ \sum_{i \in A(\mathbf{x})} \sum_{k=1}^{m} p_{kt}^i \{ f_k^{c,w} + V_{t-1}^w(x_w + w_k)\} \\
&+ \sum_{k=1}^{m} p_{kt}^s [\max\{ f_k^{s,v} + V_{t-1}^v(x_v + \upsilon_k), V_{t-1}^v(x_v)\} \\
&+ \max\{ f_k^{s,w} + V_{t-1}^w(x_w + w_k), V_{t-1}^w(x_w)\} \\
&+ \left(1 - \sum_{i \in A(\mathbf{x})} \sum_{k=1}^{m} p_{kt}^i - \sum_{k=1}^{m} p_{kt}^s\right) \{V_{t-1}^v(x_v) + V_{t-1}^w(x_w)\} \\
= &V_t^v(x_v) + V_t^w(x_w).
\end{aligned}$$

where the second inequality is obtained using the case for $t-1$, and the last inequality follows since $\max(a+b, c+d) \leqslant \max(a, c) + \max(b, d)$, for any parameters $a, b, c, d \in R$.

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