A Game Theory Approach to Demand Side Management in Smart Grids

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Abstract. With the increase in global energy awareness, smart grids improve the efficiency and peak leveling of power systems. Demand side management is the controlling scheme in such grids and it aims to optimize several characteristics using an interactive dynamic pricing scheme. In this paper we propose a game theoretic approach to the demand side management where several subscribers share one common energy supplier. In our model, users send their demand vectors for an upcoming period of time to the network and the energy provider responds by broadcasting a dynamic price vector. Energy consumers are concerned with minimizing their total energy cost per day while the power provider aims to maximize its profit while minimizing the peak to average load ratio. Converging to a unique Nash equilibrium solution using a dual constrained optimization problem, our model results motivate follow on research.

1 Introduction

With the continuous increase in electric power demand and the rise of environmental and economic concerns, the control of electric power demand is gaining a lot of focus in today's power grids. Multiple features affect the performance of power networks, for instance the high demand peaks result in severe constraints on the network, causing power supply failure in several situations. As the size of the network increases, managing and controlling the networks' parameters for best performance become a hard optimization task. In that aspect, smart grids carry the promise of achieving efficiency, reliability and robustness by implementing a powerful control scheme involving dynamic pricing. One of the mechanisms for control of such grids is the Demand Side Management (DSM). DSM aims to encourage consumers to use less energy during peak hours, or to move the time of high energy consumption to off-peak times in order to achieve not only energy savings as well as a more efficient use of the energy itself. Real-time pricing is one of the most important DSM strategies, where energy prices change frequently to reflect variations in the cost of energy supply and electricity demand over time.

Motivated to investigate load control based on a dynamic pricing of DSM consumption scheduling, we propose a novel game model where players are the consumers on one side and the energy supplier on another. The set of strategies for the consumers is the distribution of their demand across the day while it is the pricing policy

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for the energy supplier. The payoff function to be minimized for the customers is the price they will be charged for by the energy provider, while the energy supplier aims to maximize its profit payoff function while minimizing the peak-to-average ratio (PAR). We assume that users submit their tentative consumption schedule in an iterative manner, and the network adjusts the price per time slot based on the submitted demand vectors. The process is repeated with each new user entering the system or updating its bid until convergence. Once biding ends, prices per hour and users' demand distribution become fixed and get executed upon for the requested period.

The rest of this paper is structured such that section 2 surveys previous work for modeling DSM using game theory and section 3 describes our game model. In section 4, the Nash equilibrium of the proposed game is discussed, and the analytical solution of the model is detailed in section 5. Section 6 presents the experimental results obtained using our proposed technique while section 7 concludes the paper.

2 Literature Review

Recently, there has been a considerable amount of research addressing the problem of DSM. This section summarizes some of the most relevant models to our work.

In [1], DSM is modeled using a congestion game, in which time varying prices affect the demand at peak hours. The goal of each customer is to minimize its bill according to a cost function metric proposed. The model was proved to converge in a finite number of iterations to a pure Nash equilibrium solution.

Samadi et. al proposed a DSM strategy for a smart power infrastructure, [2], in which smart meters and the power grid communicate to exchange information regarding the demand and the dynamic price. A distributed algorithm optimizes the demand and prices to ensure the benefit of both consumers and supplier.

A four-stage Stackelberg game models DSM in a smart grid as shown in [3]. A retailer distributes the amount of produced energy between two sources in the first and second stage. While the real-time price and the demand profiles are updated in the third and fourth stages respectively.

In [4], users decide on their energy load profile so that PAR is minimized while the provider decides on the dynamic price. Users can also opt to charge their batteries during low-demand periods and use the charged energy during high price time slots.

A similar energy storage approach for the management of a smart grid is proposed in [5], where a price signal sent by the energy supplier controls the strategies adopted by the consumers. A Nash equilibrium was established and proven to achieve optimal results with led to an overall annual saving of nearly GBP 1.5B and savings of up to 13% on average per consumer. .

The Nash equilibrium of a distributed DSM system is developed in [6] by Mohsenian-Rad et. al, using a game theory model where the players are the users which action is their demand levels and a utility company deciding on a dynamic price. The game is an incomplete information game in which the consumers are unaware of each other actions and they tend to minimize their own cost. The total cost minimum was shown to correspond to the vector of individual minimal cost without requiring complete information.

While existing approaches have focused on modeling DSM as a non-cooperative game, Saad et. al. used coalitional game theory to model interactions between the users and the energy providers in a smart grid environment [7]. The components of this game are micro-grids supplying consumers in disjoint partitions aiming to reduce the total load on the main network as well as minimizing power losses. Surplus of power in certain coalitions are transferred to those in shortage minimizing the cost and losses.

An alternative approach in the case of incomplete information is maximizing a social welfare metric, using a Vickrey-Clarke-Groves (VCG) based approach as in [8]. The social welfare is expressed as the total energy cost subtracted from the combined utility functions of all users. Each consumer provides the supplier of its demand while the provider broadcasts a pricing strategy.

Another Stackelberg approach to DSM uses surplus stored energy to balance peak demand as described in [9]. The supplier plays the role of the leader and decides the pricing strategy and the amount of stored energy to buy from the consumers who play the follower role selling their stored energy.

Another approach presented in 10], uses power-shiftable and time-shiftable models for electrical appliances and aims to optimize cost by shifting time-shiftable appliances to particular time slots and varying power levels for power shiftable appliances to ensure the best possible operation of the network.

3 Proposed Model

3.1 Game Components

We assume that our system is a smart grid where users are equipped with a control device that enables them to manage their total electric power demand and improve some of the desirable features of an electrical power system such as efficiency, reliability, and cost.

The components of the game we adopt are as follows:

- Players:
  - N consumers connected to the smart power grid.
  - One energy provider or company responsible for generating energy
- Actions:
  - Each consumer decides on the consumption power vector over the time slots considered and which are submitted to the energy provider.
  - The energy provider decides on the pricing policy over the time slots.
- Preferences:
  - Consumers aim to minimize their power bills.
  - The energy provider aim to maximize its profit and minimize PAR.

An illustration of the system described is shown in figure 1.

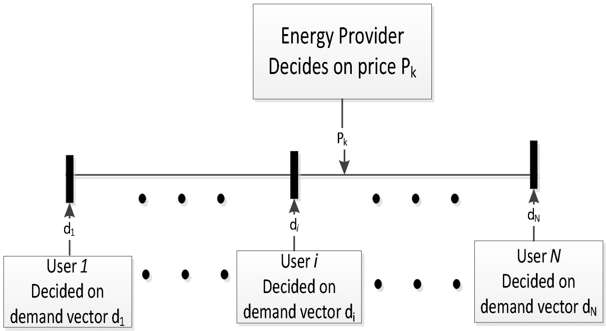

Fig. 1. Illustration of the network configuration

3.2 Utility and Cost Functions

This section aims to formulate a mathematical model of the energy consumers connected to the smart grid. For the consumers, we adopt a cost function model reflecting their daily bill while we elect a utility function to be maximized to model the generator's preference. The cost functions should satisfy the following conditions:

- The cost function is an increasing function of the price and hence the power consumption from the consumer's perspective.
- No consumption (i.e. no demand) results in a 0 cost.
- The cost function is a convex function, while the utility function is a concave function.

Denote by N the total number of users and by K the total number of time slots for which the users submit their consumption demand, also referred to by scheduling horizon. For each user i, the decision vector submitted to the energy provider is defined as:

$$d_i = \left[d_{i,1}, d_{i,2}, \ldots d_{i,k}, \ldots d_{i,K}\right]^T \tag{1}$$

Where k represents the time slot for which the i^{th} user's demand is $d_{i,k}$.

The power consumption of each user during each time slot k is constraint to a maximum value d^m pre-defined by the energy provider such as:

$$0 \leq d_{i,k} \leq d^m \tag{2}$$

At any time slot, the total power acquired from the power grid is limited to the maximum generating capacity of the network denoted by E^m.

$$\sum_{i=1}^{N} d_{i,k} \leq E^m \tag{3}$$

And assume for simplicity that $N.d^m = E^m$.

An intuitive form of cost function for energy users would be a linear function reflecting their daily bill given by the summation time slots of their demand multiplied by the unit price. Because a linear function violates the convexity condition required, we adopt a quadratic form for the cost function that users aim to minimize in response to the pricing strategy at time slot k and to the strategies adopted by consumers other than the i^{th} user d_{-i}, given in eq. (4)

$$C_i(d_i, p_k) = \sum_{k=1}^{K} \left(\beta_i d_{i,k}^2 - d_{i.k} p_k\right) \tag{4}$$

Where $\beta_i > 0$ is a user specific constant defined by consumers. Although the utility function is not a function of d_{-i} explicitly, the i^{th} user's payoff depends on the price p_k, which depends on the strategies of all consumers.

Since the energy provider aims to maximize its profit, it is convenient to model the power supplier's preferences as a utility function of the price and demand which has to be maximized. To insure a minimum PAR, we propose the following supplier preferences:

$$U_P(d_i, p_k) = \sum_{k=1}^{K} \sum_{i=1}^{N} (-\alpha_k p_k^2 + p_k d_{i,k} + PAR.p_k) \tag{5}$$

With

$$PAR = \frac{\max_k \sum_{i=1}^{N} d_{i,k}}{\frac{1}{K} \sum_{k=1}^{K} \sum_{i=1}^{N} d_{i,k}}$$

With $\alpha_k > 0$

The first term of the utility function represents a form of the cost to be paid by the users while the second term represents a form of the PAR to be minimized.

4 Nash Equilibrium

The optimal solution to our proposed game corresponds to the Nash Equilibrium a^* that ensures that "no player i can do better by choosing an action different from a_i^*, given that every other player j adheres to a_j^*" [12]. In other terms, the action profile a^* in a strategic game with ordinal preferences is a Nash equilibrium if, for every player i and every action a_i of player i, a^* is at least as good according to player i's preferences as the action profile (a_i, a_{-i}^*) in which player i chooses a_i while every other player j chooses a_j^*. Equivalently, for every player i, for every action a_i of player i,

$$u_i(a^*) \geq u_i(a_i, a_{-i}^*) \tag{6}$$

In our case, finding the Nash equilibrium of the game is equivalent to finding the strategies p_k^* adopted by the energy supplier and $d_{i,k}^*$ adopted by the consumers such as:

$$\begin{cases} C_i(d_i^*, p_k^*) \leq C_i(d_i, p_k^*) \\ U_P(d_i^*, p_k^*) \geq U_P(p_k, d_i^*) \end{cases} \tag{7}$$

Theorem 1: The previously formulated game admits one and unique Nash equilibrium

Proof: the users' cost functions $C_i(d_i, p_k)$ is strictly convex with respect to d_i resulting in a concave equivalent utility function while the supplier's utility function $U_P(d_i, p_k)$ is concave with respect to p_k; therefore the presented game is a strictly concave (N+1)-person game. In this case, the existence of a Nash equilibrium directly results from [13, Th. 1]. Moreover, the Nash equilibrium is unique due to [13, Th. 3].

Theorem 2: The unique Nash equilibrium of the game is the optimal solution to a dual maximization problem.

Proof: Consider an arbitrary subscriber $i \in \mathcal{N}$. Given d_{-i} and p_k and assuming that all other subscribers fix their energy consumption schedule according to d_{-i}, subscriber i can minimize its cost by solving the following local problem:

$$\sum_{k=1}^{K} \left(\beta_i d_{i,k}^{*2} + d_{i.k}^{*} p_k^{*}\right) \leq \sum_{k=1}^{K} \left(\beta_i d_{i,k}^{2} + d_{i.k} p_k\right) \tag{8}$$

This can be re-written as:

$$\min_{d_{i,k}} \sum_{k=1}^{K} \left(\beta_i d_{i,k}^{2} + d_{i.k} p_k\right) \tag{9}$$

subject to the constraints in (2) and (3).

Alternatively, the energy supplier aims to maximize its payoff given the users' consumption schedules. Its pricing strategy p_k^*issuch that:

$$\sum_{k=1}^{K} \sum_{i=1}^{N} (-\alpha_k p_k^{*2} + p_k^{*} d_{i,k}^{*} + PAR.p_k^{*}) \geq \sum_{k=1}^{K} \sum_{i=1}^{N} (-\alpha_k p_k^{2} + p_k d_{i,k}^{*} + PAR.p_k) \tag{10}$$

The optimization problem in (10) can be written as:

$$\max_{p_k} \sum_{i=1}^{N} (-\alpha_k p_k^{2} + p_k d_{i,k} + PAR.p_k) \tag{11}$$

Therefore the Nash equilibrium of the formulated game corresponds to the intersection of the best responses of the subscribers with that of the energy supplier also consistent with a dual constrained optimization problem given by (9) and (11).

5 Analytical Solution

In the previous section, we discussed the existence of a Nash equilibrium solution for the proposed game. We also demonstrated that finding this equilibrium is equivalent to solving a dual constrained optimization problem. In this section we present a closed form solution for the optimization problem using Lagrange multipliers.

5.1 The Cost Minimization Problem

The energy subscribers aim to maximize their payoff functions corresponding to minimizing their energy cost. This problem is expressed in (9) subject to (2) and (3). Using the KKT extension of the Lagrangian theory, we re-write the users' cost minimization problem as:

$$\max_{d_{i,k}} g(d_i) = \sum_{k=1}^{K} -\left(\beta_i d_{i,k}^2 + d_{i.k} p_k\right) \tag{12}$$

Equivalent to:

$$\max_{d_{i,k}} -\left(\beta_i d_{i,k}^2 + d_{i.k} p_k\right) \tag{13}$$

Subject to:

$$\begin{cases} d_{i,k} \geq 0 \\ d^m - d_{i,k} \geq 0 \\ E^m - \sum_{j=1, j\neq i}^{N} d_{j,k} - d_{i,k} \geq 0 \end{cases} \tag{14}$$

The corresponding Lagrange function of the previous problem relative to the i[th] user can be written as:

$$L_i\left(d_{i,k}, \lambda_1, \lambda_2\right) = -\left(\beta_i d_{i,k}^2 + d_{i.k} p_k\right) + \lambda_1\left(d^m - d_{i,k}\right) + \lambda_2\left(E^m - \sum_{i=1}^{N} d_{i,k}\right) \tag{15}$$

The solution of the problem is found by maximizing the Lagrangian with respect to $d_{i,k}$ subject to the non-negativity restrictions, and minimizing the Lagrangian with respect to the variables λ_1 and λ_2 subject to the non-negativity restrictions:

$$\frac{\partial L_i}{\partial d_{i,k}} \leq 0; d_{i,k} \frac{\partial L_i}{\partial d_{i,k}} = 0; d_{i,k} \geq 0 \tag{16}$$

$$\frac{\partial L_1}{\partial \lambda_j} \geq 0; \lambda_i \frac{\partial L_1}{\partial \lambda_j} = 0; \lambda_j \geq 0 \tag{17}$$

Equations (16) and (17) yield:

$$\begin{cases} -\left(2\beta_i d_{i,k} - p_k\right) - \lambda_1 - \lambda_2 \leq 0 \\ \left[\left(2\beta_i d_{i,k} - p_k\right) + \lambda_1 + \lambda_2\right] d_{i,k} = 0 \\ d^m - d_{i,k} \geq 0;\ \lambda_1\left(d^m - d_{i,k}\right) = 0 \\ E^m - \sum_{i=1}^{N} d_{i,k} \geq 0;\ \lambda_2\left(E^m - \sum_{i=1}^{N} d_{i,k}\right) = 0 \end{cases} \tag{18}$$

Four cases are considered:

1) $\lambda_1 = \lambda_2 = 0$
   This leads: $d_{i,k} = \frac{p_k}{2\beta_i}$ and $g(d_i) = 0$ a feasible solution.
2) $\lambda_1 = 0\,, \lambda_2 \neq 0$
   This leads $\sum_{i=1}^{N} d_{i,k} = E^m = N.d^m$, with $d_{i,k} \neq d^m$, suppose the i^{th} user submits a demand value $d_{i,k} < d^m$, another user j will submit a value $d_{j,k} > d^m$ to satisfy $\sum_{i=1}^{N} d_{i,k} = E^m$ which is not feasible.
3) $\lambda_1 \neq 0, \lambda_2 = 0$, this solution is not feasible since $\sum_{i=1}^{N} d^m = E^m$
4) $\lambda_1 \neq 0, \lambda_2 \neq 0$, leads $d_{i,k} = d^m$ and $g(d_i) = \sum_{k=1}^{K} -\left(\beta_i {d^m}^2 + d^m p_k\right) \leq 0$

Therefore the optimal action for the i^{th} user in response to a pricing strategy is $d_{i,k} = \frac{p_k}{2\beta_i}$.

5.2 The Profit Maximization Problem

The second optimization problem expressed in (13) concerns the energy supplier aiming to maximize its profit and is equivalent to

$$\max_{p_k} f(p_k) = \sum_{i=1}^{N} (-\alpha_k p_k^2 + p_k d_{i,k} + PAR.p_k) \tag{19}$$

The maximization problem is solved by setting the derivative to zero:

$$\frac{\partial f(p_k)}{\partial p_k} = 0 \Rightarrow \sum_{i=1}^{N} \left(-2\alpha_k p_k + d_{i,k} + PAR\right) = 0$$

$$\Rightarrow p_k = \frac{\sum_{i=1}^{N} d_{i,k} + PAR}{2\alpha_k} = \frac{\sum_{i=1}^{N} d_{i,k} + K \frac{\max_k \sum_{i=1}^{N} d_{i,k}}{\sum_{k=1}^{K} \sum_{i=1}^{N} d_{i,k}}}{2\alpha_k} \tag{20}$$

The optimal price obtained in (20) suggests an important property of the pricing strategy: the price is proportional to the load at each time slot, while penalizing a high PAR simultaneously.

In summary, the Nash equilibrium solution of the game model is given by:

$$\left(p_k, d_{i,k}\right) = \left(\frac{\sum_{i=1}^{N} d_{i,k} + K \frac{\max_k \sum_{i=1}^{N} d_{i,k}}{\sum_{k=1}^{K} \sum_{i=1}^{N} d_{i,k}}}{2\alpha_k}, \frac{p_k}{2\beta_i}\right) \tag{21}$$

6 Performance Evaluation

In this section we present some experimental results obtained on a possible scenario that we designed. Two algorithms are implemented: one executed by the energy supplier, the second executed by the power consumers. These algorithms are illustrated by figures 2 and 3 respectively. The system simulated is as follows:

Consider a power grid constituted of N consumers, and an energy supplier where the horizon is $K = 24\, hours$, the maximum allowed consumption per user per time slot is $50kW$, and the maximum generating capacity of the power source at any given time is $250kW$. Assume that the users and the power provider chose their parameters randomly according to the following:

- $\alpha_k \in [1\ 4]$
- $\beta_i \in [3.5\ 6.5]$

At the start of the bidding period, one user starts by requesting a uniform demand vector causing the energy supplier to broadcast an initial uniform price vector. The second user enters the bidding system and requests a uniform consumption vector. The energy provider adjusts the price vector in response to the new user entering. In return the users adjust their consumption schedule based on the provided price. The algorithm iterates until convergence and no new users enter the network.

Algorithm Executed by Energy Supplier
1. Initialization
2. Repeat
a. For each time slot $k \in K$
- Compute the best price p_k^* using (21)
- Broadcast p_k
b. Receive demand vectors d_i for all users
3. Until convergence

Fig. 2. Energy Supplier Algorithm

Algorithm Executed by Consumers
1. Initialization
2. For each time slot $k \in K$
a. Receive the broadcasted price p_k
b. Update demand value $d_{i,k}$ using (21)
c. Communicate value to network
3. End

Fig. 3. Consumers' Algorithm

The following figures show, for multiple values of consumers (N), normalized plots of:

- Initial total demand vector of all users(figure 4)
- Final demand vector of all users(figure 5)
- The evolution of the PAR until convergence (figure 6)
- The variation of the cost of a random user with iterations (figure 7)
- The variation of the utility of the power provider with iterations (figure 8)

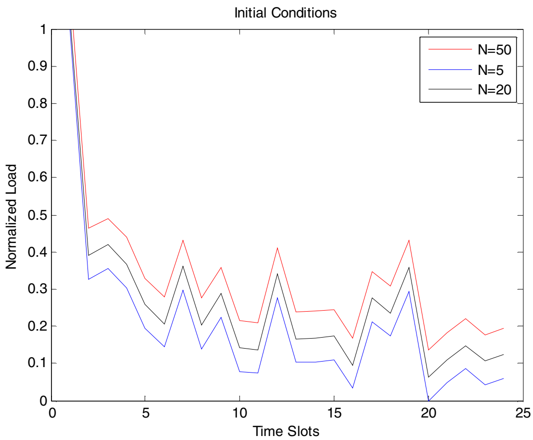

Fig. 4. Initial Demand and Prices

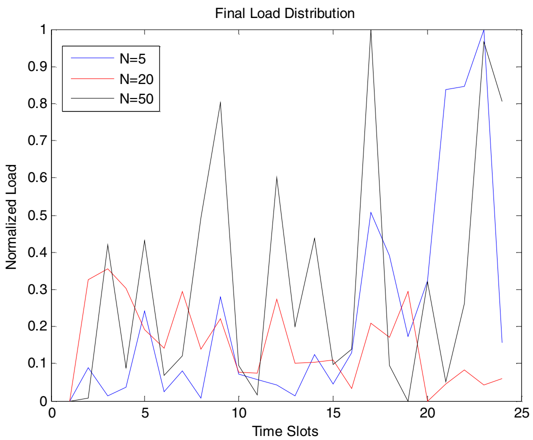

Fig. 5. Demand at Convergence

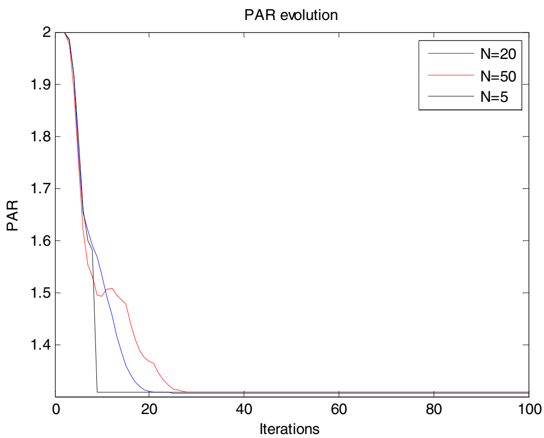

Fig. 6. Evolution of PAR

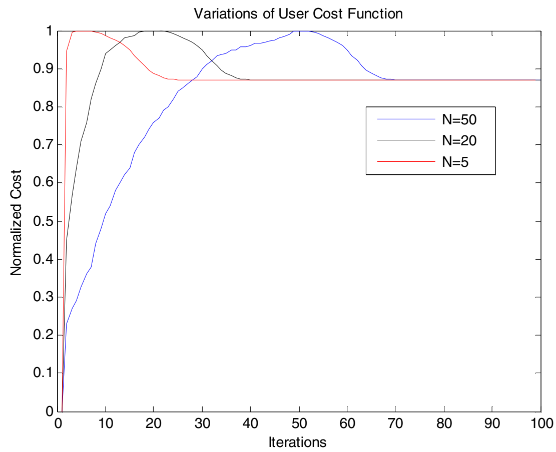

Fig. 7. Evolution of User Cost

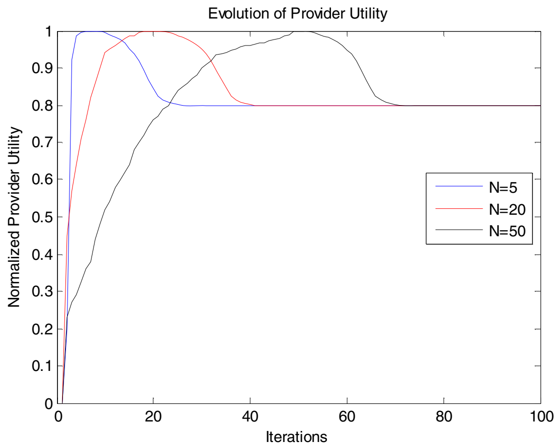

Fig. 8. Evolution of Provider Utility

The following conclusions can be drawn from figures 4-8:

- The proposed pricing scheme is consistent with the desired features as peak loads are penalized by a high pricing response.
- The proposed algorithm convergence in about 7-8 iterations for N=5 with the PAR reaching a stable minimal value.
- The proposed solution achieves simultaneously a maximum profit for the power provider and a minimum cost for energy users.
- Despite the peak observed in the utility function of the power supplier at the 3rd iteration (for N=5), however this maximum does not correspond to the optimal solution as not all the users have entered the system yet (the users enter in an iterative fashion with all users updating their demand starting the 6th iteration).

- Results obtained are consistent when increasing the number of consumers. The PAR converges slower for a larger number of consumers to the same final value as a smaller number of users. The normalized final value of the user cost is practically the same for multiple numbers of users as well as the power provider utility.

7 Conclusion

In this project we aimed to model the demand side management in smart grids using game theory concepts. We design it as a novel multi-player game where consumers aim to minimize their bill while the energy provider aims to maximize its profit and minimize the peak to average load ratio. The proposed model is proven to converge to a unique Nash equilibrium solution coinciding with the optimal solution of a dual optimization problem. Experimental results showed the efficiency of our proposed method in modeling the situation and minimizing PAR.

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