

# Fuel Cell Hybrid Electric Vehicle Sizing using Ordinal Optimization

Sami H. Karaki, Rafika Dinnawi, Rabih Jabr, and Riad Chedid  
American Univ. of Beirut

Ferdinand Panik  
University of Applied Sciences Esslingen

## ABSTRACT

An optimal design methodology is developed in this paper for fuel cell hybrid electric vehicles (FCHEV) based on ordinal optimization (OO) and dynamic programming (DP); the optimal design aims to determine the appropriate sizes of the hydrogen tank, fuel cell, battery, and motor for the purpose of minimizing investment and operational cost given some specification of the car range, the road type and its gradeability. The DP simulates the operation of the vehicle for a set of specified components' sizes for given driving cycles and provides the total vehicle cost per year. The OO method offers an efficient approach for optimization by focusing on ranking and selecting a finite set of "good enough" alternatives through two models: a simple model and an accurate model. The OO program uses the specified sizes of the components that uniformly sample the search space and evaluates these designs using a simple but fast model. As per OO theory, the evaluated designs are sorted in an ascending order and the top-*s* design solutions are selected. An ordered performance curve method is used to determine the size of the top-*s* set of "good enough" designs which are then evaluated using an "accurate model" in a kind of horse race. This model is implemented by taking the whole of the driving cycles and an increased number of states in the dynamic programming solution. In these runs the effect of varying the fuel cell, the battery, and hydrogen costs are investigated; car design results are presented with different optimal sizes for different specifications.

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## INTRODUCTION

Fuel cells with proton exchange membranes have been identified as a very promising alternative that may simplify the drive train of cars by replacing the internal combustion engines used in the transportation sector [1]. FCHEV technologies may provide a true solution for the traditional pollution problems associated with vehicles in large cities because they use the nontoxic hydrogen gas as fuel and have just water and heat as outputs. FCHEVs have two power sources, proton exchange membrane fuel cells and batteries, or super-capacitors, to supply the power-train demand load. The cost of running a FCHEV may be optimized at two levels: the design level and the operation level. An optimal design aims to determine the optimum sizes of the car components, which are the hydrogen tank, fuel cell, battery, and motor for the purpose of minimizing the investment. Optimal operation, for a given design, aims to achieve minimum hydrogen consumption and battery degradation.

Researchers initially solved the sizing problem basically by optimizing the operation using formal optimization methods and optimized the components sizing through a simple try and rank algorithm. Tate and Boyd [2] accurately approximated the optimal operation problem as a large linear program to compute the globally minimum fuel consumption over a given drive cycle. This is then used in a sizing algorithm to propose different sizes of battery and

engine and to determine for each design the minimum fuel consumption. Once the designs are obtained they are ranked and the one with overall minimum fuel consumption is selected. Bernard et al [3] followed a similar approach for the sizing but minimized fuel consumption of each design using a global optimizing algorithm based on optimal control theory. Jörg and Shröder [4] astutely indicate that the main problem associated with components sizing is the dependency of the power demand on the weight of the vehicle, and therefore on the weight of each component. They used optimal control theory and non-linear programming where the sizes were considered as control parameters of the closed loop of a control structure with an overall objective of minimizing the fuel consumption over a given drive cycle. Kim and Peng [5] approached the problem in a reverse way by using basis functions of a "pseudostochastic" dynamic programming controller to simulate near optimal operation and non-linear programming to determine optimal components sizing.

Recognizing the role of energy storage devices in characterizing hybrid or plug-in hybrid vehicles, many authors focused their attention on the sizing of energy storage devices. This was discussed by Burke [6] who concluded that the battery storage device in a charge-sustaining FCHEV is sized by the peak power during vehicle acceleration. In such designs the battery operates over a battery state-of-charge (SOC) range well within 10% which extends battery

life. Whereas for plug-in FCHEV the fuel economy can be very high because a large proportion of the energy to power the vehicle comes from grid electricity. Markel et al [7] developed a sizing methodology of battery storage devices in FC vehicles based on their role during fuel-cell start up, power assist capability, and regenerative braking.

For the problem of dependency of the power demand on the component sizing itself, researchers tried using systematic algorithms to search for “optimal” sizes of components using evolutionary methods. Yildiz et al [8] used particle swarm optimization to size vehicle components while meeting critical performance as evaluated by a simplified model of a power split plug-in vehicle developed using the Powertrain System Analysis Toolkit (PSAT) from the Argonne National Lab. Jain et al [9] used multi-objective genetic algorithms to size vehicle components to minimize overall fuel consumption using a charge-sustaining control strategy. Ordinal optimization (OO) is a method for complex optimization problems based on two tenets: ‘order’ comparison is more robust against noise than ‘value’, and ‘goal softening’ where the insistence on reaching the ‘best’ is replaced by finding ‘good enough’ solutions with high probability [10]. Jabr and Pal used OO [11] to determine the location and size of distributed generation in a power system by evaluating many alternative solutions using an approximate but fast method based on linear programming and ranking them according to this evaluation. As per OO theory, there is a high probability that the top- $s$  designs include the actually ‘good’ alternatives, which are then evaluated using a detailed optimal power flow model based on nonlinear programming.

This paper presents an ordinal optimization approach for solving the optimal design of FCHEV. The aim is to determine the suitable sizes of the different car components (hydrogen tank, fuel cell, battery, and motor) for the purpose of minimizing the investment and operational cost. The OO approach consists of three main stages. First a high number of alternatives are uniformly sampled from the large search space. Second, the total cost of operating the FCHEV per year, for each alternative is evaluated using a crude, or simple, but computationally fast model. The alternatives are then ranked in ascending cost order and the top- $s$  solutions are identified. The crude model uses a portion of the drive cycle and DP with a relatively large discretization step with simple components models. Third, the top- $s$  alternatives that have a high probability of being in the “good enough” set are then assessed via an accurate model. A “good enough” solution means that the solution has a high probability to be a good solution. OO theory allows computing the size of the selected subset such that it contains one or several designs from among the top good enough samples with a high pre-specified alignment probability. Every design is assessed using an optimizing algorithm to minimize hydrogen consumption and keep battery degradation at check using DP. The DP approach can suitably simulate charge- sustained and charge-depleting operation by proper lower bound specification and enforcement [12] and as such will be used to develop normal designs and plug-in hybrid designs.

## PROBLEM FORMULATION

A system's overview of the power train arrangement corresponding to a hybrid architecture is shown in Fig. 1 below. A FCHEV is propelled by an electric machine that receives its electric input from two dc sources through a dc/ac inverter. A dc/dc converter is used to couple the two power sources, i.e. the polymer electrolyte membrane (PEM) fuel-cell and the battery, and a dc/ac inverter is used to power the electric motor. As per current technology, their sizes will be calculated and provided as output from the ordinal optimization algorithm.

The cost is evaluated for two mixed roads, the Urban Dynamometer Driving Schedule (UDDS) and the Highway Federal Emissions Test (HWFET) drive cycles. The fuel cost ratio is taken as a weighted cost of 0.45 of that of UDDS and 0.55 of that of the HWFET cycle, which is approximately in line with what is proposed in [14]. The cost function  $J$  in \$/year is minimized over a set of designs  $n=1$  to  $N$  and is symbolically described as follows:

$$J = \min_{n=1,N} (\alpha I_C(n) + O_C(n)) \quad (1)$$

A nomenclature of the main variables is given in Appendix A at the end of the paper. The investment cost  $I_C(n)$  is determined for a given design size vector:  $D(n) = [P_{FC}^{max}, E_{BT}^{max}, H_2^{max}, P_M^{max}]$ , where the elements are the sizes of the fuel cell, the battery, the hydrogen tank, and the motor. The optimal design is determined over all  $N$  designs for a minimum investment and operation cost evaluated on a per-year basis.

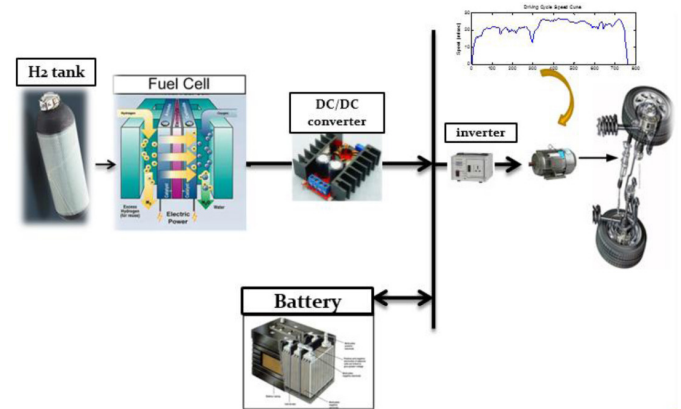


Figure 1. System Overview

So the investment cost is multiplied by the annuity factor  $\alpha$ :

$$\alpha = \frac{r(1+r)^l}{(1+r)^l - 1} \quad (2)$$

### Optimal Operation using Dynamic Programming

The operating cost in \$/trip is calculated using dynamic programming for one of the two road types and during one trip is given by:

$$O_{C1}(n) = \min \left\{ \sum_{t=1}^T \left[ \frac{1}{2} \gamma_{BT} \left( 1 + \text{sign}(P_{BT1}(t)) \right) (P_{BT1}(t)) \Delta t + \varphi_{FC}(P_{FC1}(t)) \Delta t \right] \right\} \quad (3)$$

Where  $\varphi_{FC}(P_{FC}) = a_n + b_n P_{FC} + c_n P_{FC}^2$  is the cost rate (\$/h) of operating the fuel cell at a level  $P_{FC}$ ; the corresponding curve is obtained from the polarization curve of the fuel cell and scaled by the cost of hydrogen fuel. The per-unit curve used in this paper is shown in Fig. 2 and is based on data provided in [14] and [16]. The second term is proportional to the energy delivered by the battery. The proportionality constant  $\gamma_{BT}$  is measured in \$/kWh. Thus the second term is a measure of battery degradation if one considers that a battery over its lifetime delivers a defined amount of energy, approximately. The constraints over one of the road types are:

$$P_{FC1}(t) + P_{BT1}(t) - P_{BR1}(t) = P_{D1}(t) \quad (5)$$

$$0 \leq P_{FC1}(t) \leq P_{FC}^{max} \quad (6)$$

$$P_{BT}^{min} \leq P_{BT1}(t) \leq P_{BT}^{max} \quad (7)$$

$$R_{aFC} \Delta t \leq P_{FC1}(t) - P_{FC1}(t-1) \leq R_{uFC} \Delta t \quad (8)$$

$$R_{aBT} \Delta t \leq P_{BT1}(t) - P_{BT1}(t-1) \leq R_{uBT} \Delta t \quad (9)$$

$$SOC_{min} \leq SOC_1(t) \leq SOC_{max} \quad (10)$$

$$SOC_1(t) = SOC_1(t-1) - \psi_{BT}(P_{BT1}(t)) \Delta t \quad (11)$$

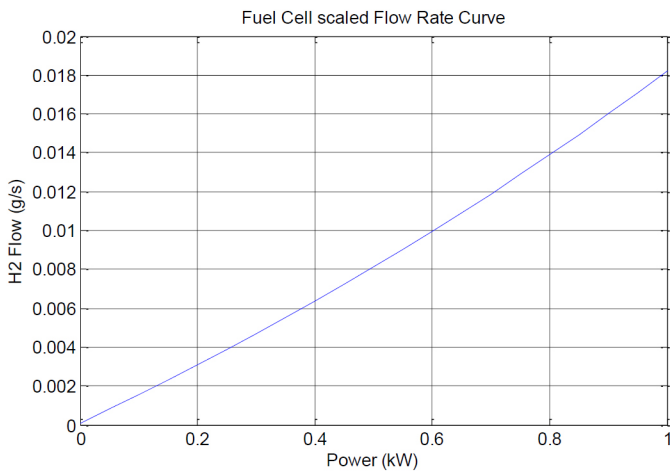


Figure 2. Fuel cell hydrogen consumption curve for 1kW

Equation (5) represents the power balance constraint over every period of the first drive cycle. Constraints (6) and (7) represent the upper and lower limits on the fuel-cell and battery powers, whereas constraints (8) and (9) represent the ramping rate limitations of the fuel-cell and battery. Constraint (10) is the upper and lower limitation on the battery state-of-charge (SOC) and constraint (11) is the dynamic update on the SOC in period  $t$  given the battery power and the SOC in period  $t-1$ .

The  $\psi_{BT}(P_{BT1})$  defines a charge power map relating internal energy rate to power output as illustrated in Fig. 3. The above equations are developed for one type of the roads, e.g. the UDDS driving cycle as indicated by subscript 1. Similarly, all the equations from (3), (5), (6), (7), (8), (9), (10), (11) are applied on the (HWFET) driving cycle in order to calculate the total cost per year,  $O_{C_{year}}(n)$ , for a given design  $D(n)$ , for the mixed two cycles. The units of  $O_{C1}$  and  $O_{C2}$  are \$/trip1 and \$/trip2 respectively, therefore the total cost of  $M$  trips per year is:

$$O_{C_{year}}(n) = M(0.45 O_{C1}(n) + 0.55 O_{C2}(n)) / 0.7 \quad (12)$$

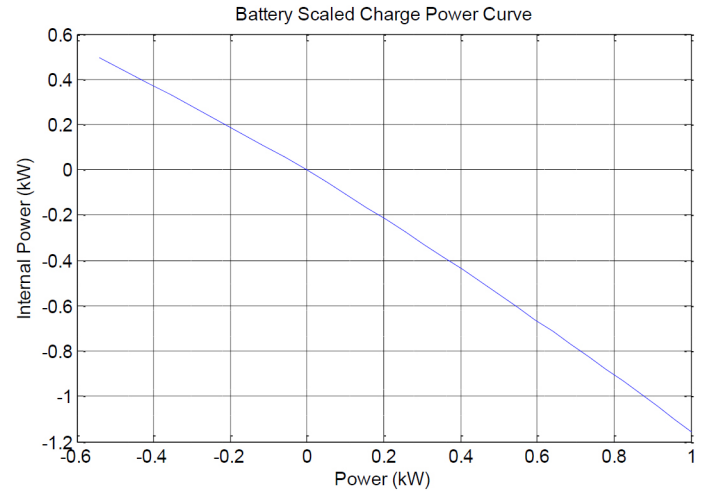


Figure 3. Battery charge map scaled to 1kW

The weighing coefficients 0.45 and 0.55 and the fuel economy adjustment factor of 0.7 to reflect real road driving conditions were adopted from [14]. The number of trips  $M$  is 700 based on two trips per day for 350 days per year. In solving the problem using DP, the power demand forecast curve is divided into stages. In each stage the fuel cell power level can be in one of several discrete states, which once fixed, for a given state, it allows the battery power to be determined from the power balance constraint (5). The objective function (3) will be calculated for all the states of stage 1. Then for each state of subsequent stages (i.e.  $t=2$  to  $T$ ), the possible transitions from states of the previous stage  $t-1$  are identified, and the minimum cumulative cost of reaching each state is built recursively. When the last stage is reached, the state with minimum cost is recognized, and the preceding states leading to it are identified by a trace-back procedure. Charged sustained (CS) operation may be obtained by setting  $SOC_{min}$  to the initial state of charge  $SOC_0$ . Whereas, charge depletion (CD) operation is obtained by reducing  $SOC_{min}$  to an appropriate value below  $SOC_0$ . During both modes of operation, the battery cost parameter  $\gamma_{BT}$  is made slightly below (i.e. 10% lower

than) the incremental fuel-cell cost calculated at the average power demand. So in CS mode when the battery gets charged due to a braking operation, its lower incremental cost will cause it to discharge on the next motoring operation until it hits its lower limit, similarly in CD operation the battery will discharge until hits its lower limit. What creates the difference between the two modes is the lower limit of the battery that is made equal to the initial SOC in a CS operation whereas it is set to a lower appropriate minimum value (e.g. 40%) for CD operation. This is discussed in detail in [12]. Moreover, the operation cost is the summation of battery degradation cost plus the hydrogen consumption through the whole year.

### Design using Ordinal Optimization

Ordinal Optimization is a technique based on two tenets [10]: firstly, “order” is much more robust against noise than “value”, and secondly, seeking “good enough” solutions with a high probability, known as goal softening, instead of insisting on reaching the best one can significantly ease the computational burden. Its purpose is not to replace existing techniques for optimization but to complement them to find solutions of problems requiring complex simulations to solve.

For instance, let  $\Theta$  be the search space of the optimization variables which represents the different sizes of hydrogen tank, fuel cell, battery, and motor sizes; and let  $\Theta_N$  be the set of selected  $N$  designs uniformly sampled from the search space, as illustrated in Fig. 4. When a large enough number of samples  $N$  (1000 or more) are selected the subset  $\Theta_N$  is used to represent the total search space  $\Theta$ . Let  $G$  be the “good enough” subset in  $\Theta_N$ , referred to as the true top- $g$  designs,  $S$  the selected subset usually made of top- $s$  designs in  $\Theta_N$ ,  $G \cap S$  the set of truly good enough designs in  $S$ ,  $AP$  the alignment probability =  $P[|G \cap S| \geq k]$  the probability that there are actually  $k$  truly good enough designs in  $S$ , and  $k$  is the alignment level. The procedure for the practical application of OO using a horse-race selection rule to a complex optimization problem is as follows:

- Step 1. Sample  $N$  designs uniformly from  $\Theta$  to form  $\Theta_N$ , as shown in Fig. 5.
- Step 2. Estimate the performance of the designs in  $\Theta_N$  using a crude but computationally efficient model and sort them in ascending order.
- Step 3. Plot the Ordered Performance Curve (OPC); for our problem a bell curve is obtained.
- Step 4. Specify the size  $g=50$  of the good enough subset ( $G$ ), the required alignment level ( $k=1$ ), and estimate the expected error bound ( $W$ ).
- Step 5. Use  $Z(k, g) = e^{Z_1 k^{Z_2} g^{Z_3}} + Z_4$  with  $Z_1 = 8.1998$ ,  $Z_2 = 1.9164$ ,  $Z_3 = 2.0250$  and  $Z_4 = 10$  obtained from a table of regression coefficients for  $AP = 0.95$ , which correspond to a low error bound of  $W = 0.5$  per unit [10]. The value of  $Z$  gives the size of the selected subset  $S$  that is 13.

- Step 6. Select the estimated top- $s$  ( $=13$ ) designs from Step 2 to form the selected subset ( $S$ ) and evaluate them using the accurate model.
- Step 7. OO theory ensures that  $S$  contains at least  $k=1$  truly good enough design with a probability level  $AP \geq 0.95$ .

Figure 5 is the OO flow chart that gives a general idea of the steps followed in this system design process. In the simple model the operation of components is simplified by taking small samples of the mixed driving cycles with appropriate scaling for the energy utilized. Moreover, the number of discrete states used in the dynamic programming is made relatively low. Design runs with and without gradeability were also investigated on a mix of the two driving cycles of UDSS (45%) and HWFET (55%) for 700 trips per year.

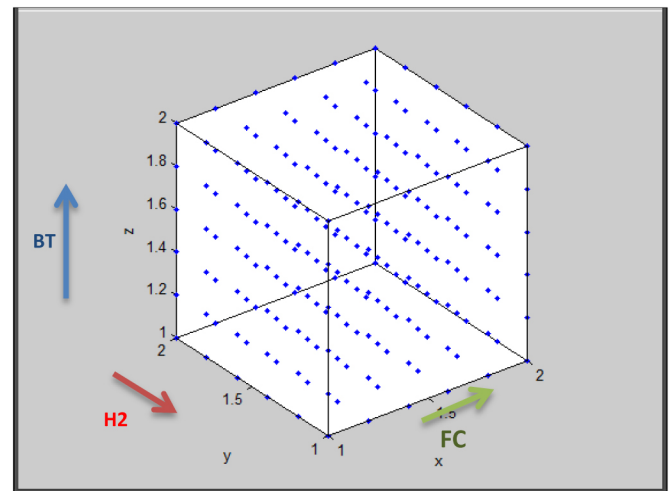


Figure 4. Search space  $\Theta$  of  $N$  designs to form  $\Theta_N$

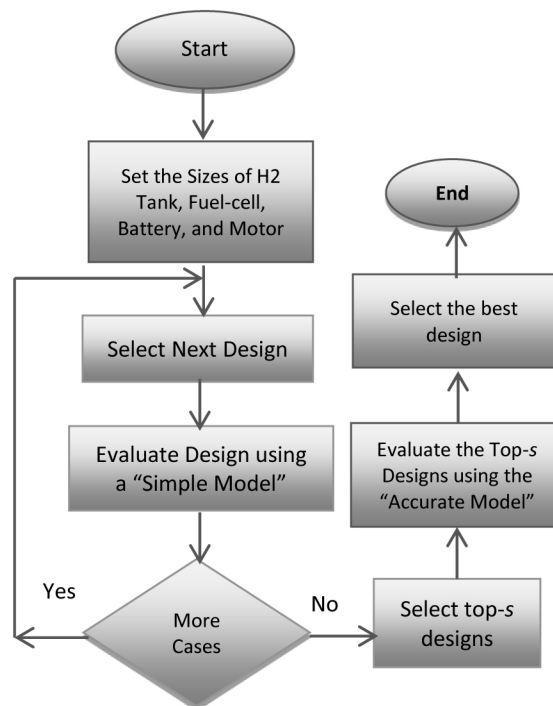


Figure 5. Ordinal optimization flow chart

## SIMULATION AND DESIGN OF FCHEV

### Modeling of Vehicle Power Demand

The load force of the vehicle consist of aerodynamic drag force  $F_w$ , rolling resistance  $F_r$ , gravitational force  $F_h$  caused by gravity when driving on non-horizontal roads (road slope), and acceleration force  $F_a$ . Therefore, the power at the wheels required for vehicle acceleration and deceleration can be written as follows:

$$P_{wheels} = (F_w + F_r + F_h + F_a)V \quad (14)$$

Where:

$$F_w = 0.5\rho A_f C_w V^2 \quad (15)$$

$$F_r = m_v g (C_{r0} + C_{r1}V) \cos(\alpha) \quad (16)$$

$$F_h = m_v g \sin(\alpha) \quad (17)$$

$$F_a = m_v a \quad (18)$$

The total electric power required from the sources is expressed as:

$$P_{req} = \begin{cases} P_{wheels}/(\eta_M \eta_D) + P_{aux} & \text{if } P_{wheels} \geq 0 \\ P_{wheels} \eta_B \eta_D + P_{aux} & \text{if } P_{wheels} < 0 \end{cases} \quad (19)$$

The vehicle technical data parameters used in the designs are presented in [Table 1](#) below, and the vehicle cost data parameters are shown in [Table 2](#). The investment cost of the components depends on the sizes and unit costs of hydrogen tank, fuel cell, battery, and motor. The hydrogen tank cost is in the range \$15-\$18/ kWh [15] and in this work is taken to be \$17/kWh. The cost of the battery unit is 400 (\$/kWh) [13]. The battery specific power and energy were calculated based on data given in [6], [13] and [14]. The fuel cell per unit cost is 47 \$/kW; it was deduced from [16] and corroborated by other references. The hydrogen cost is considered \$5/ kg, and the number of trips per year is 700 trips based on about two trips per day over the year. In addition, battery operation is modeled by a charge- rate versus power map scaled to 1 kW ([Fig. 3](#)), which was deduced from technical data of the Saft VL6A battery [13]. The fuel cell hydrogen consumption curve is based on the efficiency curve given in [14] of peak efficiency of 60%. Efficiency loss of 10% at rated value were considered to cover FC system losses as suggested in [16] and 3% of rated power when the FC is idle. The scaled 1 kW hydrogen consumption curve is shown in [Fig. 2](#). In OO several sizes of fuel cells and batteries need to be implemented, so the curves of [Fig. 2](#) and [Fig. 3](#) were scaled up in each run to the power size being simulated.

To achieve a practical design, the analysis of a FCHEV is performed with combination of two standard driving cycles and a proprietary acceleration and gradeability cycle:

1. Urban Dynamometer Driving Schedule (UDDS)
2. Highway Federal Emissions Test (HWFET)
3. For the class of cars being designed, an acceleration test from 0 to 110km in 15s is applied and was then followed by gradeability run of 5% slope at a speed of 110 km/h for 5 minutes.
4. An appropriate design range of 320 km is also specified.

Table 1. Vehicle Design Technical Data

Gravity Constant	$g$	9.81 m <sup>2</sup> /s
Air density	$\rho$	1.21 kg/m <sup>3</sup>
Glider Mass plus Car Load	$m_v$	915+320 kg
Motor Specific Power		1.25 kW/ kg
Electronics Specific Power		7.5 kW/ kg
FC Specific Power		0.519 kW/ kg
FC Transit-Time		3.75 s
Hydrogen Tank Mass		8.9 kg/ kg H <sub>2</sub>
Battery Specific Energy		
• Normal HEV		85 Wh/ kg
• Plug-in HEV		128 Wh/ kg
Battery Specific Power		
• Normal HEV		2020 W/ kg
• Plug-in HEV		1520 W/ kg
Grid-Battery Charge Efficiency		0.85
Frontal Area	$A_f$	2.18 (m <sup>2</sup> )
Aerodynamic Drag Coefficient	$C_w$	0.28
Rolling Resistance Coefficients	$C_{r0}$ $C_{r1}$	0.0055 0.00023m <sup>-1</sup> s
Auxiliary Power	$P_{aux}$	230 W
Wheel radius	$r_w$	0.314(m)
Motor-Inverter Efficiency	Motoring $\eta_M$ Braking $\eta_B$	0.9025 0.85
Drive-Line Efficiency	$\eta_D$	0.91

Table 2. Vehicle Design Cost Data

Glider Cost	$C_G$	\$23250
Motor Cost	$C_M$	\$52/ kW
FC Cost	$C_{FC}$	\$47/ kW
Hydrogen Cost	$C_{H2}$	\$5/ kg
Hydrogen Tank Cost	$C_T$	\$1700/ kg H <sub>2</sub>
Battery Cost	$C_{BT}$	\$400/ kWh
Grid Electricity Cost	$C_{EL}$	\$0.12/ kWh

These driving cycles are represented by vehicle speeds versus operating time. [Figure 6](#) shows the HWFET drive cycle speed, of which a small portion is considered to represent it; the small portion was synthesized of different parts because of the continuity of the cycle (i.e. no stops). The representative part of the simplified UDDS cycle ([Fig. 7](#)) was taken between 168 and 332s. The data of these and other drive cycles were obtained from [17].

## Simulation and Analysis

The ordinal optimization algorithm has been developed in MATLAB. The objective function that represents the total cost per year (\$/year) is evaluated using dynamic programming operated on a \$/year as described in [equation \(1\)](#). The charge sustained operation is used to design normal hybrids, whereas the charge depletion operation allows the development of plug-in hybrid designs. The different design sizes that were used amounted to 3456 cases and were as follows:

Hydrogen tank sizes= [1, 2, 3, 4, 5, 6, 7, 8] kg

Fuel cell sizes= [10, 20, 30, 40, 50, 60, 70, 80, 90] kW

Battery sizes= [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12] kWh

Motor sizes= [100, 110, 120, 130] kW

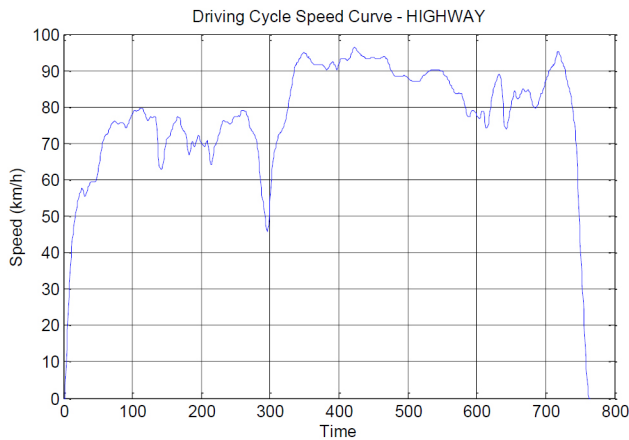


Figure 6. HWFET driving cycle speed

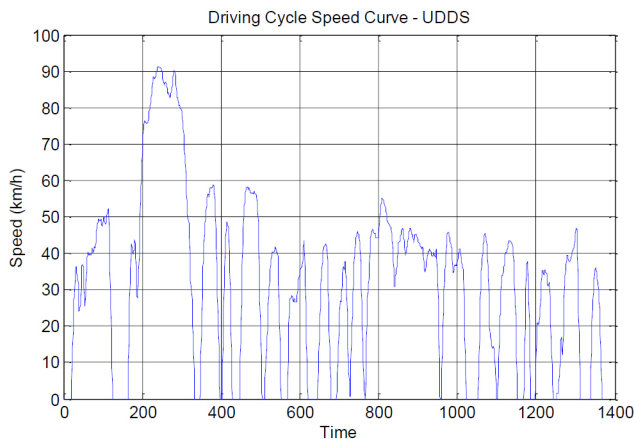


Figure 7. UDDS driving cycle speed

Lower or higher sizes in the above arrays could also have been included, but were disregarded because they just led to infeasible or uneconomic solutions. Thus the specified sizes of hydrogen tanks depend on the road range specified at 320 km, while the range of values for the fuel cell and the battery sizes are chosen according to an over-estimated peak power demand based on the upper bound of the car mass. So, to obtain feasible solutions, the size of the two sources combined should go well beyond the peak power demand. The 3456 designs were analyzed using the simple model involving a

large discretization step in the DP program and a suitable sample of the UDDS and HWFET drive cycles. The test of the acceleration and gradeability (AG) cycle were used only for a technical assessment of the design and do not include any cost evaluation. The results on operating cost are added to the annuity investment cost of each design to give a total yearly cost, to which an infeasibility penalty cost is added, in case the design is infeasible. The designs are then sorted in ascending order of cost and the best 13 are selected according to the horse race selection method. Using the crude model, the performance of the feasible designs are sorted from the lower to the higher cost and plotted with respect to the order. [Fig. 8](#) shows the ordered performance curve (OPC); the corresponding inverted performance density function is a bell curve, which gives the type of the OPC (i.e. bell type) for the base case, indicating that many mediocre designs exist. The selected designs are then evaluated using the accurate model that features the full driving cycles and a small fuel cell power discretization level in the DP.

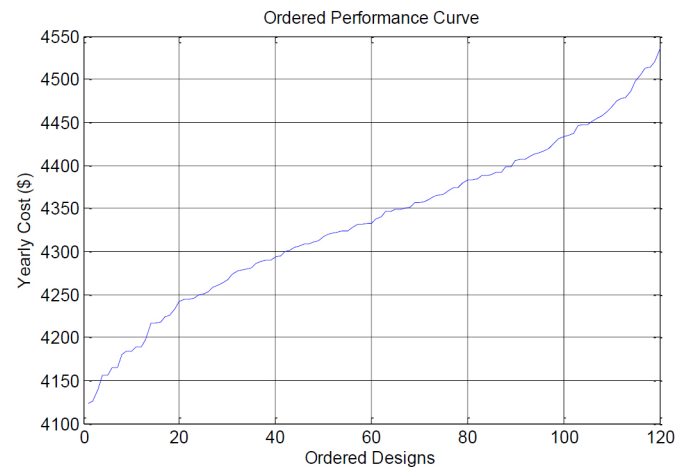


Figure 8. Ordered performance curve

## Design Runs for a Normal Hybrid Car

Several design test runs were carried out to determine “good designs” of normal hybrid cars, where the operation of each design is based on a charge sustained operation in which the battery is not being charged from the grid. The Base Case (Run #1) was done using cost values from [Table 2](#), and several variations were then conducted based on changes in the hydrogen cost (HC) to \$3/ kg (Run#2), fuel-cell investment cost (FCIC) to \$30/ kW (Run #3), HC to \$3/ kg and FCIC to \$30/ kW (Run #4), and finally battery investment cost to \$200/ kWh (Run #5). [Table 3](#) shows the best solution for each run and another good design from the top-13. The cost is dominated by the capital investment cost, for instance in the base case (Run #1) the total yearly cost is divided into investment cost of \$3207 and operating cost of \$556 per year. This clarifies the reason that no change in the order of the best two designs occurred when the H<sub>2</sub> cost was reduced to \$3/ kg instead of \$5/ kg as in the Base Case. Despite the variations in the capital cost of fuel cells (Run #3) or in the capital cost of the battery (Run #5), the top two designs of the Base Case have shown robustness and appeared in the top with a change in their relative order. The design FC= 60kW and BT= 2kWh appeared in position 15 in Run #5.

Table 3. Normal FCHEV Design Results

Run #	H2 Tank kg	FC Size kW	Battery Size kWh	Motor Size kW	Total Cost \$/year	Car Mass kg
1	5	40	3	110	3763	1500
1	5	60	2	110	3820	1532
2	5	40	3	110	3676	1500
2	5	60	2	110	3611	1532
3	5	40	3	110	3698	1500
3	5	60	2	110	3723	1532
4	5	40	3	110	3490	1500
4	5	60	2	110	3513	1532
5	5	40	3	110	3688	1500
5	5	40	4	110	3709	1512

### Design Runs for a Plug-In Hybrid Car

Several design test runs were also carried out to determine “good enough” designs of plug-in hybrid cars, where the operation of each design is based on a charge depletion operation in which the battery is charged from the grid. The size of the allowable depletion on one trip is controlled to be one half of the usable battery energy, i.e. about 30% over one trip since we are assuming 2 trips per day. An additional cost is added in this case, which is the cost of energy bought from the grid at the specified price in Table 2. In this case also, a Base Case and several variations were conducted based on changes in the of hydrogen cost (HC) to \$3/ kg (Run#2), fuel-fell investment cost (FCIC) to \$30/ kW (Run #3), HC to \$3/ kg and FCIC to \$30/ kW (Run #4), and battery investment cost being to \$200/ kWh (Run #5). In this case additional runs investigated designs at 25% lower energy cost (Run #6), then 25% higher energy cost (Run #7), and finally lower battery cost and lower energy cost (Run #8).

Table 4 shows the best solution for each run and another good design. In this case we notice that the designs have significantly smaller size of hydrogen tank and fuel cell sizes, and larger battery sizes, as plug- in hybrids rely more on electricity bought from the grid. Here also, the cost is dominated by the capital investment cost; in the base case the total yearly cost of the winning design is divided into investment cost of \$3186 and operating cost of \$541 per year. Here we notice that the size of the hydrogen tank is 2kg (i.e. 60% lower) and the winning design has a FC size of 10kW and a BT size of 8 kWh, thus favoring high battery sizes since plug-in designs are being sought. Just to compare normal hybrid designs, we also list the third best design which has a FC size of 5kW and BT size of 9kWh. The relative order of these designs change with the variation of design parameters but remain in the top-13 designs clearly demonstrating their robustness.

Table 4. Plug-In FCHEV Design Results

Run #	H2 Tank kg	FC Size kW	Bat Size kWh	Mot Size kW	Total Cost \$/year	Car Mass kg
1	2	10	8	110	3727	1435
1	2	20	7	110	3729	1449
1	2	5	9	110	3750	1432
2	2	10	8	110	3635	1435
2	2	5	9	110	3665	1432
3	2	20	7	110	3697	1449
3	2	10	8	110	3711	1435
4	2	10	8	110	3619	1435
4	2	20	7	110	3637	1449
5	2	5	9	110	3487	1432
5	2	10	8	110	3489	1435
6	2	20	7	110	3654	1449
6	2	10	8	110	3668	1435
7	2	20	7	110	3416	1449
7	2	5	9	110	3423	1432
8	2	10	8	110	3763	1435
8	2	20	7	110	3774	1449

By comparing the set of results, the best overall Plug-in design seems to be the FC= 10kW and BT= 8kWh; in Run #7 it was not among the best two, but was in position 5 in the overall ranking. So this is perhaps the winning plug-in design. Table 5 shows the detailed characteristics of the winning designs in the base case for normal and plug-in hybrid vehicle designs. As expected, the plug-in hybrids are lighter and more efficient designs with better fuel economy.

Table 5. Detailed Characteristics of Winning Designs

	Winning Hybrid Design	Winning Plug-In Design
Car Mass (kg)	1500	1435
Components Sizes/ Weights:		
• H <sub>2</sub> Tank (kg H <sub>2</sub> / kg)	5/ 44.5	2/ 17.8
• Fuel Cell (kW/ kg)	40/ 77.1	10/ 19.3
• Battery (kWh/ kg)	3/ 35.3	8/ 94.1
• Motor (kW/ kg)	110/ 88.0	110/ 88.0
Investment Cost (\$/ year)	3207	3186
Operating Cost (\$/year)	556	541
Unadjusted Fuel Economy:		
• H <sub>2</sub> (g/km)	7.14	2.94
• mpgge <sup>(1)</sup>	86.9	312
Range (km)		
• Fuel Cell Range	490	707
• Battery Range	small	50.2

(1) Miles per gallon of gasoline equivalent

### Additional Test Runs

Additional design test runs were carried out for an extended road range and for more aggressive acceleration. Their results are presented in [Table 6](#). Every two rows of the table correspond to two good designs for the shown Run number. Run #1 and Run #2 correspond to an extended range of 480 km for a normal and plug-in HEV, respectively. Whereas Runs #3 and #4 are for a more aggressive acceleration from 0 to 60 mph in 10s instead of 15s as in the design runs presented above, also for normal and plug-in HEV, respectively. In the first two runs of the extended range more or less the same winning designs recur here but with larger hydrogen tanks of 7kg instead of 5kg as obtained in [Table 3](#). The more aggressive acceleration requires larger and thus heavier motors and larger power sources leading to heavier cars. The plug-in design as expected requires smaller hydrogen tanks.

The designs reported in [16] had a slightly lower range of 250 mi (402 km). Their FC-HEV had a battery size of 1 kWh, a fuel size of 78.9 kW, a motor size of 94.3 kW, and a car mass of 1582 kg. The size of the hydrogen tank is not given in the report. The second design ([Table 6](#), Run #1) being proposed in [Table 6](#) has values which are not very far. However, the FC-PHEV-20 in [16] had a battery size of 7.6 kWh, a fuel cell size of 58.8 kW, a motor size of 94.3 kW, and an overall car mass of 1591 kg. As can be seen ([Table 6](#), Run #2) the plug-in HEV designs shown have similar sizes for the battery but much smaller fuel cell sizes, and as a result the car mass is significantly lower. The higher motor size we have is to cater for the gradeability test, which was not reported in [16].

**Table 6. Additional Design Results on extended Range and Aggressive Acceleration**

Run #	H2 Tank kg	FC Size kW	Bat Size kWh	Mot Size kW	Total Cost \$/year	Car Mass kg
1	7	40	3	110	3820	1518
1	7	70	2	110	3929	1571
2	3	10	8	110	3756	1444
2	3	20	7	110	3757	1458
3	5	50	5	170	4204	1598
3	5	70	4	170	4465	1630
4	2	40	10	170	4273	1570
4	2	60	9	170	4349	1605

Special runs were also carried out to respond to questions that arose in the review process of this paper. The first run had the number of trips doubled to 1400; in this case the yearly distance travelled by the car would be 25,795 miles/ year instead of 12398 miles/ year that correspond to 700 trips per year. The best design would still be that of the Base Case (Run#1) shown in [Table 3](#) in Row 1 and the other good design, i.e. Row 2, still featured among the top-13 designs.

Understandably, the operating cost of the car essentially doubled bringing the total yearly cost to \$4319 for the best design. The second run is to address the question of why not a fuel cell car only instead of a hybrid fuel cell car involving some storage device, in this case a battery. The best design in this case was as follows: H<sub>2</sub>-tank= 7kg, FC-size= 120kW, and BT-size= 0.5kWh, with a total car mass of 1671kg. Clearly the effect of lowering the battery size caused an increase in the fuel cell size and correspondingly the H<sub>2</sub>-tank size and made the car significantly (11.4%) heavier. Reducing the battery sizes available for the design caused further increases in the size of the fuel cell, the H<sub>2</sub>-tank and the car mass, with a corresponding increase in the yearly cost.

### CONCLUSION

This paper presented an ordinal optimization (OO) approach for finding a good solution to an optimization problem in a large space. The OO in combination with the dynamic program offers an efficient approach to determine good enough designs of the FCHEV power units for the purpose of reducing hydrogen fuel consumption and lowering operation and investment cost. Dynamic programming simulates the optimal operation of the car for a set of specified sizes on the UDDS and HWFET as mixed driving cycles and provides the total vehicle cost per year. It was used in two modes to simulate the charge sustained operation of normal hybrid designs and charge depletion operation of plug-in hybrid designs. In addition range and acceleration and gradeability specifications are used to insure that the car meets usual road operation requirements. The OO program sets the sizes of the car components to sample the search space uniformly using a fast but crude model. Different test runs were done to study the effect of the components' prices on the selected sizes of the fuel cell, battery and hydrogen tank.

Several test designs were carried out based on different variations in the parameters. The best normal hybrid design that is suitable for urban and highway driving and with suitable acceleration from 0 to 60 mph in 15s and gradeability run of 5%, and a specified range of 320 km is with hydrogen tank of 5kg, fuel cell of 40kW, battery of 3kWh, and motor of 110 kW. The best plug-in design had the following component sizes in the same order: 2 kg, 10kW, 8 kWh, and 110 kW. The two designs were very robust to variations in parameters such as the cost of hydrogen, the cost of the fuel cell, and the cost of the battery. In some variation runs they did not appear at the top of the list but always showed up in the top-13 designs. Additional test runs were carried out where the distance per year was doubled, this increased the value of the operating cost versus the investment cost. But the best designs proved to be robust and did not change. The established methodology reinforces the belief, apparent in the work of many researchers, that a hybrid design involving some kind of storage is more economic than a fuel-cell design without battery. The presence of the battery allows dynamic braking to be done and causes a reduction in the fuel-cell size that may have otherwise to meet the gradeability test on its own with its low power transient times or ramp rates.

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**APPENDIX****APPENDIX A: NOMENCLATURE**

$\Delta t$	Time interval length between steps $t$ and $t-1$
$T_1$	Number of time steps in drive cycle 1
$J$	Cost function
$I_C(n)$	Investment cost of design $n$
$O_C(n)$	Operational cost of design $n$
$\varphi_{FC}(P_{FC})$	Cost rate of operating the fuel cell at a level $P_{FC}$ in \$/h
$\psi_{BT}(P_{BT})$	Internal energy rate of battery (kW)
$\gamma_{BT}$	Battery cost parameter in \$/ kWh
$M$	Number of trips per year (700)
$p_{FC}^{max}$	Size for the fuel cell power (kW)
$p_M^{max}$	Motor power rating (kW)
$p_{BT}^{max}$	Size for the battery power (kW)
$E_{BT}^{max}$	Size for the battery energy (kW)
$H_2^{max}$	Size for the hydrogen tank (kg)
$P_{Br}$	Brake power (kW)
$P_{FC}$	Fuel Cell power (kW)
$P_D$	Demand power (kW)
$P_{BT}$	Battery power (kW)
$P_M$	Motor power (kW)
$P_{dFC} R_{uFC}$	Fuel cell ramp down and up limits (kW/h)
$P_{dBT} R_{uBT}$	Battery ramp down and up limits (kW/h)
BT	Battery
FC	Fuel cell
SOC	Battery state of charge
$\alpha$	annuity factor
$r$	Interest rate (5%)
$l$	Depreciation or life time (15 years)