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Coordinating a three-level supply chain with delay in payments and a discounted interest rate ☆

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ABSTRACT

Both researchers and practitioners recognize the importance of the interactions between financial and inventory decisions in the development of cost effective supply chains. Moreover, achieving effective coordination among the supply chain players has become a pertinent research issue. This paper considers a three-level supply chain, consisting of a capital-constrained supplier, a retailer, and a financial intermediary (bank), coordinating their decisions to minimize the total supply chain costs. Specifically, we consider a retailer managing its cash through the supplier's bank, in return for permissible delay in payments from the supplier. The bank, benefiting from increasing its cash holdings with the retailer's cash deposits, offers the supplier a discount on its borrowing rate. We show that the proposed coordination mechanism achieves significant cost reduction, by up to 26.2%, when compared to the non-coordinated model. We also find that, with coordination, the retailer orders in larger quantities than its economic order quantity, and that a higher return on cash for the retailer leads to a higher order quantity. Furthermore, we empirically validate our proposed coordination mechanism, by showing that banks, retailers, and suppliers have much to gain through collaboration. Thus, using COMPUSTAT datasets for the years 1950 through 2012, we determine the most important factors that affect the behavior of the retailers and suppliers in granting and receiving trade credit. Our results indicate that engaging into such a coordination mechanism is a win–win situation to all parties involved.

1. Introduction

The operations management literature tends to focus on the areas of inventory control and supply chain management, without considering the effects of financial constraints on a firm's operating decisions. Researchers assume that while inventory is costly, there is sufficient working capital available for its financing. However, limited working capital is a frequent constraint in procurement decisions (Buzacott & Zhang, 2004; Kouvelis & Zhao, 2012). In the presence of liquidity constraints, firms are in need of short-term financing to execute their operational decisions. The separation between operational/inventory and financial decisions simplifies complex problems, but is inevitable for firms that are capital-constrained. Several recent works have incorporated financial considerations into inventory decisions (e.g. Buzacott & Zhang, 2004; Dada & Hu, 2008; Hu & Sobel, 2005; Lee & Rhee, 2010; Raghavan & Mishra, 2011).

Banks, in general, need cash money to supply the market with more loans and to buffer any possible future unexpected runs on a bank or sudden withdrawals. With the economy downturn, banks and supply chain players are seeking ways to squeeze inefficiencies out of their systems. One area that is ripe for reevaluation is financial supply chain management. Rather than dealing solely with the actual physical/logistical aspects of business, financial supply chain management spans the planning and execution of payments between the business entities of a supply chain, from the moment a purchase order is made to the time of settlement. This concept is gaining popularity in many business environments and among banks worldwide, that will need to come up with hundred of billions of dollars in capital to comply with new regulatory requirements concerning liquidity, which will take effect over the next eight years (Wall Street Journal, 2011). The new "liquidity coverage ratio" requires banks to hold enough cash, or assets that can be converted into cash in a timely manner, to cover their expected outflows. With the investors being reluctant to lend money, funding is gradually becoming the key concern (Todayonline bloomberg, 2011). This paper presents a coordination scheme between the bank and the parties in a supply chain (retailer and supplier), that will create a win–win situation to all three parties, the bank, the supplier and the retailer.

In this paper, we consider a three-level supply chain consisting of a supplier facing liquidity constraints, (following Dada & Hu, 2008; Kouvelis & Zhao, 2012; Lee & Rhee, 2010), a retailer, and a

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financial intermediary (bank), coordinating their decisions through delay in payments and discount on the borrowing rate as we explain next. The capital-constrained supplier finances its working capital from a bank and incurs positive financial costs. During each period, the retailer places an order from the supplier. In an attempt to increase its cash holdings, the bank teams up with the supplier and offers him a discounted borrowing rate, provided that the retailer manages its cash from daily operations through the supplier's bank. As an incentive, the supplier offers the retailer an interest-free permissible delay in payments, after which, the supplier charges interest on the retailer's outstanding balance. We analyze this supply chain model where all three players coordinate their decisions to minimize their local costs and that of the supply chain.

Supply chain players are interested in managing their cash efficiently, which entails holding an optimum, neither excessive nor deficient, cash levels. There is very limited research addressing the impact of a cash management on inventory control decisions, which is one important contribution of our model. Specifically, the retailer's optimal strategy is to accumulate funds in cash for a period of time, and whenever the cash level reaches an upper level *C*, make a lump-sum purchase of securities, incurring a transactions cost (Heyman, 1973). On the other hand, if the cash balance reaches a zero or is negative, then an amount of securities is liquidated. Thus, in addition to minimizing its inventory costs, the retailer's objective is to minimize the cash related costs (which include the opportunity cost of funds left in cash, and the transactions cost of transferring cash in and out of securities).

In this research work, we solve for the retailer's inventory and financial decisions (lot size, payment time, and maximum cash level to hold), the supplier's lot size decision, and the bank's financial decision (the discount on the supplier's borrowing rate), to minimize the total inventory and financial supply chain cost. This total cost consists of the summation of the retailer's and supplier's inventory and financial costs, and the bank's financial costs. Our results indicate that the coordination scheme we propose among the retailer, supplier, and bank, reduces the supply chain costs by up to 26.2%, when compared to the no-coordination case (when there is no discount on the interest rate).

Moreover, we empirically test the importance of the coordination mechanism proposed in this paper. Thus, we develop two empirical models to determine the most important factors that affect the behavior of retailers and suppliers in their decision to grant/use trade credit. Using a sample of 6888 and 6091 US firms, we test two models for the trade credit used and granted. Using COMPUSTAT datasets of US firms, we test two models for the trade credit used and granted. Our results suggest that banks, retailers and suppliers could gain by collaborating. On one hand, by charging the suppliers lower credit rate, banks receive higher deposits, which are usually used to issue more loans. On the other hand, suppliers could use the option of dealing with the bank as a price discrimination tool that leads to a better inventory management. Finally, firms that are granted trade credit by suppliers could enhance their creditworthiness with the bank, which will improve their access to short-term and long-term borrowing in the future.

The remainder of the paper is organized as follows: Section 2 provides a review of the related literature. Section 3 formulates the model: In Section 3.1, we formulate the retailer's cost under two cases: The first case is when the permissible delay period is smaller than the retailer's cycle time, and the second case is when it is larger. In Sections 3.2 and 3.3, the supplier's and bank cost functions are formulated respectively. Section 3.4 presents the non-linear program formulation of the problem. A solution procedure for solving the non-linear program and numerical analysis to study the sensitivity of the model parameters are presented in Section 4. The empirical models are developed in Section 5. A conclusion is presented in Section 6.

2. Literature review

There is very limited research addressing financial supply chain management, and substantial literature on supply chain management and cash management as an integrated area or as independent areas of research.

A few number of papers have attempted to incorporate financial considerations into production and inventory decisions; Buzacott and Zhang (2004) incorporate asset-based lending into production decisions. Dada and Hu (2008) consider a capital-constrained newsvendor problem in which the newsvendor orders less than the optimal quantity. Hu and Sobel (2005) examine the interdependence of a firm's capital structure and its operating decisions in a dynamic newsvendor setting. Raghavan and Mishra (2011) consider a retailer and a manufacturer both approaching a lender for loan. Lee and Rhee (2010) study the impact of financing costs on different supply chain contracts. Chao, Chen, and Wang (2008) consider a dynamic inventory control problem of a self-financing retailer. Xu and Birge (2004) analyze the interactions between a firm's production and financing decisions as a tradeoff between the taxes benefits of debt and financial distress costs. Other papers followed Xu and Birge (2006), Kallberg, White, and Ziemba (1982), Babich and Sobel (2004), and Gupta and Dutta (2011).

The literature on supply chain coordination is substantial. The early work of Haley and Higgins (1973) considers a lot-sizing problem faced by a retailer under trade credit financing. In their model, they jointly determine the optimal order quantity and optimal payment time of the retailer. A sequel of papers discussing inventory control and trade credit followed (Chapman, Ward, Cooper, & Page, 1984; Daellenbach, 1986). Goyal (1985) assumes that the interest rate for credit surplus is different than that of credit deficit. Aggarwal and Jaggi (1995) study the problem of determining the optimal order quantity of deteriorating items for a retailer under permissible delay in payments using the optimal control approach. Other papers that analyze the lot-sizing problem under permissible delay in payments include Chu, Chung, and Lan (1998). Recent work on stochastic inventory model with trade credit is by Gupta and Wang (2009). Other papers on supply chain coordination include Jaber and Osman (2006) who consider a centralized two-level supply chain model with permissible delay in payments. They show that with coordination, the retailer orders in large quantities than its economic order quantity, with savings to either both players or to the entire supply chain. Abad and Jaggi (2003) consider a supplier–retailer channel in which the end demand is price sensitive. Viswanathan and Piplani (2001) propose a supply chain model of coordinating supply chain inventories through the use of common replenishment epochs. Other works include those of Lu (1995), and Goyal (1995). Readers may refer to Jaber and Zolfaghari (2008) and Glock (2012) for a review.

The cash management literature is extensive. This literature has started with Baumol (1952) who, under simplified assumptions, study the cash balance problem using inventory theory and derive an optimal cash balance level which is equivalent to the optimal lot size formula in inventory control. Tobin (1956) presents a model that balances the yield of investing cash in securities versus the cost of transactions. Robichek, Teichroew, and Jones (1965) formulate the cash balance problem as a linear program model to determine the optimal way for a firm to meet its cash needs. Eppen and Fama (1971) study a stochastic cash balance problem, in which the objective is to minimize cash holding and penalty costs. Heyman (1973) further assumes that the cash balance process changes according to random events. A comparison of our paper with the literature is provided in Table 1.

3. The mathematical model

We consider a retailer who orders Q units from a capital-constrained supplier, at a unit cost of c_r and ordering cost A_r. The retailer deposits its cash from sales in the supplier's bank, in return for an interest-free permissible delay in payments from the supplier. To pay the supplier, the retailer withdraws cash from its bank account, provided that the cash balance is positive. Following Miller and Orr (1968) and Haley and Higgins (1973), the retailer's optimal strategy is to accumulate funds in cash for a time, and whenever the cash level reaches an upper level C, make a lump-sum purchase of securities, incurring a transactions cost of purchasing securities (Heyman, 1973). Thus, the retailer enjoys a trade credit surplus due to the sales proceeds that accumulate in the bank up to level C. Each unit of positive cash incurs a holding cost, which is the opportunity cost of funds left as cash, since cash can be used to pay out dividends or as investments in earning assets. On the other hand, if the cash balance becomes zero or negative, i.e. there is a credit deficit, the retailer borrows a loan w_r from the bank at cost α_r. To proceed, we use the following notation:

Input parameters:
i = Subscript indicating the supply chain level, $i = r$ for the retailer and $i = s$ for the supplier.
D = Demand rate in units per unit time.
A_i = Order cost in dollars per period of level i.
h_i = Holding cost in dollars per unit time at level i excluding storage cost and financing charges.[a]
s_i = Storage cost in dollars per unit time at level i.
c_i = Cost in dollars per unit for level i.
p = Retailer's selling price in dollars per unit.
$r\%$ = Return on the retailer's invested cash.
$r_s\%$ = Interest charged by the supplier.
$\alpha_i\%$ = Cost of borrowing in dollars for player i.
b = Transactions cost in dollars of purchasing securities (in dollars per transaction).
w_i = Loan amount in dollars of player i.
M = Interest-free permissible delay in payment in unit time (trade credit period).

Decision variables:
α_d = The discounted borrowing rate the bank offers the supplier under coordination.
$\hat{t}$ = Retailer's payment time in unit time.
Q = Retailer's order quantity in units.
C = Maximum level of cash in dollars the retailer keeps in its bank account.
λ = Supplier's lot-size multiplier of the retailer's order quantity Q.

Consequential variables:
$t = cQ/pD$ = Time by which the retailer accumulates in its bank account the cost of goods, cQ, from sales.
T_i = Inventory cycle time for player i, $T_r = Q/D$ and $T_s = \lambda Q/D$.

[a] The holding cost is an aggregate cost that includes loss (due to damage, pilferage, and obsolescence), handling (special packaging, refrigeration, putting on pallets, and all other movements of the product), administration (store checks, computer updates, etc.), insurance and taxes.

3.1. Retailer's cost

The retailer's total cost consists of the following cost expressions: Purchasing and ordering cost, storage cost, holding cost, interest paid to the supplier, transactions cost of securities, opportunity cost of cash, and cost of borrowing. In addition, the retailer can invest the surplus funds at a return rate r and earn interest, resulting in a capital gain per cycle (a negative cost), which will be deducted from the retailer's total cost function. The purchasing and ordering cost function per cycle is simply $c_rQ + A_r$, and the storage cost per cycle is $(s_rQ/2)T_r = s_rQ^2/2D$. To compute the other cost expressions, we consider two cases: (1) $M < T_r, t > M$ and (2) $M \geqslant T_r$. Under (1), when $M < T_r, t > M$, (i.e. the time when the retailer has enough cash from sales to pay the supplier is larger than the credit period), the retailer needs to find ways to finance its inventory. Two options can be pursued: Either the retailer pays the supplier at the end of the trade credit period (i.e. $\hat{t} = M$), in which case it uses a short-term loan from the bank, or the retailer pays the supplier as soon as cash collected from sales covers the cost of goods (i.e. $\hat{t} = t$), incurring a penalty for the supplier for late payment. Thus, we subdivide this case into two subcases, case (1a) $M = \hat{t} < T_r$ and $t > M$ (borrowing at interest) and case (1b) $M \leqslant T_r, t > M$, and $\hat{t} = t$ (penalty is incurred). We note that, following Jaber and Osman (2006), we assume the total holding cost to be a composite of two cost components, one associated with opportunity cost of capital, while the other is the cost of stocking one unit of an item.

Case 1a. $M = \hat{t} < T_r$ and $t > M$.

In this case, the retailer's optimal strategy is to pay the supplier right at the end of the credit period, and before accumulating cQ in its bank account, i.e. $\hat{t} = M$, with $\hat{t} < t$. To finance the inventory investment, the retailer borrows an amount w_r at time $\hat{t}$ at interest α_r. Thus, the borrowing cost per unit time is obtained as

$$\textit{Borrowing cost} = \alpha_r w_r \frac{T_r - \hat{t}}{T_r} = \alpha_r w_r \frac{Q - D\hat{t}}{Q}. \quad (1)$$

The cash collected from sales in $(0, \hat{t})$ is deposited in the bank. Whenever the cash level exceeds the maximum cash level, C, it is transferred into securities, incurring a cost per transaction b of going into securities. The transactions cost of securities investment per cycle is $b(\text{Dollar Cash per cycle}/C) = b(pD/C)$. Per unit time, this transactions cost expression becomes

$$\textit{Transactions cost of securities investment} = b(pD/C)(\hat{t}/T_r)$$
$$= bpD^2\hat{t}/CQ. \quad (2)$$

The opportunity cost of funds left in cash can be computed per unit time as follows:

$$\textit{Opportunity cost of funds} = r * (C/2) * \hat{t}/T_r = rCD\hat{t}/2Q. \quad (3)$$

Referring to Fig. 1, there is a capital gain on the side of the retailer as he/she can invest the outstanding balance at return r. Right before paying the supplier at time $\hat{t}$, the average amount of cash is $(1/2)pDM$, which gains a capital gain of $(1/2)pDM(e^{rM} - 1)e^{r(T_r - M)}$. This cash level drops to zero, and cash is again being accumulated until T_r, with average cash equal to $(1/2)pD(T_r - M)$, for which the gain is $(1/2)pD(T_r - M)(e^{r(T_r - M)} - 1)$. Thus, the capital gain per cycle is:

$$\textit{Capital gain} = \frac{1}{2}pDM(e^{rM} - 1)e^{r(T_r - M)} + \frac{1}{2}[pD(T_r - M)](e^{r(T_r - M)} - 1)$$
$$= \frac{1}{2}pDM(e^{rT_r} - e^{r(T_r - M)}) + \frac{1}{2}[pD(T_r - M)](e^{r(T_r - M)} - 1). \quad (4)$$

Since the yearly interest has small value and is compounded daily or weekly, it is reasonable to assume that $e^{rt} - 1 \simeq rt$ (using the Taylor series first-order approximation). Thus, Eq. (4) can be written as

$$\frac{1}{2}rpDM^2 + \frac{1}{2}rpD(T_r - M)^2. \quad (5)$$

The derivation of the holding cost is provided in Appendix A. Adding up the purchasing and ordering cost, storage cost, borrowing cost in

Table 1
Comparison with the literature.

	Inventory management	Financial decisions	Supply chain coordination	Cash management
Abad and Jaggi (2003)	x		x	
Aggarwal and Jaggi (1995)	x			
Babich and Sobel (2004)	x	x		
Baumol (1952)				x
Buzacott and Zhang (2004)	x	x		
Chao et al. (2008)	x	x		
Chapman et al. (1984)	x	x		
Chu et al. (1998)	x			
Dada and Hu (2008)	x	x		
Daellenbach (1986)	x	x		
Eppen and Fama (1969, 1971)				x
Girgis (1968)				x
Goyal (1985)	x	x		
Gupta and Dutta (2011)	x	x		
Haley and Higgins (1973)	x	x		
Heyman (1973)				x
Hu and Sobel (2005)	x	x		
Jaber and Osman (2006)	x		x	
Kallberg et al. (1982)				x
Kouvelis and Zhao (2012)	x	x		
Lee and Rhee (2010)	x	x		
Lu (1995)	x		x	
Raghavan and Mishra (2011)	x	x		
Robichek et al. (1965)				x
Tobin (1956)				x
Viswanathan and Piplani (2001)	x		x	
Xu and Birge (2004, 2006)	x	x		
This paper	x	x	x	x

(1), transactions cost in (2), opportunity cost in (3), less the capital gain in (5), the total cost per unit time is

$$C_r^{1a}(Q,\hat{t},C) = c_rD + A_rD/Q + s_rQ/2 + h_r(Q-D\hat{t})^2/2Q + bpD^2\hat{t}/CQ + rCD\hat{t}/2Q + \alpha_r w_r \frac{Q-D\hat{t}}{Q} - \left[\frac{1}{2}rpDM^2 + \frac{1}{2}rpD(Q/D-M)^2\right]\frac{D}{Q}. \quad (6)$$

Case 1b. $M \leqslant T_r,\ t > M$, and $\hat{t} = t$.

In this case, the retailer settles its account with the supplier as soon as cash collected from sales covers the cost of goods purchased from the supplier, i.e. $\hat{t} = t$. The retailer incurs a penalty equal to $r_s cQ(\hat{t}-M)$, for paying beyond the credit period. This is the interest charged by the supplier on the outstanding balance over the period $\hat{t}-M$. Fig. 2 illustrates the behavior of the cash inventory level over time. The capital gain per cycle is computed as

$$\frac{1}{2}c_rQ(e^{rt}-1)e^{r(T_r-t)} + \frac{1}{2}pD(T_r-t)(e^{r(T_r-t)}-1) - c_rQ(e^{r_s(t-M)}-1)$$
$$= \frac{1}{2}c_rQ(e^{rT_r}-e^{r(T_r-t)}) + \frac{1}{2}pD(T_r-t)(e^{r(T_r-t)}-1) - c_rQ(e^{r_s(t-M)}-1) \quad (7)$$

The capital gain is to be deducted from the retailer's total cost function and can further be written as

$$\frac{1}{2}rc_rQt + \frac{1}{2}rpD(T_r-t)^2 - r_sc_rQ(t-M) \text{ or}$$
$$\frac{1}{2}rpDt^2 + \frac{1}{2}rpD(T_r-t)^2 - r_sc_rQ(t-M).$$

The other cost expressions are similar to Case 1a. We get

$$C_r^{1b}(Q,\hat{t},C) = c_rD + A_rD/Q + s_rQ/2 + h_r * (Q-D\hat{t})^2/2Q + bpD^2\hat{t}/CQ + rCD\hat{t}/2Q - \left[\frac{1}{2}rc_rQt + \frac{1}{2}rpD(Q/D-t)^2 - r_sc_rQ(t-M)\right] * \frac{D}{Q}. \quad (8)$$

Case 2. $T_r \leqslant M \leqslant \hat{t}$.

The supplier offers the retailer a credit period that is larger than the retailer's cycle time. The retailer will settle its account at time $\hat{t} \geqslant M$ when enough cash is collected to pay the supplier. Up to $\hat{t}$, this money can be invested at rate r, generating a capital gain which is computed by referring to Fig. 3. Beyond M, a penalty is

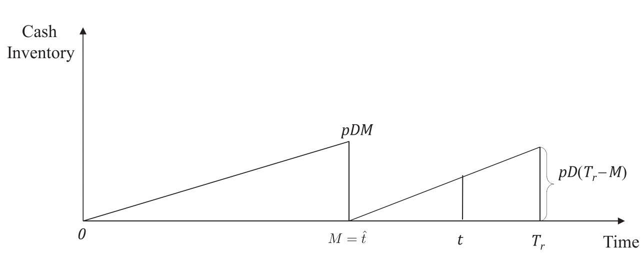

Fig. 1. Behavior of the cash inventory level when $M = \hat{t} < T_r$ and $t > M$ (Case 1a).

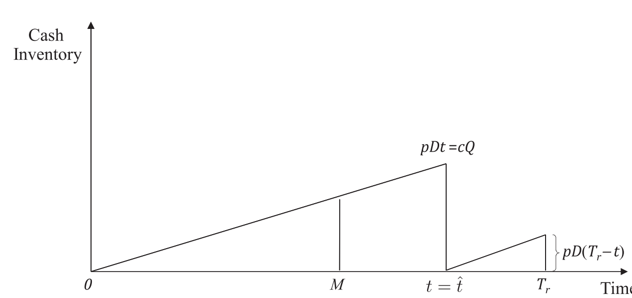

Fig. 2. Behavior of the cash inventory when $M \leqslant T_r$, $t > M$, and $\hat{t} = t$ (Case 1b).

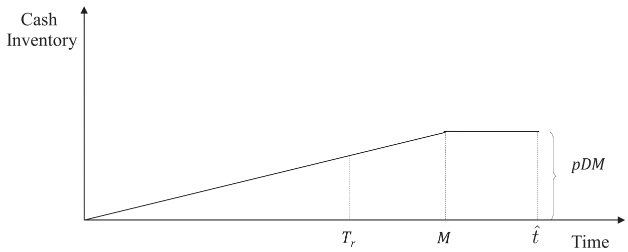

Fig. 3. Behavior of the cash inventory level when $0 \leqslant T_r \leqslant M \leqslant \hat{t}$ (Case 2).

charged by the supplier at rate r_s. Thus, the capital gain per unit time is $1/2prDT_r^2 - r_s c_r Q(\hat{t} - M)$. At time $\hat{t}$, the holding cost is zero. As a result, the retailer's cost function can be written as

$$C_r^2(Q, \hat{t}, C) = c_r D + A_r D/Q + s_r Q/2 + bpD^2\hat{t}/CQ + rCD\hat{t}/2Q - [\frac{1}{2}prDT^2 - r_s c_r Q(\hat{t} - M)] * \frac{D}{Q}. \quad (9)$$

To summarize, the retailer's cost per unit time $C_r(Q, \hat{t}, C)$ is as follows:

$$C_r(Q, \hat{t}, C) = \left\{ \begin{array}{ll} C_r^{1a}(Q, \hat{t}, C), & \text{if } M \leqslant T_r, M = \hat{t}, \text{ and } t > M, \\ C_r^{1b}(Q, \hat{t}, C), & \text{if } 0 \leqslant M \leqslant T_r, \ t > M, \text{ and } \hat{t} = t, \\ C_r^2(Q, \hat{t}, C), & \text{if } T_r \leqslant M \leqslant \hat{t}. \end{array} \right\} \quad (10)$$

3.2. Supplier's cost

The supplier delivers to the retailer shipments of size Q units every cycle. We assume that the supplier replenishes his inventory from an outside source with ample capacity. Unlike the lot-for-lot policy adopted in Abad and Jaggi (2003), which is too restrictive in nature, as discussed in the literature (e.g. Goyal, 1988), we assume that the supplier replenishes its inventory with λQ units every $T_s = \lambda Q/D$ units of time, $\lambda = 1, 2, 3, \ldots$. Once it does that, the supplier instantaneously delivers to the retailer the first shipment and thereafter every $T_r = Q/D$ units of time.

The supplier's unit cost is c_s, such that $c_s < c_r$ and order cost A_s. The first shipment of size Q is delivered to the retailer after $T_r = Q/D$. Referring to Fig. 4, the average inventory per supplier's cycle is $(\lambda - 1)QT_r + (\lambda - 2)QT_r + \cdots + QT_r + 0 = \sum_{i=1}^{\lambda}(i-1)QT_r$. For each unit in inventory, the supplier incurs two costs, h_s and s_s. Thus, the supplier's holding cost is $(h_s + s_s)\sum_{i=1}^{\lambda}(i-1)QT_r = (h_s + s_s)\lambda(\lambda - 1)QT_r/2 = (h_s + s_s)\lambda(\lambda - 1)Q^2/2D$. The supplier offers the retailer an incentive to settle its account by M units of time after delivering the shipment, thus, losing the opportunity to invest its profit per order over the permissible delay period, $(c_r - c_s)\lambda Q e^{r_s M}$, under the assumption that the supplier settles its procurement cost, $c_s \lambda Q$, at the beginning of each supplier's cycle. If the retailer settles its account with the supplier by time $\hat{t} > M$, then the supplier will charge the retailer interest r_s for a $\hat{t} - M$ period. Since the retailer settles its balance by time $\hat{t}$, the supplier will not collect the revenue for such shipment before $\hat{t}$. Then, the supplier incurs a cost of capital which is equal to $h_s \lambda Q \hat{t}$, for each shipment of size Q. In addition, the supplier obtains a loan w_s at a discounted rate α_d.

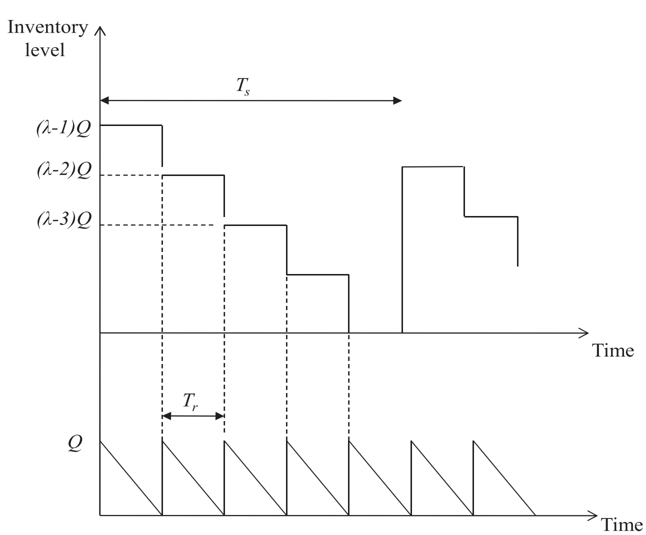

Fig. 4. Behavior of the inventory level for the supplier and retailer for $\lambda = 5$.

The supplier's cost per cycle is the sum of all costs less the savings in a cycle, and can be written as:

$$c_s \lambda Q + A_s + \frac{(h_s + s_s)}{2D} Q^2 \lambda(\lambda - 1) + h_s \hat{t} \lambda Q + (c_r - c_s) Q \lambda e^{r_s M} - c_r \lambda Q e^{r_s(\hat{t} - M)} + \alpha_d w_s.$$

Dividing the above expression by the supplier's cycle time, $T_s = \lambda Q/D$, the supplier's unit cost per unit time is given as

$$C_s(Q, \lambda, \alpha_d, \hat{t}) = c_s D + \frac{A_s D}{\lambda Q} + \frac{(h_s + s_s)}{2} Q(\lambda - 1) + h_s \hat{t} D + (c_r - c_s) D e^{r_s M} - c_r D e^{r_s(\hat{t} - M)} + \frac{\alpha_d w_s D}{\lambda Q}.$$

Theorem 1.

(i) *For a sufficiently large retailer's holding cost, $h_r > rp$, and an upper limit on the retailer's loan, specifically, $w_r \leqslant DM(h_r/2 - rp)/\alpha_r + (rC)/(2\alpha_r)$, the cost function $C_r^{1a}(Q, \hat{t}, C)$ in (6) is convex with respect to the order quantity Q.*

(ii) *For a sufficiently large retailer's holding cost, $h_r > rp$, the cost function $C_r^{1b}(Q, \hat{t}, C)$ in (8) is convex with respect to the order quantity Q.*

(iii) *The cost function $C_r^2(Q, \hat{t}, C)$ in (9) is convex with respect to the order quantity Q.*

(iv) *For $\lambda > 0$, the supplier's cost function $C_s(Q, \lambda, \alpha_d, \hat{t})$ in (11) is convex with respect to the order quantity Q and the lot multiplier λ. Furthermore, when $\lambda = 1$, this function is convex decreasing in Q.*

Proof. See Appendix B. □

3.3. The bank's problem

We next derive an expression for the discounted rate α_d. Whenever the retailer deposits cash into the bank, the latter gets a return on the cash deposits, $(\alpha_s - r)$, which is greater or equal to the cost of offering the supplier a discount on the supplier's loan, $\alpha_s - \alpha_d$. Thus, we have

$$(\alpha_s - r) * \text{retailer's average cash inventory} \geqslant (\alpha_s - \alpha_d) M w_s, \quad \text{for } M > 0. \quad (12)$$

To determine the retailer's cash inventory, we consider the three cases discussed in SubSection 3.1. In Case 1a, the retailer's average cash inventory is $1/2pDM^2 + 1/2pD(T_r - M)^2$. After some term arrangement, (12) becomes

$$\alpha_d \geqslant \alpha_s - \frac{(\alpha_s - r)\left(pD\left[M^2 + (T_r - M)^2\right]\right)}{2Mw_s} \text{ (Case 1a)}, \tag{13}$$

Similarly, replacing the retailer's cash inventory in each of the other two cases by its expression, (12) becomes

$$\alpha_d \geqslant \alpha_s - \frac{(\alpha_s - r)\left(cQt(M)/2 + pD(T_r - t(M))^2/2\right)}{Mw_s} \text{ (Case 1b)}, \tag{14}$$

$$\alpha_d \geqslant \alpha_s - \frac{pDM(\alpha_s - r)}{2w_s} \text{ (Case 2)}. \tag{15}$$

The conditions derived in Eq. (13)–(15) show the dependence of the discount rate on M. Suppose that the bank funds the loan using money received from depositors, at rate α'. Then the bank's cost function per unit time is given as

$$C_b(Q, \lambda, \alpha_d) = \frac{(\alpha_s - \alpha_d)}{\lambda Q} w_s D + \frac{w_r \alpha' D}{Q} + \frac{w_s \alpha' D}{\lambda Q}, \tag{16}$$

where α_d is given in (13)–(15).

3.4. The coordinated supply chain model

With coordination, the retailer, supplier, and the bank need to agree on the following decision variables $Q, \lambda, \hat{t}, C$, and α_d, that minimize the total supply chain cost, which is the sum of the retailer's, supplier's, and bank's costs. To find these decisions, we need to solve the following mathematical programming model

$$\text{Min } C_{\text{chain}}(Q, \hat{t}, C, \lambda, \alpha_d) = C_r(Q, \hat{t}, C) + C_s(Q, \lambda, \alpha_d, \hat{t}) + C_b(Q, \lambda, \alpha_d) \tag{17}$$

$$\text{s.t. } M < T_r,\ \hat{t} = M, \text{ and } t > M \text{ (Case 1a)}; \tag{18}$$

$$M \leqslant T_r; \hat{t} > M, \text{ and } t = \hat{t} \text{ (Case 1b)}; \tag{19}$$

$$T_r \leqslant M \leqslant \hat{t} \text{ (Case 2)}; \tag{20}$$

$$\text{Eq. (13)–(15)}; \tag{21}$$

$$\text{and } Q \geqslant 1. \tag{22}$$

Corollary 1. *For $h_r > rp$ and $w_r \leqslant DM(h_r - rp)/\alpha_r + (rC)/(2\alpha_r)$, the total supply chain's cost function in (17) is convex in Q.*

Proof. This follows from Theorem 1 and the cost in (17) being the summation of convex functions. □

Lemma 1. *For a given order quantity, as M increases, the total supply chain cost:*

(i) increases (Cases 1b and 2),
(ii) decreases up to $Q/2D + Qr_s\lambda(2c_r - c_s)/2$ and increases afterwards (Case 1a).

Proof. First, we consider Case 1a. Taking the derivative w.r.t. M, we get

$$\frac{\partial C_{\text{chain}}(Q, \hat{t}, C, \lambda, \alpha_d)}{\partial M} = -2DM/Q + 1 + (c_r - c_s)\lambda D r_s + c_r \lambda D r_s = 0.$$

The result follows. For Cases 1b and 2, taking the derivative w.r.t. M, we get $C_r r_s(\lambda - 1)D$ which is $\geqslant 0$. Thus, the total cost function is increasing in M. □

Although Lemma 1 is intuitive in Cases 1b and 2, it is not in Case 1a, when the retailer borrows from the bank. Lemma 1 implies that, when the retailer decides to borrow to fund its inventory investment, there is a unique value M^* that minimizes the total supply chain cost. The supplier needs to set $M = M^*$ to minimize the chain cost. For any other value $M < M^*$ or $M > M^*$, the supply chain cost will be higher. This is counter-intuitive, as one would expect, just as in the other cases, that the supply chain cost would increase as M increases.

4. Numerical analysis

The following solution procedure is used to solve the mathematical program in (17)–(22).

- Step 1. We set $\lambda = 1$, and consider separately each of the three cases (Case 1a, Case 1b, and Case 2), under which the retailer's cost is developed. For each case, we solve the corresponding non-linear programming problem. We get the optimal decisions and supply chain cost as follows

$$C^*_{1a}(Q^*_{1a}, \hat{t}^*_{1a}, C^*_{1a}, \alpha_d^{1a^*}, 1) = \min\{C_r^{1a}(Q_{1a}, \hat{t}_{1a}, C_{1a}) + C_s(Q, 1, \alpha_d^{1a}, \hat{t}) + C_b(Q^*_{1a}, 1, \alpha_d^{1a})\}$$

$$C^*_{1b}(Q^*_{1b}, \hat{t}^*_{1b}, C^*_{1b}, \alpha_d^{1b^*}, 1) = \min\{C_r^{1b}(Q_{1b}, \hat{t}_{1b}, C_{1b}) + C_s(Q, 1, \alpha_d^{1b}, \hat{t}) + C_b(Q^*_{1b}, 1, \alpha_d^{1b})\}$$

$$C^*_2(Q^*_2, \hat{t}^*_2, C^*_2, \alpha_d^{2^*}, 1) = \min\{C_r^2(Q_2, \hat{t}_2, C_2) + C_s(Q, 1, \alpha_d^2, \hat{t}) + C_b(Q^*_2, 1, \alpha_d^2)\}$$

- Step 2. Step 1 is repeated for subsequent values of λ, i.e for $\lambda = 1, 2, 3, \ldots$. Following Theorem 1, the supply chain cost is convex w.r.t. λ, so there exists a unique value λ^*, for each case, that minimizes the total supply chain cost. We get an optimal value of λ, along with the optimal value of the decision variables. Let these optimal values of the lot multiplier be λ^*_{1a}, λ^*_{1b}, and λ^*_2.
- Step 3. We compare the optimal costs for the three cases. The optimal solution for our problem corresponds to the lowest cost, as follows

$$C^*_{chain}(Q^*, \hat{t}^*, C^*, \lambda^*, \alpha^*_d) = \min\{C^*_{1a}(Q^*_{1a}, \hat{t}^*_{1a}, C^*_{1a}, \alpha_d^{1a^*}, \lambda^*_{1a}), C^*_{1b}(Q^*_{1b}, \hat{t}^*_{1b}, C^*_{1b}, \alpha_d^{1b^*}, \lambda^*_{1b}), C^*_2(Q^*_2, \hat{t}^*_2, C^*_2, \alpha_d^{2^*}, \lambda^*_2)\}.$$

To illustrate the significance of our model, we compare it to the non-coordinated decentralized model. Thus, we set $\alpha_d = \alpha_s$, i.e. the bank offers no discount on the supplier's loan rate. Each of the supplier and retailer makes its own optimal decisions independently by minimizing its own cost function. Since the retailer's problem features three cases, we get three optimal cost expressions for the retailer and choose the smallest one as the optimal retailer's cost. The optimal supply chain cost is the summation of the

$$\%\ \text{Savings} = \frac{\text{Optimal cost of the non-coordinated supply chain} - \text{Optimal cost of the coordinated chain}}{\text{Optimal cost of the non-coordinated supply chain}}.$$

optimal supplier, retailer and bank costs. To compare both models, we compute the percentage savings as follows:

Next, we use the solution procedure above to numerically solve our problem (17)–(22). We use the following parameter values (our base case) adopted from Jaber and Osman (2006): $D = 150,000$ units/year, $c_r = \$20$, $c_s = \$15$, $p = \$25$, $A_r = \$30$, $A_s = \$200$, $h_r = h_s = \$6$, $s_r = s_s = \$3$, $r = 10\%$, $r_s = 18\%$, $b = \$3$, $\alpha_s = 15\%$, $\alpha_r = 17\%$, $w_s = \$80,000$, and $w_r = \$40,000$.

We numerically investigate the performance of the model developed in Section 3. Table 2 summarizes the results. The numerical results in the first row of Table 2 correspond to the parameters just defined above and $M = 5$. The other three rows are for M = 10, 15, and 20. Then, we change the retailer and supplier holding and storage costs to $h_r = 3, s_r = 6, h_s = 6$, and $s_s = 3$, as indicated in the leftmost column in Table 2, keeping the other parameters constant at their base case values. We again perform the computations for different values of M, so the first row is for $M = 5$, then $M = 10$, etc. Next, we consider $h_r = h_s = 3$ and $s_r = s_s = 6$, the other parameters equal to their base case values. Lastly, we only vary the retailer's return on invested cash r from 0% to 5% from the base case values. We report the optimal decisions and supply chain cost, C_{chain}, and the optimal retailer's and supplier's costs, C_r and C_s. The optimal decisions and costs are computed for the coordinated and non-coordinated models. The values obtained in the rightmost column (savings) indicate that the proposed coordination model achieves savings that go up to 26.2% for the chosen parameters.

As a benchmark, we numerically solved the problem for the case when there is no delay in payments ($M = 0$) and no discounted rate. That is, the retailer orders its economic order quantity that minimizes its inventory and financial costs. Then, the supplier determines the optimal value of the multiplier λ that minimizes its own cost as well. For the base case, the retailer's economic order quantity is 1176.7 units and $\lambda = 2$. Comparing this value to the optimal values for Q of the base case in Table 2, we find that it is smaller, for any positive value of M, which indicates that coordination increases the optimal lot size. When we increase the retailer's holding cost to $h_r = 12$ and $s_r = 3$ (so the total holding cost, which is the summation of the opportunity cost of capital h_r and the storage cost s_r, is larger), we find that the retailer's economic order quantity is 848.5 units and $\lambda = 3$, compared to ordering 2568.4 units when $M = 5$ under coordination. Moreover, when we vary r to 0 and 5%, we find that the retailer's economic order quantity is 1000 and 1077.6 units respectively, which is smaller than the optimal quantity under coordination, as presented in Table 2. These figures indicate that coordination increases the lot size. The results corroborate those found in the literature of ordering in larger lots than the EOQ when delay in payments is exercised as a coordination mechanism among the players in a supply chain. Ordering in larger lots less frequently makes a supply chain less busy than ordering in smaller lots more frequently. It has economical, societal and environmental advantages as transportation, energy and emissions costs are minimized. Furthermore, adopting large lot policy reduces the hidden costs associated with inventory systems making it cheaper too control the flow of commodity from a production-inventory system to the market (Jaber, 2009, Chapter 9). Moreover, the large lots ordering in the banking service is contrary to the production industry due to relatively low ordering, transportation and storage of financial capital.

From Table 2, a few important observations can be made. First, our numerical analysis shows that coordination among the bank, retailer, and supplier, results in savings for the entire supply chain as high as 26.2%. Second, a higher return on cash for the retailer leads to a higher order quantity.

- All the numerical examples in Table 2 indicate that it is advantageous for the retailer, supplier, and bank to accept the coordination mechanism proposed in our model, as it results in a lower total cost compared with the no coordination case. We also notice that, as the length of the credit period increases, the bank offers a larger discount on the supplier's loan, and the supply chain savings increase as well.
- The supply chain cost is the lowest when h_r/s_r and h_s/s_s have the smallest values, i.e., when $h_r = h_s = 3$ and $s_r = s_s = 6$, which implies that $h_r/s_r = h_s/s_s = 1/2$ in our examples. Comparing the cases when $(h_r, s_r) = (3, 6)$ and $(h_s, s_s) = (6, 3)$ (i.e. $h_r/s_r = 1/2$ and $h_s/s_s = 2$), to the case when $(h_r, s_r) = (3, 6)$ and $(h_s, s_s) = (3, 6)$ (i.e. $h_r/s_r = h_s/s_s = 1/2$), we notice that the retailer's cost is the same in both cases (and its ordering decision is

Table 2
Comparing the coordinated and non-coordinated models for the following parameters (base case): $D = 150,000$ units/year, $c_r = \$20$, $c_s = \$15$, $p = \$25$, $A_r = \$30$, $A_s = \$200$, $h_r = h_s = \$6$, $s_r = s_s = \$3$, $r = 10\%$, $r_s = 18\%$, $b = \$3$, $\alpha_s = 15\%$, $\alpha_r = 17\%$, $w_s = \$80,000$, and $w_r = \$40,000$.

Case		M	Coordinated Supply chain							Non-coordinated decentralized supply chain						Savings %
			λ	Q	α_d	$\hat{t}$	C_{chain}	C_s	C_r	λ	Q	$\hat{t}$	C_{chain}	C_s	C_r	
Base		5	1	2813.9	0.1317	5	87.9	68.1	19.8	1	2568.4	5	91.1	70.9	17.2	3.5
		10	1	4109.5	0.1178	10	107.6	97.7	9.9	1	4109.5	10	114.9	97.7	10.1	6.3
		15	1	6164.3	0.1018	15	140.3	129.9	10.3	1	7705.4	15	161.1	127.3	24.9	12.9
		20	1	8219.1	0.0857	20	176.7	165.5	11.2	1	10273.9	20	205.3	163.5	29.9	13.9
$h_r, s_r = 3, 6$		5	1	2568.4	0.1329	5	98.2	70.9	27.4	1	2568.4	5	101.2	70.9	27.4	2.9
$h_s, s_s = 6, 3$		10	1	4109.5	0.1178	10	124.5	97.7	26.8	1	5136.9	10	140.2	93.7	40.5	11.2
		15	1	5000.3	0.1018	15	164.5	133.0	31.5	1	7705.4	15	191.5	127.3	55.3	14.1
		20	1	5173.3	0.0857	20	205.1	171.4	33.7	1	10273.9	20	245.9	163.5	70.4	16.6
$h_r, s_r = 3, 6$		5	1	2568.4	0.1329	5	81.3	54.0	27.4	1	2568.4	5	84.3	54.0	27.4	3.5
$h_s, s_s = 3, 6$		10	1	4109.5	0.1178	10	90.8	63.9	26.8	1	5136.9	10	106.4	59.9	40.5	14.7
		15	1	5000.3	0.1018	15	113.9	82.4	31.5	1	7705.4	15	140.8	76.6	55.3	19.1
		20	1	5173.3	0.0857	20	137.5	103.8	33.7	1	10273.9	20	178.3	95.9	70.4	22.9
r	0	5	1	2726.7	0.0966	5	90.9	69.0	21.9	1	2568.4	5	99.7	70.9	19.9	8.8
		10	1	4109.5	0.0536	10	117.6	97.7	19.9	1	5136.9	10	140.6	93.7	28.9	16.3
		15	1	5035.9	0.0055	15	156.1	132.9	23.1	1	7705.4	15	193.7	127.3	39.5	19.4
		20	2	1285.5	0.0055	15	184.4	189.8	−5.4	1	10273.9	20	249.9	163.5	50.5	26.2
	5	5	1	2787.4	0.1138	5	89.9	68.3	21.6	1	3082.1	6	95.9	68.2	20.5	6.2
		10	1	4109.5	0.0857	10	113.5	97.7	15.8	1	5136.9	10	130.9	93.7	25.3	13.3
		15	1	6164.3	0.0536	15	149.6	129.9	19.7	1	7705.4	15	178.1	127.3	32.9	16.0
		20	1	7028.0	0.0215	20	189.2	167.2	22.0	1	10273.9	20	228.3	163.5	40.9	17.1

the same) for any value of M, whereas the supplier's cost decreases as h_s/s_s decreases, which decreases the total supply chain cost.

- A higher return on the retailer's cash (r increases from 0% to 5% in Table 2) leads to a higher retailer's order quantity. The return on cash is computed as the risk-free rate plus some additional percentage. The risk-free rate is a proxy for the growth in the economy, so a higher return on cash implies a boom in the economy due to a higher risk-free rate. Consequently, we expect retailers to sell more and order in larger quantities.

Finally, the maximum level C that the retailer should keep in its account can be found by minimizing the retailer's cost function with respect to C. Thus, $C = \$15,000$ for the base case parameters. As we increase (decrease) r, the maximum cash level decreases (increases). For example, when $r = 12\%$, C reduces to \$13693.1 On the other hand, reducing r to 5% increases the cash level to \$21,213. This indicates that it is optimal for the retailer to invest more (less) cash into securities, thus reducing (increasing) the cash level, as the return on investment becomes higher (lower).

Next, we provide sensitivity analysis in Table 3 of the optimal solution with respect to changes in some of the important model parameters. In this analysis, we set $M = 5$. Inline with findings in the literature, the optimal order quantity increases as the retailer's order cost A_r increases, which increases the optimal supply chain's and retailer's costs. Large order quantities imply large cash deposits in the bank, so the bank offers the supplier a larger discount on the supplier's borrowing rate, so α_d decreases, which decreases the supplier's cost. Similarly, as the supplier's order cost A_s increases, the optimal order quantity that minimizes the total supply chain cost increases, along with the retailer's and supplier's costs. The bank offers the supplier higher discounts on the loan, as a result of the coordination scheme, in return for higher cash deposits by the retailer. We interpret the changes with respect to the retailer's unit cost c_r by comparing it to the unit price p, which is equal to 20. When $c_r = 20 < p = 25$, the optimal order quantity is 2813.9 units, which increases as $c_r = p = 25$, since the opportunity cost of the supplier, $c_r - c_s$, due to the delay in payments, increases. As c_r exceeds p, it becomes unprofitable for the retailer to order in large quantities, so Q decreases, and so does the retailer's cost. When the unit price p increases, the cash deposits are higher, and the resulting capital gain also increases. Thus, the retailer's and supply chain costs decrease. Finally, a higher loan amount for the retailer, w_r implies a higher borrowing cost per unit time $\alpha_r w_r(1 - D/Q)\hat{t}$. To keep the retailer's cost low, Q has to decrease. Since the cash deposits get smaller because of a smaller Q, the supplier receives a larger borrowing rate, α_d.

Several research papers discuss how the net benefit should be shared among the various entities in a supply chain (the supplier, retailer, and bank). We follow the compensation policy developed in Goyal (1988) and adopted in Lee and Moon (2006), which shares the gains and losses according to the ratio of the individual cost per unit time with respect to the chain's non-coordinated cost per unit time. Specifically, letting C_s^0, C_r^0, and C_b^0 be the costs per unit time of the supplier, retailer, and bank, under the case of no coordination, and C_{chain} the total chain cost under coordination, we get the following ratios

$$Z_s = \frac{C_s^0}{C_s^0 + C_r^0 + C_b^0} \text{ and cost of supplier} = Z_s C_{chain}, \tag{23}$$

$$Z_r = \frac{C_s^0}{C_s^0 + C_r^0 + C_b^0} \text{ and cost of retailer} = Z_r C_{chain}, \tag{24}$$

$$Z_b = \frac{C_s^0}{C_s^0 + C_r^0 + C_b^0} \text{ and cost of bank} = Z_b C_{chain}, \tag{25}$$

Table 3
Sensitivity analysis with respect to A_r, A_s, c_r, p, amd w_r, assuming $M = 5$.

	λ	Q	α_d	C_{chain}	C_s	C_r
A_r						
10	1	2644.8	0.1326	84.9	69.9	14.9
15	1	2688.1	0.1324	85.6	69.4	16.2
20	1	2730.7	0.1322	86.4	69.0	17.4
25	1	2772.6	0.1320	87.2	68.5	18.6
30	1	2813.9	0.1317	87.9	68.1	19.8
A_s						
200	1	2813.9	0.1317	87.9	68.1	19.8
250	1	3112.6	0.1297	95.5	71.9	23.7
300	1	3463.6	0.1264	101.8	74.5	27.3
350	1	3782.1	0.1226	107.5	76.9	30.6
400	1	4075.8	0.1184	112.7	79.2	33.5
c_r						
20	1	2813.9	0.1317	87.9	68.1	19.8
25	1	2819.2	0.1317	93.0	73.1	19.9
30	1	2054.8	0.1339	100.5	89.0	11.5
35	2	1467.7	0.1326	104.2	100.2	4.0
40	2	1284.2	0.1317	108.2	107.0	1.2
p						
30	1	3082.2	0.1259	87.2	65.5	21.7
35	1	3595.9	0.1149	87.3	61.7	25.6
40	1	2054.8	0.1243	87.2	78.9	8.4
45	1	2054.8	0.1211	86.2	78.9	7.3
50	1	2054.8	0.1179	85.0	78.9	6.2
w_r						
25,000	1	3087.2	0.1299	85.8	65.5	20.3
30,000	1	2998.8	0.1306	86.5	66.3	20.3
35,000	1	2907.8	0.1312	87.2	67.1	20.1
40,000	1	2813.9	0.1317	87.9	68.1	19.8
45,000	1	2716.7	0.1323	88.5	69.1	19.4

Table 4
The total cost per unit time for each party in the non-coordinated chain.

M	Supplier	Retailer	Bank	Total
Non-coordinated model				
5	70.9	17.2	3	91.1
10	97.7	10.1	7.1	114.9
15	127.3	24.9	8.9	161.1
20	163.5	29.9	11.9	205.3

Table 5
The total cost per unit time for each party in the coordinated chain with compensation policy.

M	Supplier	Retailer	Bank	Total
Coordinated model				
5	68.4	16.6	2.9	87.9
10	91.5	9.5	6.6	107.6
15	110.9	21.7	7.8	140.3
20	140.7	25.7	10.2	176.7

with $Z_s + Z_r + Z_b = 1$. Tables 4 and 5 illustrate that by applying the compensation policy in (23)–(25), the benefits of reducing the total supply chain's cost is shared among the supplier, retailer, and bank. For example, when $M = 5$, the chain' cost under coordination is \$87.9. Thus, using (23)–(25), we get the following ratios and results

$$Z_s = 0.7782,\ \text{supplier's cost} = 87.9 * Z_s = \$68.4,$$
$$Z_r = 0.1888,\ \text{retailer's cost} = 87.9 * Z_r = \$16.6,$$
$$Z_b = 0.0329,\ \text{bank's cost} = 87.9 * Z_b = \$2.9.$$

5. Empirical models

A vital question that our model poses is whether banks, suppliers, and retailers are interested in coordinating in order to reduce their supply chain costs. Why would a retailer, for instance, be interested in depositing its cash in a specific bank in order to reduce the costs of suppliers? On the other hand, why would a bank be interested in charging the supplier lower interest rates in exchange for cash deposits? In order to validate the implementation of our theoretical model, we propose two models, "Use of Trade Credit" model and "Trade Credit Granted" model. These two models will help us understand the behavior of suppliers and retailers by determining the important factors behind the decision of granting and receiving more trade credit. We show that these major determinants such as inventories, price discrimination, and access to capital markets are the key drivers of our theoretical model. Our sample comprises COMPUSTAT US companies for the years 1950 through 2012. We exclude from the sample firms that behave differently from other firms in terms of trade credit. Therefore, we remove from the sample used to test the "Use of Trade Credit" model, transportation and public utility firms (SIC codes from 4000 to 4999), finance, insurance and real estate firms (SIC codes from 6000 to 6999), and service firms (SIC codes from 7000 to 8999). For the "Trade Credit Granted" model, we exclude further the retailer sector (SIC codes from 5000 to 5999). For robustness, we choose only a sample of manufacturing firms (SIC codes between 2000 and 3999) that use uniform practices of trade credit as these firms could play two roles: The first is to deliver products to retailers, and the second is to receive raw material and intermediate manufacturing goods from other suppliers. Also we use a more updated sample for robustness that starts at 1980, since trade credit key drivers might have changed over time with the development of financial theories. We end up with a sample of 34,051 yearly observations for 6888 firms to test the "Trade Credit Used" model, and 31,149 yearly observations for 6091 firms when testing the "Trade Credit Granted" model. These numbers get reduced to 13,191 observations for 3778 firms when we use sample years starting from 1980 for the manufacturing sample. We use accounts receivable to sales and accounts payable to cost of goods sold, as proxies for trade credit granted and used respectively, as in Petersen and Rajan (1997) and Shenoy and Williams (2011). These ratios measure the supply and demand of trade credit respectively and their duration. Consequently, throughout this section, we use better terms to imply higher levels and longer maturities.

It is well known in the literature that a supplier may play a more efficient role than banks in offering trade credit, since it can better assess the credit worthiness of its customers. Moreover, with the use of ongoing transactions, a supplier can control its customers better than a bank (Brennan, Maksimovics, & Zechner, 1988; Emery, 1984; Schwartz & Whitcomb, 1977; Smith, 1987). Also, suppliers can enforce debt repayment efficiently by confiscating the inventory of their defaulting customers and reselling it to other customers with whom they deal frequently (Mian & Smith, 1992). Consequently, when compared to banks, suppliers have greater ability to reduce the risk of giving credit. These financially sound suppliers are usually characterized by having low cost of debt, low cost of capital, and can finance their trade credit more easily (Long, Malitz, & Ravid, 1993; Petersen & Rajan, 1997; Shi & Zhang, 2013). For these suppliers, the best-fitting trade credit financing is specific to short term loans, which are usually too costly for risky suppliers (Long et al., 1993). Consequently, we expect suppliers that are financially healthy to use short term debt to finance their trade credit offerings. However, it is unlikely that firms would finance short term maturity trade credit with longer maturity and more costly debt as firms tend to match the maturity of their assets and liabilities (e.g. Diamond, 1991). Therefore, we propose the following two hypotheses:

H1 : Firms with higher short-term debt tend to grant better trade credit terms.

H2 : Firms with long term debt tend to avoid using the proceeds to finance trade credit.

We use the total current liabilities to sales as a measure of short term debt financing in the "Trade Credit Granted" model. We expect this short term debt measure to be positively associated with trade credit offering. On the other hand, we use a dummy variable that takes the value of 1 if the firm has a long term debt with an investment grade rating of Baa and above, and 0 otherwise. This dummy variable signals a better financial position for suppliers, as it conveys that suppliers have a low cost of debt and access to capital markets. We expect the coefficient of the credit rating dummy variable to be insignificant, as firms with long term borrowing tend to veer away from using the proceeds to finance their trade credit offerings (Elliehausen & Wolken, 1993). Similarly to the two hypotheses that we propose for the "Trade Credit Granted" model, we develop in the same context two hypotheses related to the "Trade Credit Used" model. We believe that short term financing will affect the use of trade credit. Firms tend to maintain a cap on their usage of short term debt that does not exceed short term assets, in order to maintain a positive net working capital, as these firms tend to match their current assets with current liabilities. Consequently, a higher net working capital will encourage firms to rely more on accounts payable and short term debt financing. On the other hand, better access to long term debt and capital markets signals creditworthiness for firms that use trade credit. Inversely, firms with higher cost of debt and limited access to capital will be offered more tightened credit terms, despite the increasing need for this source of financing (Choi & Kim, 2005). Therefore, we propose the following two additional hypotheses:

H3 : Firms with higher working capital tend to use more trade credit.

H4 : Firms with lower cost of debt and better access to capital markets usually receive better terms when using trade credit.

We use the net working capital to sales (excluding accounts payables) to measure the effect of the working capital cap on the trade credit used. We expect this variable to have a positive coefficient. In testing the fourth hypothesis, we expect the coefficient of the credit rating dummy to be positive, as more trade credit will be offered when a firm signals its quality by getting access to capital markets. A different measure of financial power is retained earnings. Firms that reinvest more of their net income in the firm, tend to grow more and have a better financial position. Consequently, we expect retained earnings to play a major role in determining the financing options of a firm. The Pecking Order Theory (Myers, 1984) posits that firms prefer to use first their internally generated funds before seeking any other form of external financing, since it comes with a cheap cost. Therefore, it would be most likely that retailers with high retained earnings tend to rely less on external financing and trade credit. Conversely, suppliers that have more money reinvested in the firm enjoy a longer financial arm and have less financial constraints related to financing their trade credit offerings (Choi & Kim, 2005). Thus,

H5 : Firms with higher retained earnings levels offer better trade credit terms.

H6 : Firms with higher retained earnings tend to use less trade credit.

We use retained earnings to sales as a variable that captures the effect of internally generated funds on trade credit. We expect to have a positive relationship between retained earnings to sales and trade credit granted, and a negative relationship between the ratio and the use of trade credit. The decision about granting trade credit is also related to the optimal inventory level that the supplier is targeting (Choi & Kim, 2005; Emery, 1987). The literature has shown that in order to reduce warehousing costs and liquidity carrying costs, suppliers might follow an aggressive trade credit policy to regain the optimal inventory level (Banerjee, Dasgupta, & Kim, 2004). On the other hand, high inventory turnover levels (equivalently low inventory levels) motivate retailers to use more trade credit. This is because trade credit could be used to minimize the transaction costs of paying bills, by aggregating the payments instead of making many single payments after each delivery of goods (Ferris, 1981). Thus, we expect retailers to be granted better and longer terms when they have high inventory turnover levels and low inventory levels. Therefore,

H7 : The higher the inventory level of the supplier, the higher the tendency to grant better credit.

H8 : The higher the inventory level of firms, the lower the likelihood that they will be granted better trade credit terms.

We use the inventory to sales ratio as a proxy for inventory levels. A high inventory to sales signals that products tend to deteriorate as they are stored in the warehouse. Therefore, high inventory levels should motivate suppliers to offer more trade credit to get rid of the excess inventory. On the contrary, in the "Trade Credit Used" model, high inventory to sales means the customer will not buy frequently from the supplier. Consequently, fewer buying transactions would lead to the reduction of trade credit usage terms. It follows that we expect a positive sign between inventory to sales and trade credit offering, but a negative coefficient with trade credit usage.

Intuitively, a supplier has a vested interest to invest in high growth customers. Building a solid relationship with such customers and helping them to grow would positively affect the current and future level of sales of the supplier. Hence, suppliers tend to offer more trade credit to customers that have high growth opportunities (Petersen & Rajan, 1997), as the future growth of these customers will equivalently imply future growth to suppliers. Therefore, we develop a ninth hypothesis that pertains solely to the use of "Trade Credit Used" model,

H9 : The higher the growth of the firm, the better the term credit offered.

Similarly to Shi and Zhang (2013), we compute the annual growth of assets as a measure for annual growth, using the three year compound annual growth rate of total assets. This growth rate is highly correlated with the total assets variable that we use as a control variable in our regressions, since firms with low total assets tend to have high growth rates. Therefore, we use a dummy variable to capture the growth effect of firms that use trade credit, where the dummy takes the value of 1 if it is a high growth firm with a double digit growth greater or equal to 10%, and takes the value of 0 otherwise. We expect high growth firms to be granted more trade credit, and we expect the variable to have a positive coefficient.

Suppliers could use trade credit offerings to give more trade credit to financially constrained customers in the market, as a price discrimination tool (Banerjee et al., 2004; Brennan et al., 1988; Mian & Smith, 1992; Schwartz & Whitcomb, 1977). Financially-constrained customers would benefit from more trade credit. Accordingly, these suppliers can boost their sales level and lower their variable costs further. Usually, firms with higher sales to cost margins can afford to offer more term credit with longer terms, without fearing serious financial consequences. We, therefore, propose a final hypothesis,

H10 : Firms with higher gross margins tend to offer more and longer trade credit terms.

Thus, we expect the gross margin to sales in our model to have a positive relationship with trade credit granted. We propose the following fixed effect panel data regressions that analyze the term credit granted and used respectively:

$$\begin{aligned} \text{logars}_{it} &= \beta_0 + \beta_1(\text{loginvs}) + \beta_2(\text{logres}) + \beta_3(\text{logms}) + \beta_4(\text{logsdebts}) \\ &\quad + \beta_5(\text{logta}) + \alpha_{it}, \qquad (26) \\ \text{logapcogs}_{it} &= \beta_0 + \beta_1(\text{loginvs}) + \beta_2(\text{logres}) + \beta_3(\text{rtgdummy}) \\ &\quad + \beta_4(\text{wcs}) + \beta_5(\text{gdummy}) + \beta_6(\text{logta}) + \alpha_{it}, \end{aligned}$$

where

logars = log of accounts receivable/sales.

logapcogs = log of accounts payable/cost of goods sold.

α = the intercept.

loginvs = log of inventory/sales.

logres = log of retained earnings/sales.

logms = log of (sales-cost of goods sold)/sales.

logsdebts = log of current liabilities/sales.

logta = log of total assets.

rtgdummy = a dummy variable which is 1 if the firm has an investment grade rating of Baa and above, and 0 otherwise.

wcs = current assets − (current liabilities other than accounts payable).

gdummy = a dummy variable which is 1 if the growth of assets is greater than10% and 0 otherwise.

Table 6 reports the descriptive statistics of the variables in models (26) and (27). "ars", "apcogs", "invs", "res", "ms", "sdebts", "wcs" are ratios, while "ta" is in millions of dollars, and "g" is a growth rate. "rtg" is a continuous rating with $Aaa = 1, Aa1 = 2$,$Aa2 = 3, Aa3 = 4, \ldots, C = 22$.

Table 7 reports the correlation between the explanatory variables of Eqs. (26) and (27). All these correlations fall below the cut-off point of 0.5 correlation with the highest correlation exhibited between the rating dummy and log of total assets, as large firms tend to have investment grade bonds. All the above explanatory variables are used in the panel regressions in lagged terms to avoid potential endogeneity problems. Also, we take the natural log of all variables except variables that could take negative values as well as null values. Additionally, we control for total asset size as it is frequently done in the literature. We expect the coefficient of this variable to be positive in both models, since larger firms tend to offer and use more trade credit, given that their creditworthiness is usually higher as a result of high cash flows and limited growth opportunities (Petersen & Rajan, 1997). Finally, we chose fixed effect models since the Hausman test for fixed versus random effect models shows that using fixed effect models is more appropriate.

Our empirical results support our hypotheses. Table 8 shows that inventory levels play a very important role in determining

Table 6
Descriptive statistics.

Variable	Panel A: All sample firms				Panel B: Manufacturing firms			
	Mean	Std. dev.	Min	Max	Mean	Std. dev.	Min	Max
ars	0.163	0.12	0.000095	7.1	0.175	0.10	0.000287	6.2
apcogs	0.151	0.53	0.000121	101.0	0.132	0.14	0.000121	11.1
invs	0.184	0.15	0.000296	10.1	0.190	0.11	0.000642	3.1
res	0.262	0.50	0.000024	91.8	0.263	0.29	0.000024	28.2
ms	0.322	0.16	0.000127	1.0	0.327	0.15	0.000127	1.00
sdebts	0.062	0.13	0.000002	9.6	0.059	0.10	0.000002	9.6
ta	1403.649	9248.23	1.010000	479920.0	1553.988	10295.06	1.010000	479920.0
rtg	13.652	6.01	1.000000	22.0	13.478	6.20	1.000000	22.0
g	106.946	288.84	0.000000	6986.7	109.354	301.71	0.000000	6986.7
wcs	0.510	2.48	−16.759860	455.9	0.500	1.21	−16.759860	241.6

Table 7
Correlation matrix of the explanatory variables of the trade credit empirical models.

	rtgdummy	loginv	logres	logms	logsdebts	logta
Panel A: Trade credit granted model						
rtgdummy	1					
loginv	−0.0752***	1				
logres	0.0982***	0.0824***	1			
logms	0.0568***	0.0887***	0.293***	1		
logsdebts	0.00271	0.232***	−0.00292	0.00952*	1	
logta	0.338***	−0.0951***	0.209***	0.0170***	−0.0469***	1
Panel B: Trade credit used model						
rtgdummy	1					
loginv	−0.0705***	1				
logres	0.0859***	0.0585***	1			
wcs	−0.0210***	0.0140***	0.167***	1		
gdummy	0.0140***	−0.00043	0.0157***	0.00614	1	
logta	0.333***	−0.0898***	0.179***	0.0121**	0.0479***	1

* $p < 0.05$.
** $p < 0.01$.
*** $p < 0.001$.

Table 8
Empirical determinants of trade credit granted and used models.

Panel A: All sample firms starting 1950			Panel B: All sample firms starting 1980			Panel C: Manufacturing firms starting 1950			Panel D: Manufacturing firms starting 1980		
Trade credit granted model											
rtgdummy	−0.02245	−1.33	rtgdummy	−0.01085	−0.64	rtgdummy	−0.02115	−1.4	rtgdummy	−0.01577	−1.05
loginvs	0.106118	7.61*	loginvs	0.124618	8.18*	loginvs	0.106828	7.24*	loginvs	0.127269	7.91*
logres	0.030136	5.39*	logres	0.027105	4.68*	logres	0.024388	4.42*	logres	0.022255	3.86*
logms	0.105037	6.15*	logms	0.10026	5.7*	logms	0.074276	4.32*	logms	0.074001	4.18*
logsdebts	0.043572	13.48*	logsdebts	0.042125	12.42*	logsdebts	0.038636	12.1*	logsdebts	0.038125	11.37*
logta	0.024626	4.76*	logta	0.023737	4.5*	logta	0.028721	5.13*	logta	0.029236	5.03*
intercept	−1.45197	−34.25*	intercept	−1.432	−32.07*	intercept	−1.52562	−35.16*	intercept	−1.49822	−32.95*
Trade credit used model											
rtgdummy	0.116225	4.89*	rtgdummy	0.078454	4.04*	rtgdummy	0.137746	5.27*	rtgdummy	0.080882	3.62*
loginvs	0.002287	0.19	loginvs	−0.01588	−1.1	loginvs	−0.02362	−1.9***	loginvs	−0.03671	−2.23**
logres	−0.02203	−3.54*	logres	−0.01389	−2.21**	logres	−0.03659	−5.67*	logres	−0.02034	−2.89**
wcs	0.09626	2.41**	wcs	0.072984	2.35**	wcs	0.205171	9.06*	wcs	0.156762	4.76*
gdummy	0.123177	6.23*	gdummy	0.118445	4.12*	gdummy	0.146988	6.65*	gdummy	0.142303	4.04*
logta	0.03266	7.97*	logta	0.024795	4.38*	logta	0.033461	7.54*	logta	0.024594	3.54*
intercept	−2.53046	−52.62*	intercept	−2.42256	−42.73*	intercept	−2.69187	−63.3*	intercept	−2.51619	−37.21*

the terms of the trade credit granted. All panel data regressions indicate that inventories are the most important driving factor for trade credit granted, since suppliers use trade credit to boost their sales and decrease their warehousing costs. On the other hand, inventory discourages firms from using trade credit, as their number of transactions diminishes when inventory slacks. Higher retained earnings tend to enhance trade credit offerings due to their cheap cost, and discourage firms from using trade credit as they prefer not to borrow when they can finance their transactions from internally generated funds. Also, Table 8 shows that access to long term debt capital markets does not affect trade credit, as firms tend to finance trade credit with short term debt when necessary. However, access to long term and short term capital markets increases the reliance on the use of trade credit, because improved credit worthiness of the firm favorably affects the terms offered by suppliers. Furthermore, higher gross margins are associated with more trade credit offerings. However, when we use a sample of solely manufacturing firms, the gross margin of granting firms remains positively associated with trade credit offerings, but its importance is diminished even though it remains significant at the 1% level. Unsurprisingly, total assets is also a major determinant, as firms with higher asset size tend to use and grant more trade credit. Lastly, the growth of the granted firm seems to be one of the most important determinants of trade credit used, as suppliers invest in their future sales by financing the trade credit of high growth firms.

To conclude, it seems that banks, retailers, and suppliers have much to gain by collaborating. On one hand, by compromising and charging the suppliers a lower credit rate, banks will receive higher deposits that could be used to generate more loans and more income. On the other hand, suppliers can use the option of dealing with the bank as a price discrimination tool and inventory management tool that can generate more sales and better inventory management. Finally, firms that are granted trade credit by suppliers can invest in their future by establishing creditworthiness with the bank. This will improve their access to short term and long term debt in the future which is vital for the financial stability of the firm. In conclusion, the collaboration mechanism proposed in this paper is a winning situation to all parties involved.

6. Conclusion

In this paper, we propose a coordination mechanism with permissible delay in payments and discounted interest rate for a three-level supply chain (retailer–supplier–bank). We formulate a model that simultaneously finds the financial and inventory decisions, in order to minimize the total chain cost. Our paper extends the traditional supply chain literature in that it analyzes the interactions of the financial and inventory decisions in a coordinated supply chain. This paper assumes that the length of the delay period is an input parameter, then investigates the factors affecting the delay period using an empirical model.

We first develop the retailer's cost function, which includes the traditional inventory cost expressions (purchasing, ordering, and inventory costs) and the financial costs associated with cash management (cost of funds in cash in the retailer's account, cost of purchasing and selling securities, borrowing cost and cost of capital) by considering two cases: whether the credit period is smaller or larger than the retailer's cycle time. We further divided the first case into two subcases, depending on whether the retailer pays its inventory investment using a loan or with cash collected from sales whenever it covers the cost of goods sold. We also develop the supplier's inventory and financial cost, and the bank's cost. We derive conditions on the discounted interest rate that the bank offers the supplier so that the latter is willing to coordinate. The optimal inventory and financial decisions are obtained by minimizing the total supply chain cost subject to the conditions on the discounted rate. We numerically solve our model for different combination of input parameters. Our results indicate that the coordination mechanism we propose significantly reduces the supply chain costs, (up to 26.2% in our numerical example, in comparison to a non-coordinated chain). Also, we find that the retailer orders in larger lot sizes compared to the no coordination case, with no delay in payments and no discounted interest rate, which is consistent with results found in the literature of ordering larger lots than the EOQ when delay in payments is exercised as a coordination mechanism among the players in a supply chain. Finally, using panel data regressions, we determine the most important factors that dictate the behavior of firms that grant and receive trade credit. These factors validate the notion of collaboration among all parties involved in our theoretical model.

The model developed in this paper can be immediately extended by integrating environmental and financial aspects of sustainability into a supply chain model. This could be done by bringing the concept of a retailer–bank–supplier cash flow model developed in this paper into the works of Jaber, Glock, and El Saadany (2013). One limitation in this paper is that we treat the length of the credit period as an input. An extension can be developed in which the credit period is a decision variable. Also, among the limitations of our model is that we do not account for the retailer's default risk when it does not have the capacity to pay back the supplier. Despite that, our analysis sheds the light on the importance of simultaneously making the financial and inventory decisions.

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Appendix A

At time $\hat{t}, \hat{t} > t$, cash from sales is used up to pay the supplier, and the cash balance reduces to 0. To compute the holding cost, we refer to Fig. 5A which illustrates the inventory level over time. When the retailer pays the supplier, the inventory level is $Q - DM$. In $(0, \hat{t})$, the retailer incurs no cost of capital. In $(\hat{t}, T_r)$, the retailer owns the inventory and thus incurs a cost of capital which we account for. The holding cost per cycle can be computed by multiplying h by the shaded area. Thus we have

$$\begin{aligned}\text{Holding cost per cycle} &= h_r * (T_r - M)(Q - DM)/2 \\ &= h_r * (Q - DM)^2/2D \\ \text{Holding cost per unit time} &= h_r * (T_r - M)(Q - DM)/2 * (1/T_r) \\ &= h_r * (Q - DM)^2/2Q. \end{aligned} \tag{27}$$

Similarly, the holding cost when $0 \leqslant M \leqslant T_r$, $\hat{t} > M$, and $t = \hat{t}$ (i.e. Case 1b) can be computed as follows:

$$\begin{aligned}\text{Holding cost per cycle} &= h * (T_r - \hat{t})(Q - D\hat{t})/2 = h * (Q - D\hat{t})^2/2D \\ \text{Holding cost per unit time} &= h * (T_r - \hat{t})(Q - D\hat{t})/2 * (1/T_r) \\ &= h * (Q - D\hat{t})^2/2Q.\end{aligned}$$

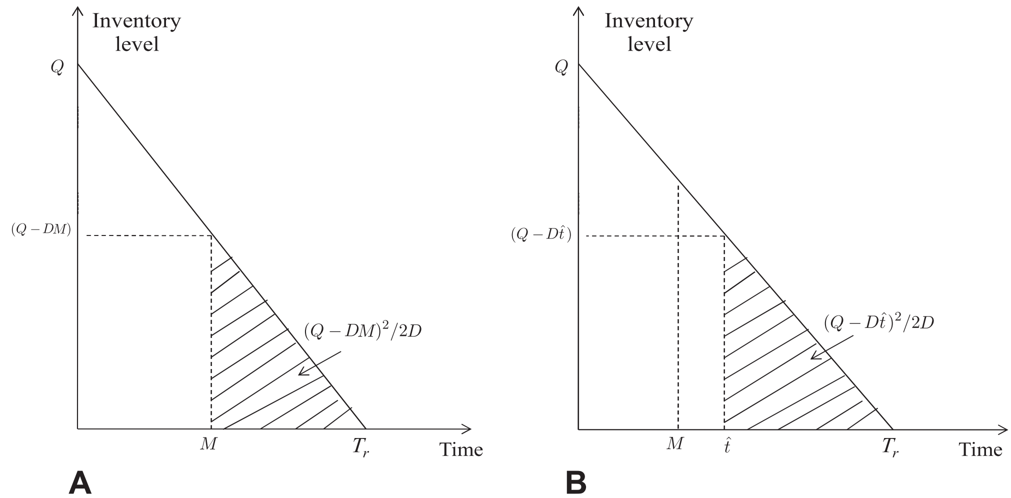

Fig. 5. Computing the retailer's holding cost.

Appendix B

Proof of Theorem 1. We first prove (i). Taking the partial derivative with respect to Q of the cost function in (6), we get after some simplification the following expression:

$$\frac{\partial C_r^{1a}(Q,\hat{t},C)}{\partial Q} = -A_rD/Q^2 + s_r/2 + (h_r/2)(1 - D^2\hat{t}^2/Q^2) - (1/Q^2) \times (bpD^2\hat{t}/C + rCD\hat{t}/2) + \alpha_r w_r D\hat{t}/Q^2 + rpD^2M^2/Q^2 - rp/2.$$

The second derivative of the above expression becomes

$$\begin{aligned}\frac{\partial C_r^{1a^2}(Q,\hat{t},C)}{\partial Q^2} &= (2/Q^3)\left\{A_rD + (h_r/2)D^2\hat{t}^2 + bpD^2\hat{t}/C + rCD\hat{t}/2 - \alpha_r w_r D\hat{t} - rpD^2M^2\right\} \\ &= (2/Q^3)\left\{A_rD + D^2M^2(h_r/2 - rp) + bpD^2M/C + DM(rC/2 - \alpha_r w_r)\right\}. \qquad (28)\end{aligned}$$

The equality in (28) follows since, in Case 1a, we have $\hat{t} = M$. We next show that (28) is positive. To do that, all we need to show is $D^2M^2(h_r/2 - rp) + DM(rC/2 - \alpha_r w_r) \geqslant 0$, which after some manipulation gives $w_r \leqslant DM(h_r/2 - rp)/\alpha_r + (rC)/(2\alpha_r)$. This completes the proof of (i). To interpret this inequality, notice that for $h_r > rp$, the right hand side of this expression is a large number, thus we get a large upper bound on the loan amount w_r, which holds in general. For illustration, consider the parameters that we will use in the numerical analysis in Section 4: Let $D = 150{,}000$, $M = 20$, $h_r = 6$, $r = 10\%$, $p = 25$, $C = 15{,}000$, $\alpha_r = 15\%$. Then the right hand side of the inequality is 9,004,500, which is a large upper bound on w_r.

To show (ii), we take the partial derivative with respect to Q of the cost function in (8), we get after some simplification the following expression:

$$\frac{\partial C_r^{1b}(Q,\hat{t},C)}{\partial Q} = -A_rD/Q^2 + s_r/2 + (h_r/2)(1 - D^2\hat{t}^2/Q^2) - (1/Q^2) \times (bpD^2\hat{t}/C + rCD\hat{t}/2) + rpD^2t^2/Q^2 - rp/2.$$

The second derivative of the above expression becomes

$$\begin{aligned}\frac{\partial C_r^{1b^2}(Q,\hat{t},C)}{\partial Q^2} &= (1/Q^3)\left\{2A_rD + h_rD^2\hat{t}^2 + 2(bpD^2\hat{t}/C + rCD\hat{t}/2) - rpD^2t^2\right\} \\ &= (1/Q^3)\left\{2A_rD + D^2t^2(h_r - rp) + 2(bpD^2t/C + rCDt/2)\right\}. \qquad (29)\end{aligned}$$

The last equality (29) follows since, in Case 1b, we have $\hat{t} = t$. Since $h_r > rp$, (28) is positive, which completes the proof.

We next show (iii). Taking the partial derivative with respect to Q of the cost function in (9), we get after some simplification the following expression:

$$\frac{\partial C_r^2(Q,\hat{t},C)}{\partial Q} = -A_rD/Q^2 + s_r/2 - (1/Q^2)(bpD^2\hat{t}/C + rCD\hat{t}/2) - rp/2.$$

The second derivative of the above expression becomes

$$\frac{\partial C_r^2(Q,\hat{t},C)}{\partial Q^2} = (1/Q^3)\left\{2A_rD + 2(bpD^2\hat{t}/C + rCD\hat{t}/2)\right\},$$

which is positive. Hence, the cost function in (9) is convex.

Finally, we show (iv). To show the convexity w.r.t. Q, we take the partial derivative with respect to Q of the cost function in (11), we get after some simplification the following expression:

$$\frac{\partial C_r^{1a}(Q,\hat{t},C)}{\partial Q} = -A_rD/Q^2 + s_r/2 + (h_r/2)(1 - D^2\hat{t}^2/Q^2) - (1/Q^2)(bpD^2\hat{t}/C + rCD\hat{t}/2) + \alpha_r w_r D\hat{t}/Q^2 + rpD^2M^2/Q^2 - rp/2.$$

Clearly, when $\lambda = 1$, the first derivative is decreasing in Q. The second derivative of the above expression becomes

$$\frac{\partial C^{s^2}(Q,\lambda,\hat{t},C)}{\partial Q^2} = 2A_sD/(\lambda Q^3) + 2\alpha_d w_s D/(\lambda Q^3),$$

which is positive. Thus, the supplier's cost function in (11) is convex w.r.t. Q, and is decreasing for $\lambda = 1$. Taking the derivative of the cost function w.r.t. λ, the first and second partial derivatives are respectively $-A_sD/(\lambda^2 Q) + (h_s + s_s)Q/2 - (-\alpha_d w_s D/(\lambda^2 Q)$ and $2A_sD/(\lambda^3 Q) + 2\alpha_d w_s D/(\lambda^3 Q)$. Clearly the second partial derivatives is positive for $\lambda > 0$, which completes the proof. □

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