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Promoting impulse buying by allocating retail shelf space to grouped product categories

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This paper addresses a problem where a retailer seeks to optimize store-wide shelf-space allocation in order to maximize the visibility of products to consumers and consequently stimulate impulse buying. We consider a setting where the retailer, because of product affinities or the retailer's historical practice, has pre-clustered product categories into groups each of which must be assigned to a shelf. On the basis of its location in the store layout, each shelf is partitioned into contiguous shelf segments having different anticipated customer traffic densities. The retailer seeks to assign each group of product categories to a shelf, to determine the relative location of product categories within their assigned shelf, and to specify their allocated total shelf space within given lower/upper bounds. We propose a 0–1 integer programme that takes into account expected customer traffic densities within the store, groups of product categories, their relative profitability, and the desirability to keep certain product groups in the same aisle, with the objective of maximizing the impulse buying profit. The proposed model is grounded in a preprocessing scheme that explores feasible assignments of subsets of product groups to available aisles by iteratively solving an $\mathcal{NP}$-hard subproblem and is numerically observed to greatly outperform an alternative mixed-integer programming formulation. We demonstrate the usefulness of and the enhanced tractability achieved by the proposed approach using a case study motivated by a grocery store in New England and a variety of simulated problem instances.

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Keywords: Shelf-space allocation; impulse buying; product shelf visibility; mixed-integer programming; optimization and modelling

1. Introduction and literature review

The retail industry is a vital sector for economies worldwide and is marked by high competition, tight profit margins, and demanding consumers. The continuous flux and change in retailing, coupled with the availability of point-of-sale data, has motivated a growing trend in retail analytics and optimization that seeks new managerial directives to improve operations efficiency and business profitability. Key decisions in retail analytics include store location, supplier selection, product assortment, pricing strategies, inventory management, store layout and shelf-space management (Hariga *et al*, 2007). Some of these decisions are interrelated and require integrated planning, as in the case of product assortment and pricing problems (Irion *et al*, 2011; Ghoniem *et al*, 2014a; Ghoniem and Maddah, 2015). An effective allocation of shelf space, a scarce asset (Amrouche and Zaccour, 2007), can provide retailers with a competitive advantage (Lim *et al*, 2004) and improve the financial performance of stores (Buttle, 1984). The present work focuses on shelf-space allocation (Cox, 1970; Borin *et al*, 1994; Amrouche and Zaccour, 2007) that involves the following hierarchy of decisions: (i) Assigning product categories to certain areas, aisles, and shelves of the store; (ii) locating a product category within a shelf (eg, in the middle of a shelf or towards its ends); (iii) allocating a shelf space to a category within given lower/upper bounds; and (iv) building detailed planograms that determine the number of facings for specific items. At a tactical level, this paper examines the first three intertwined decisions that can serve as an input to derive store planograms at a more operational level.

Our investigation of tactical retail shelf-space allocation is motivated by the practice of grocery stores and is predicated on the notion that improved product visibility can stimulate substantial *impulse buying* revenues (Dreze *et al*, 1994). Our research can be also applied to other retailing contexts beyond grocery with straightforward amendments. Impulse purchases correspond to unplanned, 'on-the-spot' purchases (Piron, 1991) that are triggered by in-store stimuli (Clover, 1950; Piron, 1991) and may account for over 50% of purchases in supermarkets (Kollat and Willett, 1967). Making a product more visible to the shopper is a function of the product shelf-space location/allocation and constitutes an important in-store stimulus for impulse buying (Chen *et al*, 2006). This relationship between 'product exposure' as a precursor for impulse purchases (Abratt and Goodey, 1990) has been documented in the

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marketing literature. As such, our objective is not to empirically reveal this phenomenon; rather it is to examine how retail shelf-space allocation can be tactically optimized at a store level in order to increase the average profit per customer from impulse buying. While we focus on shelf-space allocation and the resulting product visibility as a key determinant of impulse buying, offering and advertising promotions periodically can also serve as an in-store stimulus that impacts impulse buying. The latter consideration is admittedly beyond the scope of this work and is relegated to future research.

The relationship between retail shelf-space allocation and the profit because of impulse buying can be elucidated by the following factors: (i) Sales data for products, their relative 'impulse potential', and the behaviour and profile of the shopper (Kollat and Willett, 1967; Bellenger *et al*, 1978; Iyer, 1989) and (ii) the store layout (Iyer, 1989). Deciding about items that must receive greater in-store display and expanded shelf-space allocation can be informed by the impulse potential of a product. For example, products such as milk, bread, or egg commonly appear in customers' shopping lists and tend to have relatively low impulse potentials. In contrast, some products (eg, candies and desserts) do not constitute shopping essentials and tend to have greater impulse potential (Kollat and Willett, 1967). In general, *hedonic* products are associated with 'procuring pleasure' and tend to have higher impulse purchase rates (Inman *et al*, 2009). Marketing studies indicate that the sales of products having high impulse purchase rates tend to be more sensitive to changes in shelf space (Curhan, 1972; Desmet and Renaudin, 1998). It is, therefore, in the retailer's interest to optimize the shelf space in order to ensure higher visibility to products that can result in lucrative impulse purchases. These products may garner more customer attention by altering: (i) the location of the product within a display; (ii) the area (facings) devoted to the product; (iii) product adjacencies; and (iv) aesthetic elements such as size and colour coordination and special signage (Dreze *et al*, 1994). Noting that different areas of a store and different segments of the same shelf are not equally visited by or visible to the shopper, Ghoniem *et al* (2014b) proposed a 0–1 mixed-integer programme (MIP) that optimizes the shelf-space allocation of *individual* product categories in order to maximize the average impulse buying profit per customer. The work was motivated by a medium-size supermarket in Beirut, Lebanon, for which an increase in the average impulse purchase profit was foreseen using the optimized solution in lieu of the existing shelf-space allocation.

In the present paper, we consider a setting similar to that in Ghoniem *et al* (2014b). The main distinction resides in the fact that the retailer seeks to allocate pre-grouped product categories, as opposed to individual categories, to shelves and aisles. In the tactical context of our study, a product category denotes a set of substitutable items that serve the same need for the customer (eg, Tea is viewed as one product category) and the terms 'product category' and 'product' are used interchangeably. In this work, product categories are pre-grouped based on their relative affinity (eg, pasta and pasta sauce) or to preserve conformity to the retailer's historical practice by grouping products that the shopper is used to finding on the same shelf or in the same aisle. To optimize the store-wide assignment of product groups to shelves, it is possible to further specialize the MIP formulation in Ghoniem *et al* (2014b), as discussed in Section 2, by introducing product affinity constraints that require any pair of categories that belong to the same group to be assigned to the same shelf. However, our experience indicates that this approach can become computationally prohibitive because of the combinatorial nature of the problem, especially for larger store layouts.

In the present paper, we propose an alternative modelling and optimization solution approach to the problem of allocating grouped product categories to store shelves and aisles. The following decisions are jointly optimized: (i) The assignment of grouped product categories to shelves and aisles; (ii) determining the location of a specific product in the group within its assigned shelf; and (iii) setting the size of the allocated shelf space of each product within the assigned shelf. An illustration of two alternative allocations is provided in Figure 1. Note, for example, that the groups composed of products {A, B, C} and {D, E} have been assigned to different shelves. In addition, in Allocation 2, the locations of Products A and C have been swapped within their group/shelf. Moreover, Product E has been granted a larger space, at the expense of Product D.

To our knowledge, such integrated shelf-space decisions have not been considered in the literature, except for the recent works of Botsali (2007) and Ke and van Ryzin (2011). However, both Botsali (2007) and Ke and van Ryzin (2011) utilize heuristics and simulation approaches for addressing such decisions (as opposed to modelling and optimization techniques). Our work is, however, distinctive in that we provide a practical optimization framework that is highly tractable for large-scale problem instances. A key element of our solution methodology consists of viewing the problem at hand as a generalized assignment problem for which a set partitioning-type formulation is explored. In doing so, we construct by total enumeration all the columns of the model by iteratively solving a single-shelf, single-group subproblem that is proved to be $\mathcal{NP}$-hard. Much of the computational difficulty associated with the adjusted model from Ghoniem *et al* (2014b) is thus

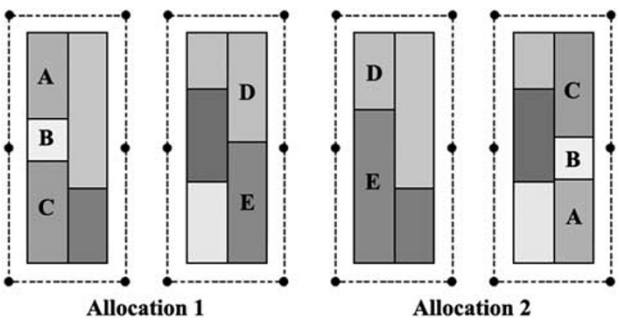

Figure 1 Alternative shelf space allocation.

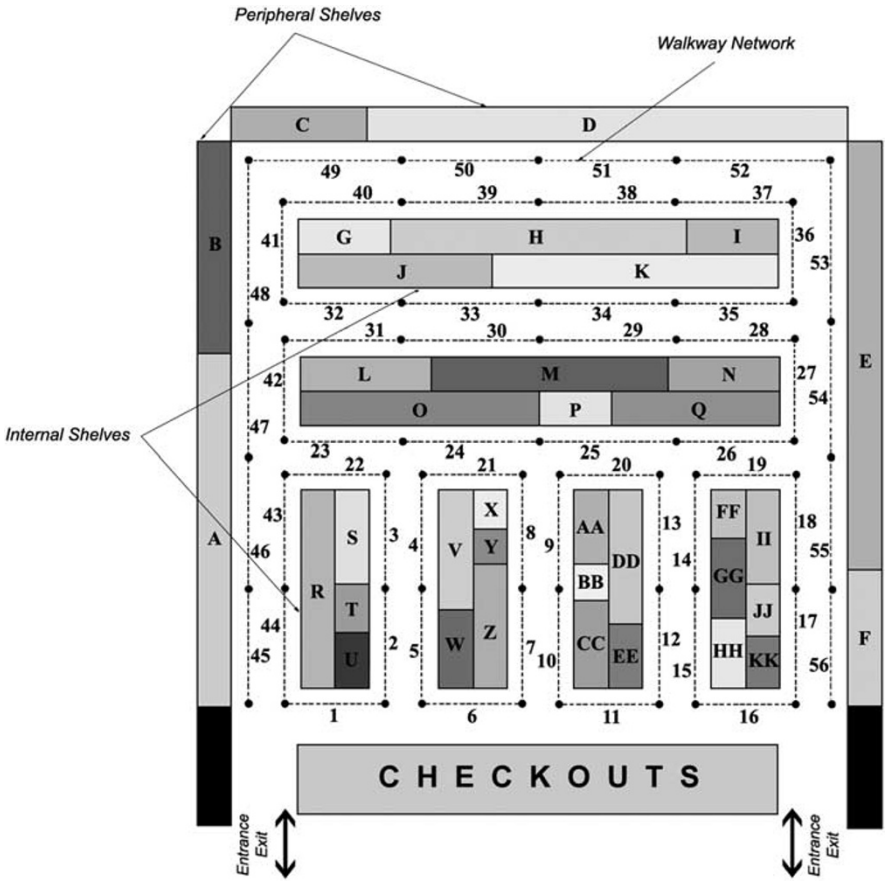

Figure 2 Grocery store grid layout.

circumvented because of the adopted decomposition technique and exact solutions to instances of practical size are obtained in manageable times.

The remainder of this paper is organized as follows. Section 2 offers a formal problem statement and proposes an optimization model for impulse purchase maximization via the allocation of shelf space to grouped product categories. The usefulness of the proposed model is demonstrated in Section 3 using a popular grocery store in New England in the United States. Section 4 reports extensive computational results for a set of problem instances generated over various grid layouts. Section 5 concludes the paper with a summary of our findings and directions for future research.

2. Mathematical programming formulation

Section 2.1 provides a problem statement and elaborates on key assumptions that are made in this work. In Section 2.2, we introduce our notation and demonstrate how the product-based model in Ghoniem *et al* (2014b) can be adjusted to address the setting, where product groups are pre-formed. This then serves as a benchmark for the alternative formulation and algorithmic schemes that are proposed in this paper in Sections 2.3 and 2.4.

2.1. Problem statement and assumptions

Consider a retailer who seeks to assign product categories to available shelves and aisles in the store in a way that can attract shoppers to products and result in impulse purchases. Owing to product affinities, cross-selling opportunities, or current shelf-space allocation in the store the retailer has already clustered product categories into groups. The scope of actions considered by the retailer consists of assigning groups of product categories to different aisles of the store, where each group is assigned to a single shelf, internally shuffling the position of certain products in their designated shelves, and to possibly adjust the relative space allocated to different products. This tactical allocation of shelf space to grouped product categories is guided by a managerial directive to maximize the profit because of impulse buying by improving the visibility of products having a high impulse potential.

Figure 2 illustrates a grid layout, with peripheral and internal shelves, as commonly found in grocery stores. A walkway network in Figure 2 represents different shelf segments along which customers can walk in the store. Each shelf segment is characterized by its expected traffic density or the likelihood that a shopper who enters the store would walk by it. Figure 2

also illustrates a specific allocation of shelf space to different product categories (represented by letters). Each product category has the following attributes: (i) its overall profitability; (ii) its impulse purchase potential, defined as the likelihood of being purchased if it is made visible to the consumer; and (iii) the minimum/maximum requirements on the shelf space it should be allocated, based on historical practices, demand forecasting (Dreze *et al*, 1994), or marketing strategies that create a particular store image (Corstjens and Doyle, 1981), stimulate interest in certain (new) products (Corstjens and Doyle, 1981; Borin *et al*, 1994; Amrouche and Zaccour, 2007), or allocate shelf space to update the product mix (Yang, 2001).

In our study, the concept of 'layout' relates to shelf configurations and locations without taking into account any product allocation. We make the simplifying assumption that customer traffic densities are exogenous, based on the store layout, and independent from the shopping lists of customers. An ongoing study for a grocery store in Lebanon (Ghoniem *et al*, 2014b) seeks to empirically identify so-called 'traffic drivers', that is, high-planned product categories that are mainly in the shopping list of a customer, and to judiciously allocate them in a way that guides in-store traffic and strengthens the expected impulse buying. The literature proposes alternatives for estimating traffic densities. For example, Hui *et al* (2009) and Larson *et al* (2005) analysed customer in-store shopping behaviour for a given layout using Radio Frequency Identification, while Farley and Ring (1966) proposed a Markov chain/regression model to estimate traffic densities. Moreover, Hui *et al* (2009) investigated the effect of the deviations from the path, which is formed by the customer's shopping list, on impulse buying. Hui *et al* (2013) examined impulse buying decisions as a result of walking more to locate promoted/discounted products. Once traffic densities are estimated based on one of the approaches in the literature, our work in this paper focuses on promoting impulse buying by assigning impulsive products to relatively attractive areas of a store (eg, near entrance/exit and perimeter) and within a shelf (close to end caps), thereby making them more visible to shoppers.

At last, it is worthwhile to underscore that we adopt a product-centric, store layout-based point of view for the tactical allocation of shelf space. In this sense, our work does not immediately address consumer behaviour issues at a more operational level. For example, the impulse purchase rate for a product, measuring the likelihood of a product category to result in an unplanned purchase when it is made visible to consumers, is assumed to be a constant. Although this constant may be viewed as an aggregate value across customer segments, it misses the more subtle situation where the impulse purchase rate may vary for the same customer over multiple trips. The latter situation might arise when a customer is budget-constrained and the dynamic filling of her basket during one shopping trip impacts her perception of the impulse purchase rate of unplanned items. Moreover, the model is constructed with an estimated unit profit per product. The retailer can, however, examine the average number of units a customer purchases from a given product/category and deem this to be the buying unit, thereby scaling the profit accordingly (eg, two water packs may be viewed as a unit with an appropriately scaled profit).

2.2. Notation and base MIP formulation

For completeness and comparative purposes, we adjust the impulse buying maximization model in Ghoniem *et al* (2014b) to the context where product have been pre-grouped and certain groups have to be assigned to the aisle. To this end, consider the following notation:

Sets:

- $\mathcal{A}$: Set of aisles.
- $\mathcal{B}$: Set of shelves.
- $\mathcal{N}_a$: Set of shelves located in the same aisle a, $\forall a \in \mathcal{A}$.
- $\mathcal{E}_b \equiv \{\alpha_b, \ldots, \beta_b\}$: Set of edges (shelf segments) associated with shelf b, $\forall b \in \mathcal{B}$. For example, the set of edges associated with the top internal shelf in Figure 2 are $\{37,38,39,40\}$.
- $\mathcal{E} \equiv \cup_{b\in\mathcal{B}}\mathcal{E}_b$: Set of all walkway edges numbered consecutively from 1 to $|\mathcal{E}|$.
- $\mathcal{P}$: Set of all product categories.
- $\mathcal{L}$: Set of product pairs $(p,p') \in \mathcal{P}^2$ that should not be allocated to the same shelf because of a lack of affinity (eg, detergents and food products).
- $\mathcal{H}_1$: Set of product pairs $(p,p') \in \mathcal{P}^2$ that must be allocated to the same shelf because of high affinity (eg, pasta and pasta sauce).
- $\mathcal{H}_2$: Set of product pairs $(p,p') \in \mathcal{P}^2$ that must be allocated to the same aisle because of high affinity.

Input parameters:

- c_e: Shelf space available along edge (shelf segment) e, $\forall e \in \mathcal{E}$.
- $k_e \in [0,1]$: Customer traffic density along edge e, $\forall e \in \mathcal{E}$. This is the likelihood of edge e to be visited by a customer and can be estimated based on its overall location in the store. For example, k_e may have higher values for edges in the vicinity of store entrances or at the end of aisles near busy walkways.
- ℓ_p, u_p: Minimum/maximum shelf-space requirement for product category p, $\forall p \in \mathcal{P}$. It is assumed that the minimum shelf-space requirement ℓ_p is set by the retailer in a manner that minimizes the risk of stock-outs. Therefore, a more generous shelf-space allocation to one product cannot be detrimental to the availability of another.

- ϕ_p: Smallest product facing length within product category p, $\forall p \in \mathcal{P}$.
- ρ_p: Profit margin for product category p, $\forall p \in \mathcal{P}$. The average unit profit margin for a category that we consider in this paper is a weighted average of the gross profit margins of SKUs (stock keeping units) in the category with the weights being the sales (in monetary terms) of each SKU.
- $i_p \in [0,1]$: Impulse purchase rate for product category p, $\forall p \in \mathcal{P}$. This represents the likelihood of a product being purchased impulsively when noticed by the customer.

Decision Variables:

- $x_{pb} \in \{0,1\}$: $x_{pb} = 1$, if and only if product category p is assigned to shelf b, $\forall p \in \mathcal{P}, b \in \mathcal{B}$.
- $y_{pe} \in \{0,1\}$: $y_{pe} = 1$, if and only if product category p is assigned to edge e, $\forall p \in \mathcal{P}, e \in \mathcal{E}$.
- s_{pe}: Shelf space allocated to product category p along edge e, $\forall p \in \mathcal{P}, e \in \mathcal{E}$.
- v_{pe}: The product visibility of product category p on edge e, as a function of the space it is allocated and its attractiveness, $\forall p \in \mathcal{P}, b \in \mathcal{B}, e \in \mathcal{E}_b$, defined as follows:

$$v_{pe} \equiv \frac{k_e s_{pe}}{c_e} \tag{1}$$

  This represents the likelihood of a product category to be noticed on edge e, when it is visited.
- v_p: The total product visibility of product category p, $\forall p \in \mathcal{P}$, defined as follows:

$$v_p \equiv \sum_{b \in \mathcal{B}} \sum_{e \in \mathcal{E}_b} v_{pe} \tag{2}$$

The visibility in (2) assumes that a product may be purchased multiple times, from each edge where it is assigned. (As such, it is a metric whose value may exceed 1.) This is a reasonable assumption as typically a product is allocated to one or two edges at most. Moreover, the visibility in (2) can be seen as a first-order approximation to a more refined, probability-based, measure that takes into account the walking pattern of a customer along a shelf. A more refined, probability-based expression of the visibility of a product is derived in Appendix A, but is computationally intractable. For this reason, we adopt (2) as a reasonable, and more tractable, approximation. The concept of visibility is expressed in the spirit of Botsali (2007), where $v_p \equiv \min\{1, (\sum_e k_e s_{pe})/T\}$ and where T is length of an aisle. One problematic aspect of the latter definition is that it deems two products to be equally visible, in fact visible with certainty, when $(\sum_e k_e s_{pe})/T > 1$. In contrast, we express the visibility of a product category v_p as the summation of the probabilities v_{pe} of product category p to be noticed along edge e.

- h_{pe}: Auxiliary non-negative variable that assumes a value of 1, if product category p is assigned to the contiguous edges e and $e+1$, $\forall p \in \mathcal{P}, b \in \mathcal{B}, e = \alpha_b, \ldots, \beta_b - 1$.

The impulse buying profit-maximization model in Ghoniem *et al* (2014b) can be adapted as follows to ensure that all product categories that belong to the same group must be assigned to the same shelf and related groups are assigned to the same aisle:

$$\text{SSAP}: \text{ Maximize } \sum_{p \in \mathcal{P}} \rho_p i_p v_p \tag{3}$$

$$\text{subject to } v_{pe} = \frac{k_e s_{pe}}{c_e}, \qquad \forall p \in \mathcal{P}, b \in \mathcal{B}, e \in \mathcal{E}_b \tag{4}$$

$$v_p = \sum_{b \in \mathcal{B}} \sum_{e \in \mathcal{E}_b} v_{pe}, \qquad p \in \mathcal{P} \tag{5}$$

$$\sum_{b \in \mathcal{B}} x_{pb} = 1, \qquad \forall p \in \mathcal{P} \tag{6}$$

$$\ell_p \leqslant \sum_{b \in \mathcal{B}} \sum_{e \in \mathcal{E}_b} s_{pe} \leqslant u_p, \qquad \forall p \in \mathcal{P} \tag{7}$$

$$\sum_{p \in \mathcal{P}} s_{pe} \leqslant c_e, \qquad \forall b \in \mathcal{B}, e \in \mathcal{E}_b \tag{8}$$

$$y_{pe} \leqslant x_{pb}, \qquad \forall p \in \mathcal{P}, b \in \mathcal{B}, e \in \mathcal{E}_b \tag{9}$$

$$\sum_{e \in \mathcal{E}_b} y_{pe} \geqslant x_{pb}, \qquad \forall p \in \mathcal{P}, b \in \mathcal{B} \tag{10}$$

$$\phi_p y_{pe} \leqslant s_{pe} \leqslant \min\{c_e, u_p\} y_{pe}, \quad \forall p \in \mathcal{P}, b \in \mathcal{B}, e \in \mathcal{E}_b \tag{11}$$

$$s_{pi} \geqslant c_i (y_{pe} + y_{pj} - 1), \qquad \forall p \in \mathcal{P}, b \in \mathcal{B}, e, i, j \in \mathcal{E}_b | e < i < j \tag{12}$$

$$h_{pe} \geqslant y_{p,e+1} + y_{pe} - 1, \qquad \forall p \in \mathcal{P}, b \in \mathcal{B}, e = \alpha_b, \ldots, \beta_b - 1 \tag{13}$$

$$\sum_{p \in \mathcal{P}_t} h_{pe} \leqslant 1, \qquad \forall b \in \mathcal{B}, e = \alpha_b, \ldots, \beta_b - 1 \tag{14}$$

$$x_{pb} + x_{p'b} \leqslant 1, \qquad \forall (p, p') \in \mathcal{L}, b \in \mathcal{B} \tag{15}$$

$$x_{pb} - x_{p'b} = 0, \qquad \forall (p, p') \in \mathcal{H}_1, b \in \mathcal{B} \tag{16}$$

$$\sum_{b\in\mathcal{N}_a} x_{pb} - \sum_{b\in\mathcal{N}_a} x_{p'b} = 0, \qquad \forall (p,p') \in \mathcal{H}_2, a \in \mathcal{A} \quad (17)$$

$$x, y \text{ binary}, s, h \geqslant 0, 0 \leqslant v \leqslant 1. \quad (18)$$

The objective function (3) maximizes the impulse buying profit function per customer. Constraint (4) defines the visibility of a product category as a function of the space it is allocated and its attractiveness. By definition, the visibility of any product category on any edge e is bounded between 0 and 1. This represents the likelihood of a product category to be noticed on edge e, when it is visited. Constraint (5) defines a metric of the total visibility of a product category, that is the summation of the visibilities of a product category over all edges. Constraint (6) ensures that each product category is assigned to a single shelf. Constraint (7) ensures that the total shelf space allocated to a product category lies between its minimum and maximum space requirements. Constraint (8) requires the shelf space along any edge not to exceed its capacity. Constraint (9) guarantees that a product cannot be assigned to an edge unless it is allocated to its associated shelf. Constraint (10) requires any product group that is assigned to a shelf to be assigned to at least one of its edges. Constraint (11) enforces conditional lower and upper bounds on the space a product may be allocated along any edge. Constraint (12) requires any product that is assigned to a pair of edges, $e<j$, along the same shelf to entirely fill any intermediate edge i ($e<i<j$). Constraints (13)–(14) reflect that at most one product category can run over two contiguous edges. Constraint (15) prevents pairs of product categories having a lack of affinity from being assigned to the same shelf, whereas Constraints (16)–(17) require product pairs having high affinity to be assigned to the same shelf and the same aisle, respectively. Constraint (18) enforces logical binary and non-negativity restrictions on the decision variables.

SSAP belongs to a challenging class of generalized assignment problems with location/allocation considerations that is $\mathcal{NP}$-hard, even for its single-shelf single-group variant as discussed in Proposition (Section 2.3). Each shelf is a discretized knapsack with shelf segments of varying attractiveness and, in contrast with classical generalized assignment problems (Martello and Toth, 1990; Cattrysse and Van Wassenhove, 1992; Öncan, 2007), the specific location of a product on a shelf and its total space allocation impact the objective reward.

Estimating the shopping effort. In our setting, alternative shelf-space allocations are likely to influence the consumer shopping experience and convenience. Such trade-offs between impulse buying and shopping convenience can be accommodated in the proposed optimization methodology in the form of a shopping effort target (as in goal programming) or as a second objective term. We, however, recommend such developments for future research in concert with additional empirical analysis pertaining to customer segments, shopping lists, and in-store customer shopping paths (greedy, optimized, or random). That said, the proposed model partially takes into account the notion of shopping convenience by requiring certain pairs of products that have high sales affinity (eg, pasta and pasta sauce) to belong to the same shelf/aisle, as in Constraints (16)–(17).

The following metric may be used to quantify the overall shopping effort: $\mathcal{C} \equiv \sum_{p\in\mathcal{P}} \sum_{q\in\mathcal{P}:p<q} \chi_{pq}\Delta_{pq}$, where χ_{pq} captures the relative *association* between product categories p and q and Δ_{pq} estimates the distance between p and q. The affinity level between p and q, χ_{pq}, can be expressed as the frequency of buying p and q together. The distance Δ_{pq} can be approximated as the distance between the aisles where p and q are located (ie, the shorter the distance between aisles, the shorter the distance between p and q). As such, metric $\mathcal{C}$ can be used in a post-solution stage to estimate the shopping effort associated with a solution, or it can be cast as a quadratic constraint that is integral to Model SSAP by defining $\Delta_{pq} \equiv \sum_{a\in\mathcal{A}} \sum_{a'\in\mathcal{A}} \sum_{b\in\mathcal{N}_a} \sum_{b'\in\mathcal{N}_{a'}} \Delta_{aa'} x_{pb} x_{qb'}$, where $\Delta_{aa'}$ is the distance between aisles a and a'.

2.3. Single-shelf, single-group subproblem

As a precursor to the group-based model proposed in Section 2.4, we first present in this section a single-shelf, single-group model that specializes Model SSAP and optimizes the allocation of a specific group of product categories g to a specific shelf b in the store. The model aims at determining the optimal location of product categories along this shelf and their respective space allocation within their lower/upper bounds in order to maximize the impulse buying objective function. Let $\mathcal{Z}_g$ be a set of product categories in group g. Formally, we introduce the following model, denoted by SSAP(g,b):

$$\text{SSAP}(g,\ b):\ \text{Maximize} \quad \sum_{p\in\mathcal{Z}_g} \rho_p i_p v_p \quad (19)$$

$$\text{subject to} \quad v_{pe} = \frac{k_e s_{pe}}{c_e}, \qquad \forall p \in \mathcal{Z}_g, e \in \mathcal{E}_b \quad (20)$$

$$v_p = \sum_{e\in\mathcal{E}_b} v_{pe}, \qquad \forall p \in \mathcal{Z}_g \quad (21)$$

$$\ell_p \leqslant \sum_{e\in\mathcal{E}_b} s_{pe} \leqslant u_p, \qquad \forall p \in \mathcal{Z}_g \quad (22)$$

$$\sum_{p\in\mathcal{Z}_g} s_{pe} \leqslant c_e, \quad e \in \mathcal{E}_b \quad (23)$$

$$\phi_p y_{pe} \leqslant s_{pe} \leqslant \min\{c_e, u_p\} y_{pe}, \qquad \forall p \in \mathcal{Z}_g, e \in \mathcal{E}_b \quad (24)$$

$$s_{pi} \geqslant c_i(y_{pe} + y_{pj} - 1), \qquad \forall p \in \mathcal{Z}_g, e,i,j \in \mathcal{E}_b | e<i<j \quad (25)$$

$$h_{pe} \geqslant y_{p,e+1} + y_{pe} - 1, \qquad \forall p \in \mathcal{Z}_g, e = \alpha_b, \ \ldots\ , \beta_b - 1 \quad (26)$$

$$\sum_{p\in\mathcal{Z}_g} h_{pe} \leqslant 1, \quad e = \alpha_b, \ \ldots\ , \beta_b - 1 \tag{27}$$

$$y \text{ binary}, s, h \geqslant 0. \tag{28}$$

The objective function and constraints in (19)–(28) of SSAP(g,b) can be understood in a similar manner to those of model SSAP in Section 2.2.

Proposition 1 *The single-shelf single-group subproblem, modeled in* SSAP*(g,b), is* $\mathcal{NP}$*-hard.*

Proof The proof is established by reduction from PARTITION problem—a notoriously $\mathcal{NP}$-complete (Karp, 1972) problem.

PARTITION: Given a set S of n positive integers $a_1,\ldots,a_n$ with $\sum_{j=1}^{n} a_j = 2c$, decide whether there exists a partition of S into two disjoint subsets S_1 and S_2 such $\sum_{j\in S_1} a_j = \sum_{j\in S_2} a_j$.

Given an instance Π of PARTITION, it is possible to construct an instance $\mathcal{I}$ of the single-shelf single-group subproblem. To this end, consider the set $\mathcal{N}$ that includes $n+1$ items with the following characteristics:

- $\rho_j = 1, \phi_j = 0, \ell_j = u_j = a_j, \qquad \forall j = 1,\ldots,n.$
- $\rho_j = \epsilon$ (with $0 \leqslant \epsilon < 1$), $\phi_j = 0, \ell_{n+1} = u_{n+1} = c.$

The knapsack corresponding to the single-shelf includes three edges of equal size, ie, $c_e = c, \forall\, e = 1,2,3$. The traffic density of the inner edge is 0, whereas that of each outer edge is 1; in our notation, $k_1 = 1$, $k_2 = 0$, $k_3 = 1$.

We claim that this single-shelf single-group instance $\mathcal{I}$ has an optimal value $2c$ if and only if the PARTITION instance Π has a 'yes' answer.

Indeed, if there exists a partition of S into two disjoint subsets of S_1 and S_2 then the space that is required by each subset is exactly c and therefore the items of S_1 and S_2 can be assigned to Edges 1 and 3, respectively. In addition, item $n+1$ is assigned to Edge 2. This yields a total reward of $2c$.

Conversely, if a solution of the single-shelf single-group instance $\mathcal{I}$ has a corresponding reward $2c$, then item $n+1$ is necessarily assigned to Edge 2. Therefore, items in $\mathcal{N} \setminus \{n+1\}$ are assigned to Edges 1 and 3. Thus, the partition instance Π has a 'yes' answer. □

2.4. Group-based model

In this section, we propose a group-based model that allocates product category groups to shelves and subsets of groups to aisles that provides an alternative formulation to Model SSAP as discussed in Section 2.2. Consider the following input parameters:

- $\mathcal{J}$: Set of all pre-formed product category groups.
- $\mathcal{Z}_g$: Set of product categories in group g, $\forall g \in \mathcal{J}$.
- $\mathcal{G}_b$: Set of product groups that can be feasibly allocated to shelf b, because the total minimum space requirements of its product categories does not exceed the shelf capacity, that is, $\sum_{p\in\mathcal{Z}_g} \ell_p \leqslant \mathcal{C}_b, \forall b \in \mathcal{B}$.
- μ_{gb}: Impulse purchase profit for allocating group g to shelf b, $\forall b \in \mathcal{B}, g \in \mathcal{G}_b$. To determine μ_{gb}, we solve Model SSAP(g,b) for product group g and shelf b, for example, $\mu_{gb} = \sum_{p\in\mathcal{Z}_g} \sum_{e\in\mathcal{E}_b} (k_e \bar{s}_{pe}/c_e) i_p \rho_p$ where $\bar{s}_{pe}$ is the optimal shelf-space values of product categories in group g obtained by solving Model SSAP(g,b) from Section 2.3. Let $|\mathcal{B}| = |\mathcal{J}| = m$ be the number of shelves and groups, the computation of the μ_{gb} coefficients, $g = 1,\ldots,m$, $b = 1,\ldots,m$, requires solving the $\mathcal{NP}$-hard subproblem SSAP(g,b) $O(m^2)$ times.
- $\mathcal{Q}_a$: Set of ordered $|\mathcal{N}_a|$-tuples of product groups that can be allocated to aisle a that has $|\mathcal{N}_a|$ ordered shelves, $a \in \mathcal{A}$, where the kth group in the tuple is assigned to the kth shelf in aisle a. Here, ordering is based on the space requirements: The kth group of $\mathcal{Q}_a$ belongs to $\mathcal{G}_{b_k}$, where b_k is the kth shelf in aisle a, $k = 1, 2, \ \ldots\ , |\mathcal{N}_a|$. In particular, if $|\mathcal{N}_a| = 1$, we are considering a single-shelf aisle (eg, aisles {4,6} in Figure 3), and $\mathcal{Q}_a$ lists individual product groups that may be allocated to the single shelf in aisle a. Likewise, if $|\mathcal{N}_a| = 2$, we are considering two-shelf aisles (eg, aisles {2,3,5,7,8,9,10} in Figure 3) that can accommodate a pair of product groups and if $|\mathcal{N}_a| = 3$, we are considering three-shelf aisles (eg, Aisle 1 in Figure 3). Generalizing this process, an aisle with $|\mathcal{N}_a|$ shelves has an associated set $\mathcal{Q}_a$ of admissible $|\mathcal{N}_a|$-tuples of product groups. For example, in Figure 3, assume that {1,2} are a set of product groups that can be only assigned to one of the peripheral shelves, and {3,4} are a set of product groups that can be assigned to internal small shelves, respectively, because of their total

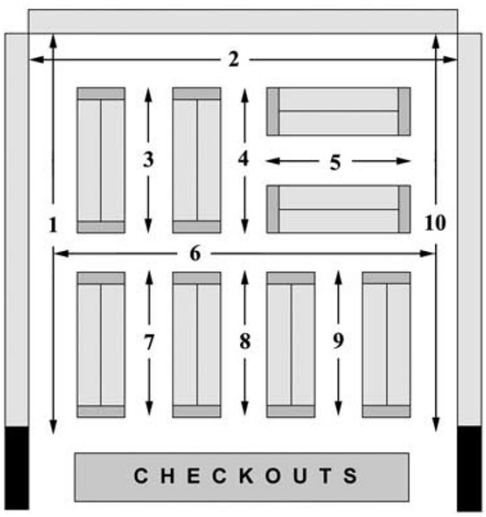

Figure 3 Grid layout displaying multi-shelf aisles.

minimum requirements. In this case, Aisle 1 in Figure 3 can accommodate only tuples, where one group is assigned to a peripheral shelf and the other two groups are assigned to a small shelf. Accordingly, for Aisle 1, $\mathcal{Q}_1 = \{(1,3,4),(1,4,3),(2,3,4),(2,4,3)\}$. In general, the generation of $\mathcal{Q}_a$ can be achieved by enumerating all groups in sets $\mathcal{G}_b$, $b \in \mathcal{N}_a$. Algorithm 1 formalizes the generation of $\mathcal{Q}_a$ for any aisle a.

- ψ_{wa}: The total impulse purchase profit for assigning an ordered $|\mathcal{N}_a|$-tuple of product groups, denoted by w, to aisle a, $\forall a \in \mathcal{A}, w \in \mathcal{Q}_a$. This impulse purchase profit is calculated as follows:

Algorithm 1 Generating $\mathcal{Q}_a, a \in \mathcal{A}$
1: Define the shelves in a as $b_{a,1}, b_{a,2}, \ldots, b_{a,|\mathcal{N}_a|}$.
2: Set $\mathcal{Q}_a \leftarrow \varnothing$.
3: **for all** $g_1 \in \mathcal{G}_{b_{a,1}}$ **do**
4: **for all** $g_i \in \mathcal{G}_{b_{a,i}} \big| \cup_{k=1}^{i-1}\{g_k\}, \quad \forall i \in 2 \ldots |\mathcal{N}_a|$ **do**
5: Set $w \leftarrow \{g_1, g_2, \ldots, g_{|\mathcal{N}_a|}\}$.
6: Set $\mathcal{Q}_a \leftarrow \mathcal{Q}_a \cup \{w\}$.
7: **end for**.
8: **end for**.

Algorithm 2 Generating ψ_{wa}, $w \in \mathcal{Q}_a, a \in \mathcal{A}$
1: **for all** $a \in \mathcal{A}$ **do**
2: **for all** $w \in \mathcal{Q}_a$ **do**
3: Define the groups in tuple w as $g_{w,1}, g_{w,2}, \ldots, g_{w,|\mathcal{N}_a|}$.
4: **for all** $k = 1$ to $|\mathcal{N}_a|$ **do**
5: Set $\psi_{wa} \leftarrow 0$.
6: Solve SSAP($g_{w,k}$,$b_{a,k}$).
7: Obtain the optimal objective value $\mu_{g_{w,k},b_{a,k}}$.
8: $\psi_{wa} \leftarrow \psi_{wa} + \mu_{g_{w,k},b_{a,k}}$.
9: **end for**.
10: **end for**.
11: **end for**.

$$\psi_{wa} \equiv \sum_{k=1}^{|\mathcal{N}_a|} \mu_{g_{w,k}b_{a,k}}, \tag{29}$$

where the $\mu_{g_{w,k}b_{a,k}}$ values are obtained from Model SSAP(g,b), for $g = g_{w,k}$, the k^{th} group in tuple w, and $b = b_{a,k}$, the k^{th} shelf in aisle a. Algorithm 2 can be used to estimate ψ_{wa}.

Setting up the group assignment problem involves the computation of 'input' parameters $\mathcal{Q}_a$ and ψ_{wa}, $\forall a \in \mathcal{A}, w \in \mathcal{Q}_a$, using Algorithms 1 and 2. This determines the subsets of groups (group tuples) that can be allocated to an aisle and the associated impulse buying reward. In Algorithm 1, to determine the set of product category groups that can be feasibly allocated to each shelf b, $\mathcal{G}_b$, can be calculated using $|\mathcal{B}||\mathcal{J}|$ operations while determining the subsets of group tuples that can be allocated to an aisle, $\mathcal{Q}_a$, requires $|\mathcal{J}|^{|\mathcal{N}_a|}$ operations. Hence the effort associated with Algorithm 1 can be expressed as $O(|\mathcal{B}||\mathcal{J}| + |\mathcal{J}|^{|\mathcal{N}_a||})$, that is, $O(|\mathcal{J}|^2 + |\mathcal{J}|^{|\mathcal{N}_a||})$ as the total number of shelves is equal to the total number of product category groups ($|\mathcal{B}| = |\mathcal{J}|$). Hence, the complexity of Algorithm 1 is $O(|\mathcal{J}|^{|\mathcal{N}_a|})$ when $|\mathcal{N}_a|>2$ or $O(|\mathcal{J}|^2)$ when $|\mathcal{N}_a| \leqslant 2$. In Section 3, the proposed case study involves 31 product groups ($|\mathcal{J}| = 31$), and the total number of shelves on each aisle, $|\mathcal{N}_a|$, varies between 1 and 2. In this case, the run time of Algorithm 1 was about one CPUs. Algorithm 2 requires solving Model SSAP(g,b) in an iterative fashion ($|\mathcal{A}||\mathcal{Q}_a||\mathcal{N}_a|$ times) that, in our experience, was achieved in manageable times for relatively large instances using modern-day computing capabilities.

Define the following binary variable for assigning product groups to aisles:

$\theta_{wa} \in \{0,1\}$: $\theta_{wa} = 1$ if and only if w, an $|\mathcal{N}_a|$-tuple of product groups, is assigned to an aisle a, $\forall a \in \mathcal{A}, w \in \mathcal{Q}_a$.

The group assignment problem, denoted by GAP, is a set partitioning formulation of the generalized assignment problem (Savelsbergh, 1997) that is known to be $\mathcal{NP}$-hard and is stated as follows:

$$\text{GAP}: \text{ Maximize} \quad \sum_{a \in \mathcal{A}} \sum_{w \in \mathcal{Q}_a} \psi_{wa}\theta_{wa} \tag{30}$$

$$\text{subject to} \quad \sum_{a \in \mathcal{A}} \sum_{w \in \mathcal{Q}_a : g \in w} \theta_{wa} = 1, \qquad \forall g \in \mathcal{J} \tag{31}$$

$$\sum_{w \in \mathcal{Q}_a} \theta_{wa} = 1, \qquad \forall a \in \mathcal{A} \tag{32}$$

$$\sum_{a \in \mathcal{A}} \sum_{w \in \mathcal{Q}_a : g, g' \in w} \theta_{wa} = 1, \qquad \forall (g, g') \in \mathcal{R} \tag{33}$$

$$\theta \text{ binary}. \tag{34}$$

The objective function of the model (30) maximizes the impulse purchase reward. Constraint (31) guarantees that every product category group must be assigned to one aisle. Constraint (32) guarantees that every aisle a (having $|\mathcal{N}_a|$ shelves) is assigned exactly one $|\mathcal{N}_a|$-tuple of product groups. Constraint (33) requires group pairs having high affinity to be assigned to the same aisle. Finally, Constraint (34) enforces logical binary restrictions on the decision variables.

A final note should be made regarding Model GAP. Whereas Model GAP undertakes the effort of forming and evaluating the reward of group tuples that can be feasibly assigned to aisles, it is conceivable to alternatively develop an assignment-type model using the coefficient μ_{gb} and binary variables for assigning a group to a shelf. Our aisle-based modelling choice in GAP, however, is better suited for future extensions that can take into account synergies because of cross-selling effects or traffic drivers within groups that are assigned to the same aisle.

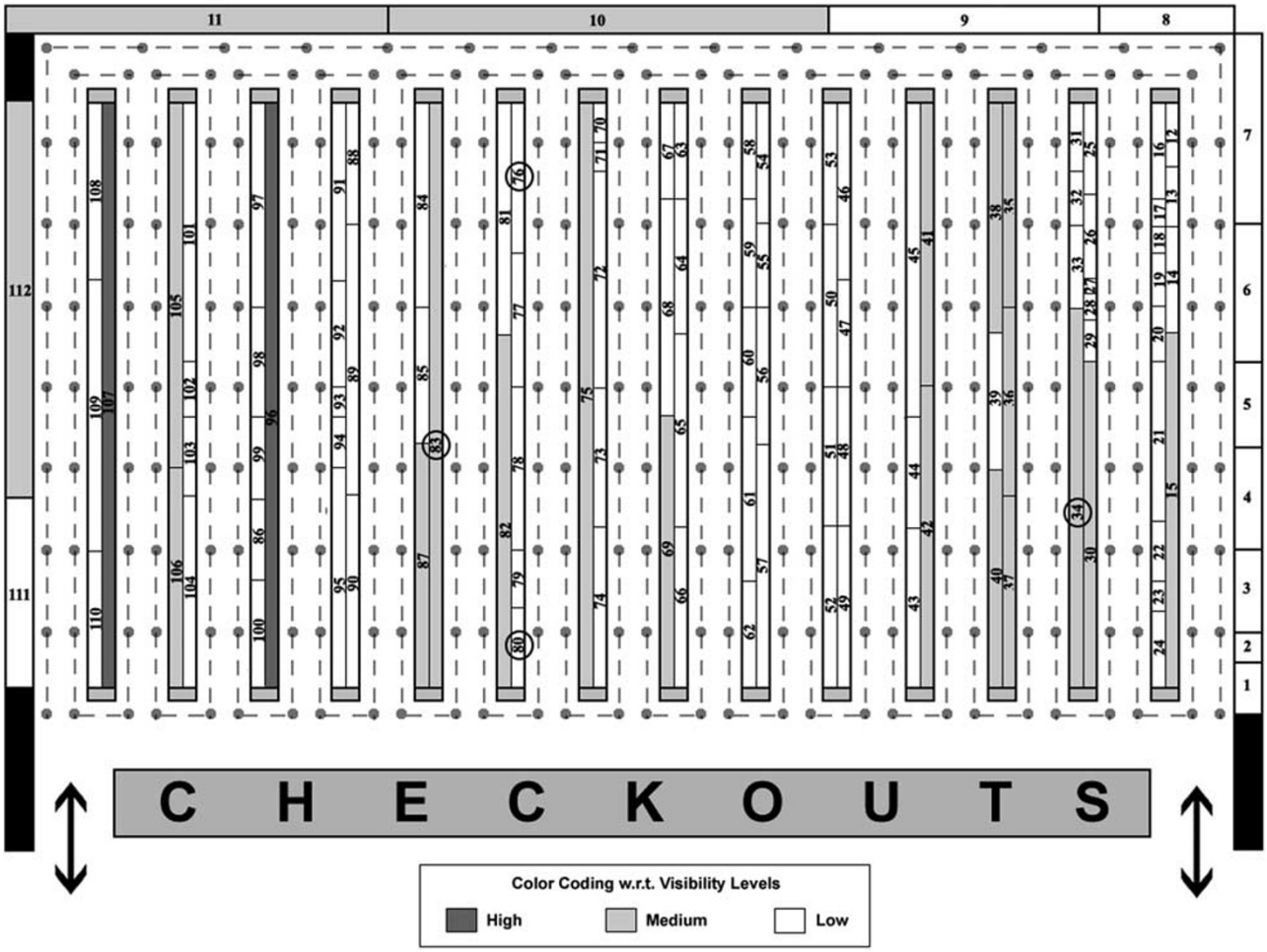

Figure 4 Shelf space allocation in a major grocery store in New England, USA.

3. Case study of a grocery store in New England

The usefulness of the proposed group-based model, GAP, is demonstrated using a grocery store in New England for which using the product-based MIP model in Section 2.2 is computationally prohibitive. Managerial insights into the structure of the optimal solution obtained by Model GAP are also discussed.

3.1. Store description and input data

Figure 4 presents the current shelf-space allocation in the store that is used as a benchmark solution in our study. For the convenience of the reader, some product categories are identified by a circle to facilitate the discussion of our observations and managerial insights in Section 3.2. As illustrated in Figure 4 and summarized in Table 1, this grocery store has a grid layout and comprises 31 parallel shelves. The problem involves 31 groups of product categories that need to be allocated to the store shelves with the objective of enhancing product visibility, thereby maximizing impulse buying. The product category groups consist of 112 product categories as listed in Table 1. Owing to the retailer's practice, product categories that are presently on the same shelf are deemed to form a group, as displayed in Table 1 and Figure 4. Moreover, the following pairs of groups are required to belong to the same aisle under the current, or any future, configuration: (8,9), (10,11), (12,13), (14,15), (20,21), (26,27) and (30,31). For instance, product groups 10 and 11 should be allocated to the same aisle, because both have products including dog needs.

In Figure 4, product categories have received a colour that reflects their current visibility levels to the consumer that, based on our discussion in Section 2, are estimated using the layout-based attractiveness of shelf segments (the parameter k_e for edge e) and the amount of space a product category p has received along some edge e, as dictated by the decision variable s_{pe}. Shelf segments were assigned one of three attractiveness levels (low, medium, or high) based on their location with numerical values k_e as detailed in the Appendix B.

The average unit profit of a product category, ρ_p, was estimated based on a limited data set we obtained. Although the unit profits could not be shared, we report in the Appendix B the unit revenues. The impulse purchase rate of a product

Table 1 Product category groups in a retail store in New England, the United States

Group	*Product category*
1	Eggs (1), Biscuits (2), Butter (3), Cheese (4), Sour cream (5), Yogurt (6), Juice (7)
2	Coffee cream (8), Milk (9), Quick meals (10), Fresh meat (11)
3	Pies & Toppings (12), Desserts & Fruits (13), Novelties (14), Ice cream (15)
4	Vegetables (16), Fries & Onions (17), Ethnic meals (18), Organic (19) Waffles (20), Breakfast (21), Bread & Bagels (22), Juice (23), Ice (24)
5	Poultry & Meats (25), Dinners (26), Entrée (27), Side dishes (28), Snacks (29), Pizza (30)
6	Stuffing & Coating (31), Bread crumbs (32), Gravies (33), Bread (34)
7	Peanut butter & Jelly (35), Stationery (36), Light bulbs (37)
8	Cups & Plates (38), Facial tissue (39), Bath tissue (40)
9	Wraps & Bags (41), Paper towels (42)
10	Automotive (43), Insecticides (44), Dry dog food (45)
11	Seasonal/Promo (46), Pet toys (47), Wet dog food (48), Dog treats (49)
12	Bird seed & food (50), Pet supplies (51), Kitty litter (52), Candles (53)
13	Hardware & Closet (54), Shoe care (55), Dry cat food (56), Wet cat food (57)
14	Bath needs (58), Mops & brooms (59), Fabric softeners (60), Household cleaner (61), Bleach (62)
15	Cleaning wipes (63), Air fresheners (64), Laundry detergents (65), Dish detergents (66)
16	Soup (67), Prepared foods (68), Canned vegetables (69)
17	Oil & Shortening (70), Vinegar (71), Condiments (72), Pickles & Olives (73), Salad dressings (74)
18	Soda (75)
19	Water (76), Seltzer (77), Juice (78), Cocoa & Ice tea (79), Tonic (80)
20	Snacks (81), Candy & Gum (82)
21	Potato chips (83)
22	International food (84), Asian food (85), Rice (87)
23	Canned fish (88), Pasta sauce (89), Pasta (90)
24	Instant coffee (91), Creamers & Filters (92), Hot tea (93), Rice cakes (94), Canned fruit (95)
25	Cookies & Cracker (96)
26	Spices (86), Kitchen utensils (97), Bakeware (98), Cake decorations (99), Sugar (100)
27	Foilware (101), Honey & Syrups (102), Flour (103), Cake mixes (104)
28	Fruit juice (105), Bottled juice (106)
29	Cereal (107)
30	Baby food (108), Baby needs (109), Toys (110)
31	Greeting cards (111), Diapers (112)

category, i_p, was estimated based on earlier studies in the literature (Kollat and Willett, 1967; Inman *et al*, 2009; Kacen *et al*, 2012) that suggest that hedonic products have higher impulse purchase rates than non-hedonic products. The findings by Kacen *et al* (2012) also indicate that low-priced products, products on sale, and hedonic products are more likely to have higher impulse purchase rates. Accordingly, products in our study were divided into three impulse purchase rate levels: low, medium, or high with numerical values ranging over [0,0.05], [0.1,0.15], and [0.2,0.45], respectively.

3.2. Qualitative insights into optimized versus benchmark solution

The current shelf-space allocation in the store, shown in Figure 4, serves as a benchmark for our study and yields an average impulse purchase profit metric of 128.9/customer in the objective function (30). Model GAP is used to prescribe a worthwhile re-allocation of shelf space to product groups. In doing so, the model also optimizes the internal shuffling of product categories inside their designated shelf in a fashion that maximizes the impulse purchase reward. The optimized solution is illustrated in Figure 5 and yields an average impulse purchase profit metric of 170.7/customer, or a 32% improvement over the benchmark solution in Figure 4. The optimized solution was obtained using our proposed methodology within 25 CPUs, whereas Model SSAP in Section 2 failed to produce any feasible solution to this problem within 1 CPU h using CPLEX 12.4. Further, the shopping effort, $\mathcal{C}$, increased by 8.1% in the optimized solution over the benchmark solution. That is, the optimized solution provides significantly improved profitability with a relatively small reduction in shopping convenience. The following qualitative insights are also derived by comparing the optimized solution against the benchmark solution:

- In the benchmark solution, most of the product categories that have high-planned demand are assigned to the shelf segments (edges) having high customer traffic. This can be suboptimal from an impulse buying viewpoint, because product categories having high-planned demand have relatively low impulse purchase rates. However, in the optimized solution, product categories having high demand are assigned to less attractive shelves, thereby saving the more visible shelf segments for product categories that have higher impulse purchase rates. For instance, Bread (#34), a high-planned-demand product in our study, is allocated to shelf segments close to the end of its aisle and the entrance, as illustrated in Figure 4. However, this product category was assigned to less attractive shelf segments in our optimized solution as in Figure 5.
- In the benchmark solution certain product categories having high impulse purchase rates are allocated to less attractive shelves. In the optimized solution, such products are assigned to more visible shelf segments (eg, closer to entrances/end of aisles). For instance, Potato Chips (#83) is allocated to a shelf that has a medium visibility level in the benchmark, whereas in the optimized solution its visibility is increased to high level since it is allocated to a more attractive shelf, near the entrance. Note that allocating Bread (#34), and its associated products in its group, to a less attractive shelf allowed Potato Chips (#83) to be allocated there instead, thus increasing its visibility.

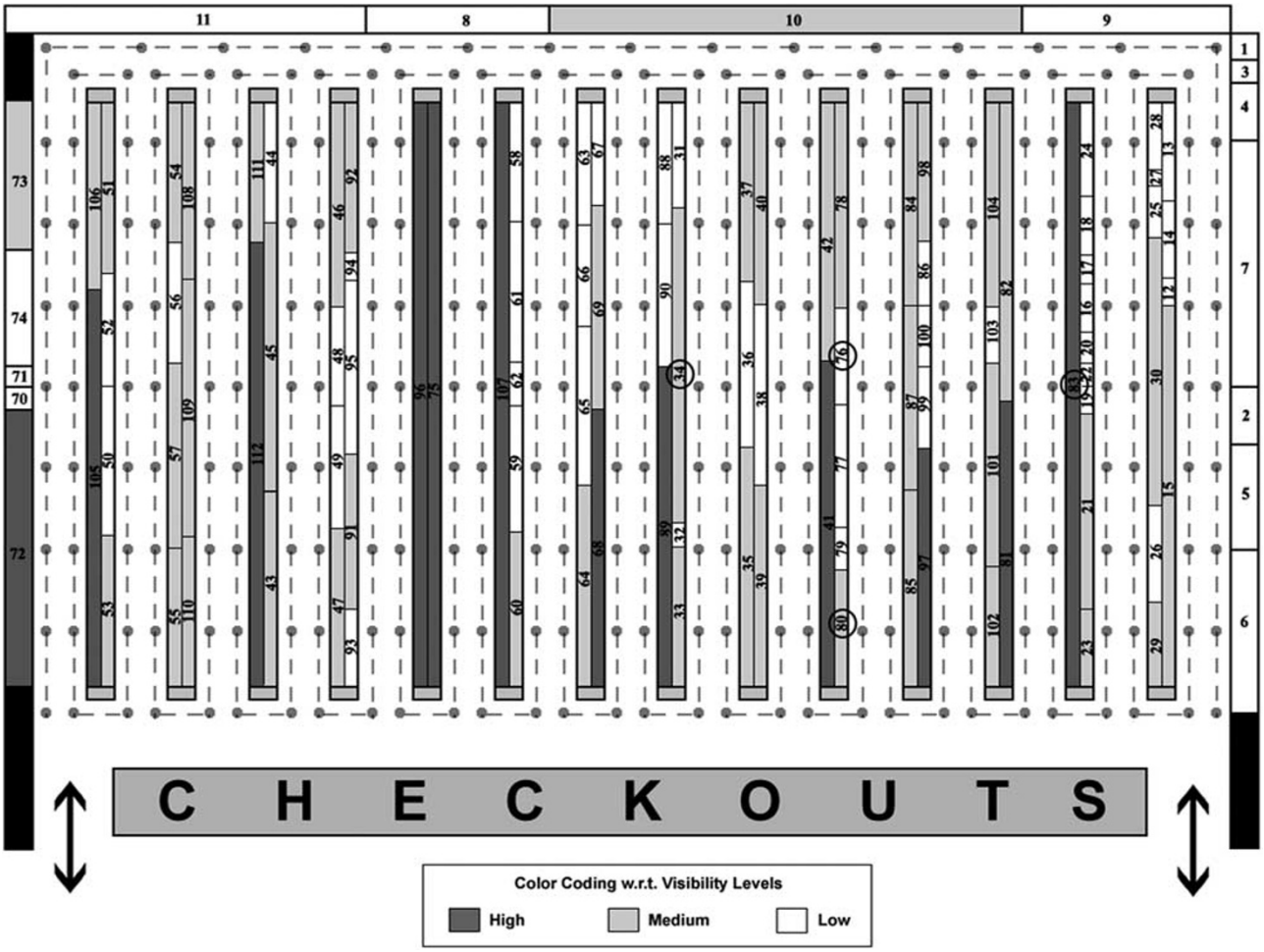

Figure 5 Optimized shelf space allocation for a major grocery store in New England, USA.

- Product categories having high sales volumes should be accommodated with a shelf space close to their minimum requirement. This allows high-impulse categories to be allocated more shelf space, thereby improving their visibility. For example, in the optimized solution, Water (#76) is allocated with its minimum requirement to allow more space for Tonic (#80) that have a higher impulse purchase rate. Hence, the visibility level of Tonic (#80) increases from low to medium. Here, it is worth mentioning that as for the chance of stock-outs, as a result of reducing the space allocated to non-impulsive products, the minimum space requirements that are incorporated in our model serve to reduce this risk. The purpose of enforcing the minimum space requirements is to avoid stock outs (Dreze *et al*, 1994), and to balance holding cost and fixed ordering (replenishment) cost. In fact, classical inventory theory can be used to set this minimum requirement by developing a lower bound on the space of each SKU in the category, and then aggregating. Note that allocating a product category to its minimum requirement might require more frequent replenishment. However, typically, fixed replenishment costs are not high in retailing, especially, that shelf replenishment is often done from a back room, as it is the case in supermarkets.

4. Computational study

In this section, the computational tractability of Model GAP is compared against that of Model SSAP in Section 2.2 using a set of instances generated over three grid layouts. For Model SSAP, affinity constraints, of the type (16) and (17), (see Section 2.2), have been adopted to ensure that (i) the same groups of products will be assigned to shelves as in Model GAP, and (ii) that groups with high affinity that should be assigned to the same aisle are consistent between GAP and SSAP. All mathematical programs were coded in AMPL and solved using CPLEX 12.4 on a Dell XPS 8300 workstation having a Intel Core(TM) i7-2600 CPU 3.40GHz processor and 12GB of RAM. We imposed a time limit of 1 CPUh within CPLEX 12.4.

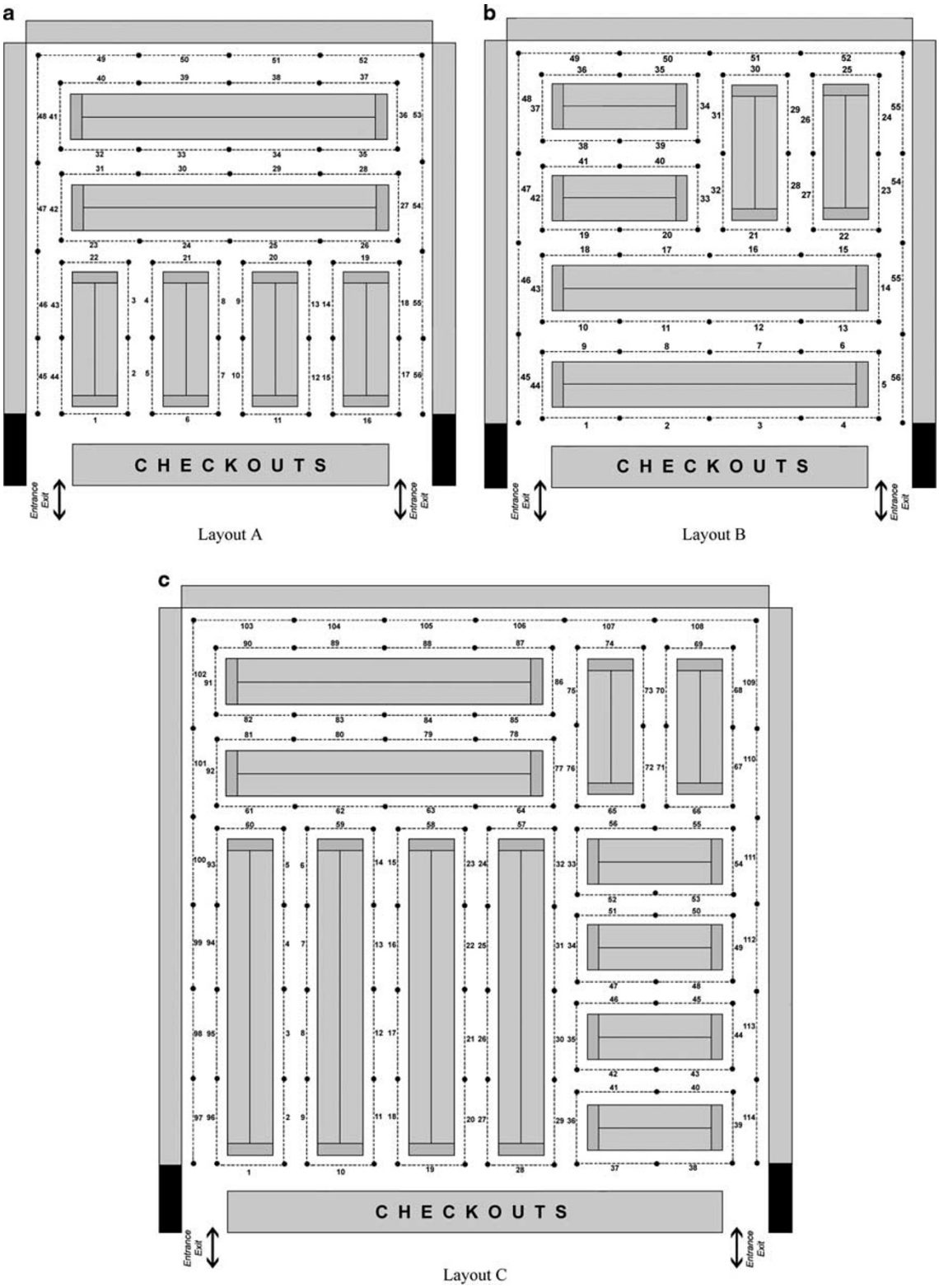

Figure 6 Grid layouts.

Table 2 Results for three grid layouts, three data sets for Model SSAP and Model GAP

Layout	$\lvert\mathcal{P}\rvert$	$\lvert\mathcal{J}\rvert/\lvert\mathcal{B}\rvert$	*Number of Tuples*	*Instance*	*Model SSAP*			*Model GAP*	
					CPU(s)	*B&B*	*Gap%*	*CPU(s)*	*B&B*
A	200	15	146	1	437.5	13296	0	2	0
			149	2	584.8	42174	0	2.2	0
			149	3	3600	1541590	0.01	2	0
			149	4	620.4	6544	0	2	0
			149	5	523.6	5094	0	2.3	0
B	200	15	95	1	1021.9	6042	0	1.5	0
			95	2	3600	619866	0.01	2.3	0
			84	3	2816.8	66993	0	1.9	0
			95	4	3600	513115	0.01	2	0
			95	5	2497.3	95033	0	2.4	0
C	250	27	550	1	3600	2151	3.8	4.2	0
			550	2	3600	1918	2.6	4.1	0
			550	3	3600	665	(unsolved)	4.5	0
			550	4	3600	2890	4.4	4.4	0
			550	5	3600	1259	4	5.2	0

4.1. Data generation

For each layout in Figure 6, one data set is considered, each comprising five problem instances. Key elements of our data generation scheme are summarized as follows: (i) Minimum and maximum shelf-space requirements of products were randomly generated using uniform distributions over ranges inspired from the case study in Section 3; (ii) impulse purchase rates were generated using a uniform distribution over the interval [0.4,0.5] for high impulse rates, [0.17,0.35] for medium impulse rates and [0.005,0.15] for low impulse rates; and (iii) a hierarchy of traffic densities related to shelf segments was generated, with values ranging from 0 to 1, depending on the segment location in the store.

4.2. Computational results for the group-based model

Table 2 reports our computational results using Layouts A – C. The first column provides the reference of the layout. The second column accounts for the total number of product categories to be allocated. The third and fourth columns report the number of available groups/shelves and the total number of feasible tuples that were generated, respectively (see Section 2.4 for details on tuple generation.) The fifth column indicates the instance number. The following three columns report CPU times (in seconds), the number of branch-and-bound/cut nodes, and the solver optimality gap(%) at termination when the time limit of 1 CPUh was reached for Model SSAP. The remaining two columns report CPU times (in seconds) and the number of branch-and-bound/cut nodes for Model GAP, respectively.

Our computational results demonstrate that Model GAP is well-suited for solving large-scale problem instances with pre-grouped product categories. It offers an effective alternative for Model SSAP that, because of the high combinatorial nature of the problem, turns out to be computationally intractable. All problem instances in our testbed were solved to optimality using Model GAP in under 5 CPUs, including the preprocessing time for tuple generation and computing input parameter values. In contrast, it was not possible to obtain an optimal solution for most of these problem instances using Model SSAP within 1 CPUh. The optimality gap observed for Model SSAP within CPU hour worsened for larger problem instances. For such instances, Model SSAP yielded a 3.7% optimality gap at an average, whereas all these instances were solved to optimality using GAP within 4s at an average. The results demonstrate that pre-calculating each optimal group-shelf allocation *a priori* as in Model GAP significantly improves the tractability of the problem and alleviates the combinatorial challenge it poses.

5. Conclusions and directions for future research

We have examined a shelf-space allocation problem where a retailer is interested in allocating grouped product categories to aisles and shelves in a fashion that maximizes the average impulse purchase profit per basket. Although the problem could be tackled by adapting an MIP in Ghoniem *et al* (2014b), its combinatorial nature makes this approach intractable. We propose an alternative algorithmic and optimization scheme that defeats the computational burden

of this problem by pre-computing the value of assigning certain grouped categories to aisles and shelves. This preprocessing task turns out to be highly effective and serves as an input to a group-based assignment formulation that enables exact solutions to large-scale problem instances within a few CPU seconds.

The usefulness of the proposed optimization model is demonstrated by examining a case study motivated by a grocery store in New England, the United States. The optimized solution was compared against the current shelf-space allocation in the store and revealed the following insights. First, product categories having high sales volumes should be assigned to less attractive shelves in order to reserve more attractive shelf segments for products having high impulse purchase rates. This increases the visibility of high-impulse products and, hence, the overall impulse purchase profits. Second, the shelf space allocated to a product category having high sales volumes should be close to its minimum requirement. This, in turn, reserves a more generous shelf space to the high-impulse categories, thereby improving their visibility. Third, grouped categories that include a high number of high-impulse products should be allocated to the shelves that have higher customer traffic density. These insights may be used as a guidance for the retailers in practice and the proposed optimization model can be employed to conveniently produce high-quality solutions.

At an operational level, it would be interesting to study the generation of detailed planograms based on the tactical, aggregate solutions our model produces. At a strategic level, we recommend for future investigation the integrated planning of layout decisions and tactical shelf-space allocation in a fashion that improves the customer flow in the store, the product visibility to shoppers, and the average retailer profit.

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References

Abratt R and Goodey SD (1990). Unplanned buying and in-store stimuli in supermarkets. *Managerial and Decision Economics* **11**(2): 111–121.

Amrouche N and Zaccour G (2007). Shelf-space allocation of national and private brands. *European Journal of Operational Research* **180**(2): 648–663.

Bellenger DN, Robertson DH and Hirschman EC (1978). Impulse buying varies by product. *Journal of Advertising Research* **18**(6): 15–18.

Borin N, Farris PW and Freeland JR (1994). A model for determining retail product category assortment and shelf space allocation. *Decision Sciences* **25**(3): 359–384.

Botsali AR (2007). Retail facility layout design, PhD Dissertation, Texas A&M University, USA.

Buttle F (1984). Retail space allocation. *International Journal of Physical Distribution and Material Management* **14**(4): 3–23.

Cattrysse D and Van Wassenhove L (1992). A survey of algorithms for the generalized assignment problem. *European Journal of Operational Research* **60**(3): 260–272.

Chen YL, Chen JM and Tung CW (2006). A data mining approach for retail knowledge discovery with consideration of the effect of shelf-space adjacency on sales. *Decision Support Systems* **42**(3): 1503–1520.

Clover VT (1950). Relative importance of impulse-buying in retail stores. *Journal of Marketing* **15**(1): 66–70.

Corstjens M and Doyle P (1981). A model for optimizing retail space allocations. *Management Science* **27**(7): 822–833.

Cox KK (1970). The effect of shelf space upon sales of branded products. *Journal of Marketing Research* **7**(1): 55–58.

Curhan RC (1972). The relationship between shelf space and unit sales in supermarkets. *Journal of Marketing Research* **9**(4): 406–412.

Desmet P and Renaudin V (1998). Estimation of product category sales responsiveness to allocated shelf space. *International Journal of Research in Marketing* **15**(5): 443–457.

Dreze X, Hoch SJ and Purk ME (1994). Shelf management and space elasticity. *Journal of Retailing* **70**(4): 301–326.

Farley JU and Ring LW (1966). A stochastic model of supermarket traffic flow. *Operations Research* **14**(4): 555–567.

Ghoniem A and Maddah B (2015). Integrated retail decisions with multiple selling periods and customer segments: Optimization and insights. *Omega* **55**: 38–52.

Ghoniem A, Maddah B and Ibrahim A (2014a). Optimizing assortment and pricing of multiple retail categories with cross-selling. *Journal of Global Optimization*,doi:101007/s10898-014-0238-3.

Ghoniem A, Flamand T and Maddah B (2014b). *Maximizing Impulse Buying via Store-Wide Shelf Space Allocation of High-Impulse Products and Traffic Drivers*. Isenberg School of Management, University of Massachusetts Amherst.

Hariga MA, Al-Ahmari A and Mohamed AA (2007). A joint optimisation model for inventory replenishment, product assortment, shelf space and display area allocation decisions. *European Journal of Operational Research* **181**(1): 239–251.

Hui SK, Fader PS and Bradlow ET (2009). The traveling salesman goes shopping: The systematic deviations of grocery paths from TSP optimality. *Marketing Science* **28**(3): 566–572.

Hui SK, Inman JJ, Huang Y and Suher J (2013). The effect of in-store travel distance on unplanned spending: Applications to mobile promotion strategies. *Journal of Marketing* **77**(2): 1–16.

Inman JJ, Winer RS and Ferraro R (2009). The interplay among category characteristics, customer characteristics, and customer activities on in-store decision making. *American Marketing Association* **73**(5): 19–29.

Iyer ES (1989). Unplanned purchasing: Knowledge of shopping environment and time pressure. *Journal of Retailing* **65**(1): 40–57.

Irion J, Lu JC, Al-Khayyal F and Tsao YC (2011). A hierarchical decomposition approach to retail shelf space management and assortment decisions. *Journal of the Operational Research Society* **62**(10): 1861–1870.

Kacen JJ, Hess JD and Walker D (2012). Spontaneous selection: The influence of product and retailing factors on consumer impulse purchases. *Journal of Retailing and Consumer Services* **19**(6): 578–588.

Karp RM (1972). Reducibility among combinatorial problems. In Miller RE and Thatcher JW (eds.) *Complexity of Computer Computations*: *Proceedings of a Sympposium on the Complexity of Computer*

Computations. The IBM Research Symposia Series, New York, NY: Plenum Press, pp 85–103.

Ke W and Van Ryzin G (2011). *Optimization of Product Placement in a Retail Environment*. Graduate School of Business, Columbia University.

Kollat DT and Willett RP (1967). Customer impulse purchasing behaviour. *Journal of Marketing Research* **4**(1): 21–31.

Larson JS, Bradlow ET and Fader PS (2005). An exploratory look at supermarket shopping paths. *International Journal of Research in Marketing* **22**(4): 395–414.

Lim A, Rodrigues B and Zhang X (2004). Metaheuristics with local search techniques for retail shelf-space optimization. *Management Science* **50**(1): 117–131.

Martello S and Toth P (1990). *Knapsack Problems Algorithms and Computer Implementations*. John Wiley & Sons: New York.

Öncan T (2007). A survey of the generalized assignment problem and its applications. *INFOR* **45**(3): 123–141.

Piron F (1991). Defining impulse purchasing. In: Holman RH and Solomon MR (eds). *Advances in Consumer Research*. Vol. 18, Association for Consumer Research: Provo, UT, pp 509–514.

Savelsbergh M (1997). A branch-and-price algorithm for the generalized assignment problem. *Operations Research* **45**(6): 831–841.

Yang MH (2001). An efficient algorithm to allocate shelf space. *European Journal of Operational Research* **131**(1): 107–118.

Appendix A

Total visibility of a product, v_p

The current form of v_p,

$$v_p = \sum_{b \in \mathcal{B}} \sum_{e \in \mathcal{E}_b} \frac{k_e s_{pe}}{c_e}, \tag{A.1}$$

assumes that product p can be bought *multiple times* along each edge where it is allocated. This can be seen as a reasonable approximating assumption since a product is typically allocated to one or two edges at most.

Exact form under an assumption on the direction of walking along a shelf

Assume that a customer walks along Shelf b according to the order of edges, $1,2,3,\ldots,|\mathcal{E}_b|$. Then, a customer either buys Product p from Edge 1, with probability $(k_1 s_{p1})/c_1$. Otherwise, if the customer does not buy p from Edge 1, the

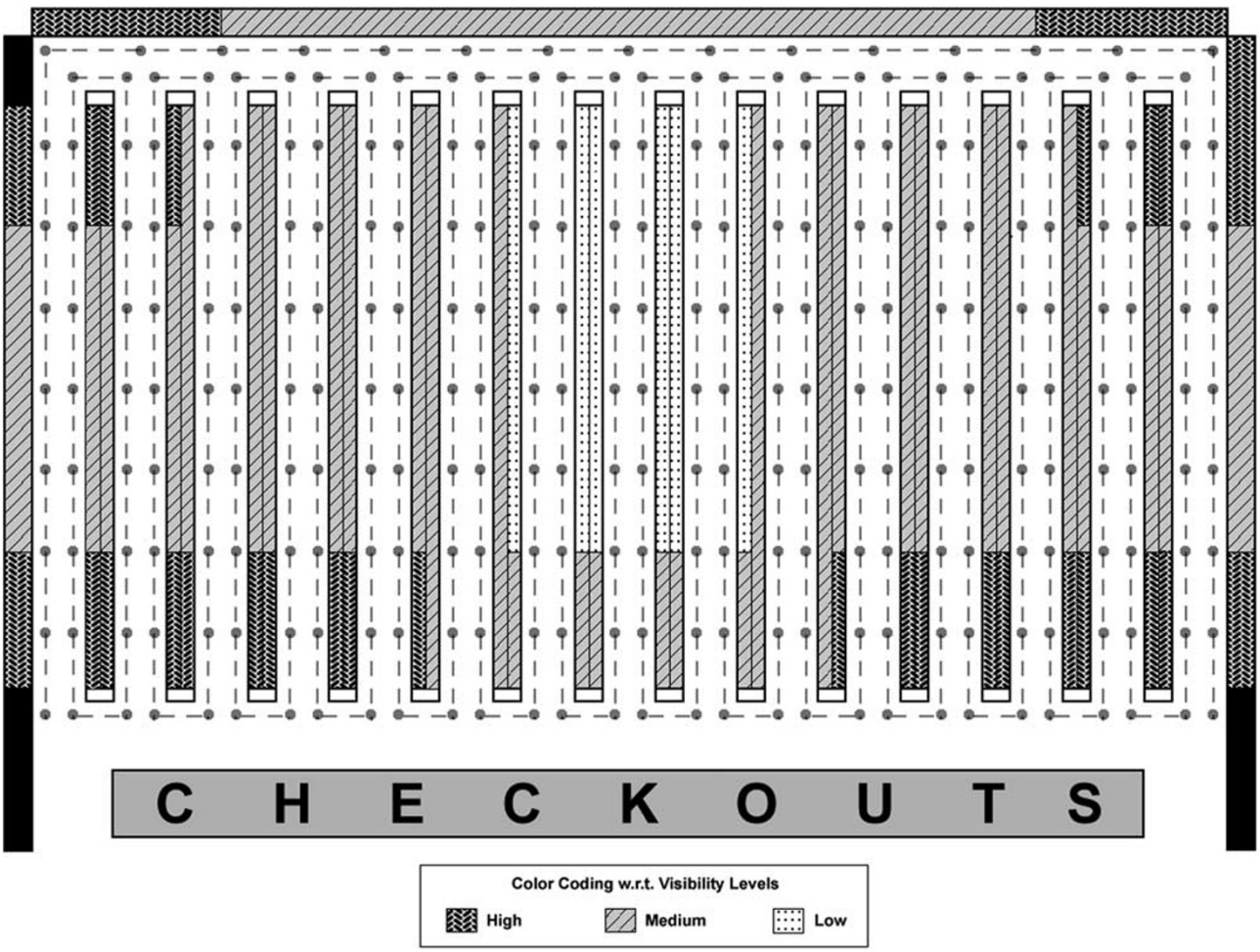

Figure B1 Shelf segment attractiveness / visibility levels.

customer could buy p from Edge 2, $(1-(k_1 s_{p1})/c_1)(k_2 s_{p2})/c_2$. In addition, if the customer does not buy p from Edges 1 and 2, the customer could buy p from Edge 3, $(1-(k_1 s_{p1})/c_1)(1-(k_2 s_{p2})/c_2)(k_3 s_{p3})/c_3$, and so on. The probability that the customer buys p from edge i is:

$$\left[\prod_{j=1}^{i-1}\left(1-\frac{k_j s_{pj}}{c_j}\right)\right]\frac{k_i s_{pi}}{c_i}.$$

This gives a total visibility of

$$v_p=\sum_{b\in\mathcal{B}}\sum_{i=1}^{|\mathcal{E}_b|}\left[\prod_{j=1}^{i-1}\left(1-\frac{k_j s_{pj}}{c_j}\right)\right]\frac{k_i s_{pi}}{c_i}, \qquad \text{(A.2)}$$

where sums over empty sets are set equal to 0.

Comparing the exact and the approximate form

Note that (35) and (36) can be rewritten as:

$$\tilde{v}_p=\sum_{b\in\mathcal{B}}\sum_{i=1}^{|\mathcal{E}_b|}\frac{k_i s_{pi}}{c_i},$$

$$v_p=\sum_{b\in\mathcal{B}}\left[\sum_{i=1}^{|\mathcal{E}_b|}\frac{k_i s_{pi}}{c_i}-\sum_{i_1=1}^{|\mathcal{E}_b|}\sum_{i_2=1,i_2\neq i_1}^{|\mathcal{E}_b|}\left(\frac{k_{i_1} s_{pi_1}}{c_{i_1}}\right)\left(\frac{k_{i_2} s_{pi_2}}{c_{i_2}}\right)-\cdots\sum_{i=1}^{|\mathcal{E}_b|}\cdots\sum_{i_j=1,i_j\neq i_{j-1}\neq\ \ldots\ i_1}^{|\mathcal{E}_b|}\prod_{l=1}^{i_j}\left(\frac{k_l s_{pl}}{c_l}\right)-\cdots\prod_{l=1}^{|\mathcal{E}_b|}\left(\frac{k_l s_{pl}}{c_l}\right)\right].$$

This gives another justification for (35) as a first-order approximation of (36). For example, if a product p is allocated

Table B1 Unit revenues and impulse purchase rates of product groups for a grocery store in New England, the United States

p	*Unit revenues*	i_p	p	*Unit revenues*	i_p	p	*Unit revenues*	i_p
1	3.09	0.03	39	3.94	0.12	77	4.2	0.14
2	1.62	0.24	40	4.56	0.04	78	4.03	0.22
3	4.32	0.009	41	3.86	0.14	79	2.57	0.23
4	3	0.04	42	8.52	0.01	80	6.86	0.33
5	3.4	0.12	43	7.13	0.21	81	3.47	0.45
6	3.22	0.22	44	5.75	0.13	82	1.31	0.45
7	6.88	0.04	45	6.35	0.0001	83	2.5	0.45
8	4.25	0.13	46	3.5	0.41	84	4.39	0.24
9	6.35	0.005	47	4.58	0.42	85	9.2	0.24
10	4.8	0.21	48	1.16	0.11	86	8.57	0.12
11	5.34	0.03	49	1.91	0.31	87	6.26	0.01
12	4.35	0.31	50	8.75	0.038	88	2.67	0.13
13	4.2	0.43	51	11.15	0.22	89	2.175	0.22
14	4.21	0.42	52	5.8	0.02	90	1.95	0.04
15	7.32	0.4	53	16.15	0.42	91	4.69	0.15
16	3.93	0.12	54	5.59	0.14	92	7.02	0.15
17	4.38	0.15	55	10.68	0.13	93	15.05	0.12
18	3.04	0.22	56	6.9	0.008	94	2	0.1
19	4.84	0.13	57	1.36	0.11	95	1.98	0.23
20	1.64	0.2	58	3.72	0.24	96	2.86	0.4
21	4.36	0.24	59	5.37	0.03	97	14.25	0.11
22	1.94	0.21	60	9.79	0.13	98	8.8	0.13
23	6.73	0.2	61	3.35	0.04	99	4.58	0.24
24	34.6	0.02	62	2.52	0.03	100	4.8	0.03
25	4.05	0.01	63	6.14	0.12	101	1.86	0.14
26	6.04	0.23	64	3.74	0.34	102	12.58	0.24
27	4.95	0.22	65	10.56	0.01	103	6.1	0.03
28	5.83	0.24	66	3.88	0.01	104	2.34	0.14
29	5.18	0.32	67	2.04	0.1	105	10	0.23
30	3.05	0.34	68	4.23	0.34	106	6.96	0.23
31	2.97	0.14	69	1.28	0.14	107	6	0.11
32	3.06	0.1	70	7.73	0.02	108	4.39	0.02
33	10.23	0.12	71	5.24	0.008	109	7.37	0.002
34	3.25	0.01	72	9.6	0.23	110	7.1	0.42
35	3.7	0.11	73	6.76	0.22	111	3.13	0.03
36	2.88	0.02	74	3.94	0.22	112	34.31	0.02
37	7.52	0.03	75	1.61	0.14			
38	4.3	0.02	76	1.83	0.001			

to Edges 1 and 2 of a shelf, then:

$$\tilde{v}_p = \frac{k_1 s_{p1}}{c_1} + \frac{k_2 s_{p2}}{c_2},$$

$$v_p = \frac{k_1 s_{p1}}{c_1} + \frac{k_2 s_{p2}}{c_2} - \left(\frac{k_1 s_{p1}}{c_1}\right)\left(\frac{k_2 s_{p2}}{c_2}\right).$$

Appendix B

The generation of traffic densities, unit revenues, and impulse purchase rates

Figure B1 demonstrates a heat map for the grocery store in New England, examined in Section 3. As illustrated in Figure B1, shelf segments that are close to the end-of-aisles/entrances tend to have higher customer traffic density, whereas shelf segments located in the middle of aisles usually have low customer traffic. On the basis of that, to estimate the numerical values of customer traffic density k_e, shelf segments were divided into three categories, having low, medium, high customer traffic density as shown in Figure B1. Then the numerical values were randomly generated using the following intervals for illustrative purposes:

- $k_e \in [0.05, 0.25)$ for shelf segments with low traffic density.
- $k_e \in [0.35, 0.65)$ for shelf segments with medium traffic density.
- $k_e \in [0.75, 1)$ for shelf segments with high traffic density.

Moreover, unit revenues and impulse purchase rates of product categories are listed in Table B1. Unit profit margins of product categories are confidential and, therefore, not provided. Instead, we displayed unit revenues for interested readers.

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