Applied Mathematical Modelling 78 (2020) 217–231

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Temporary price increase during replenishment lead time

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ARTICLE INFO

Article history:
Received 29 March 2019
Revised 25 July 2019
Accepted 25 September 2019
Available online 8 October 2019

Keywords:
Pricing
Continuous review
Lost sales
Inventory control
Replenishment policy

ABSTRACT

Pricing and inventory management make up together revenue management, which is a significant effort to boost revenues out of available resources. Firms use various forms of dynamic pricing, including personalized pricing, markdowns, promotions, coupons, discounts, and clearance sales, to respond to market fluctuations and demand uncertainty. In this paper, we study a temporary price increase policy, a form of dynamic pricing, for a non-perishable product, a practice used by several giant retailers such as Amazon, Walmart, and Apple. We develop a continuous review inventory model that allows for joint replenishment and pricing decisions, where the lead time is not zero. A replenishment decision controls supply, while a pricing decision controls demand. A manager exercises a temporary price increase to slow demand and avoid a stock-out situation while waiting for a shipment, which may not necessarily increase revenues, but decrease stock-out costs. The problem is to solve for the optimal replenishment and the pricing policy parameters that maximize the long-run expected profit. That is, when and how much to order and when to raise the price. In this paper, the inventory level and time trigger a price increase. We solve many numerical examples and perform extensive sensitivity analyses. Our results show that compared to a model that focuses on fixed pricing, our model brings an additional increase in profit of about 13%.

1. Introduction

Matching supply and demand has always been one of the challenging issues faced by practitioners and supply chain researchers. Vitasek et al. [1] suggest accounting for demand volume and variability in chronic supply-demand mismatch situations. Over the years, developments in both customer and supply sides lead to even increasing demand variability, thus more challenges in demand satisfaction. On the customer side, one factor is to increase the demand variation coefficient per product is the increase in product variants (Ton and Raman [2]). Another factor is the easy access to more sellers regardless of distance through online selling (Forman et al. [3]). On the supply side, long lead times result in higher demand uncertainty during the lead time (Allaire and Firsirotu [4]). Firms across the world have been heavily relying on non-domestic outsourcing partners to lower their procurement costs, increase flexibility, and access to new technology, talent, and proven

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https://doi.org/10.1016/j.apm.2019.09.050
0307-904X/

provider offerings (Fersht [5]). While there was a peak in total global outsourcing size in 2014 with 105 billion USD, the total value is still over 85 billion USD in 2018 (Mazareanu [6]). Non-domestic outsourcing freight may add on average four to six weeks to the delivery time (Quint and Shorten [7]). Moreover, high average lead time is generally accompanied by high lead time variability (Wacker [8]). Long lead time and high lead time variability increase supply uncertainty and stock-out risk for the customers. According to Corsten and Gruen [9], only 15% of customers realizing stock-outs will postpone the purchase to a later time when the item is back in stock.

Despite intense efforts, there is no clear-cut solution for a perfect match, mainly due to the unavoidable variability in both demand and supply. To counteract the demand and supply mismatch, firms may employ various emergency actions such as emergency orders from the supplier, expediting outstanding orders, or transshipping from a same-echelon partner. It is often very costly to reorder, expedite, or reallocate inventory from one store to another, while it is easier for retailers to change prices. Bhargava et al. [10] state that "short-lived price changes can be announced quite costlessly online". Price change is one of two main wings of revenue management that allows a firm to control its sales to get more revenues out of available supply (Talluri and van Ryzin [11]), where the second wing is inventory allocation. Firms use various forms of dynamic pricing, including personalized pricing, markdowns, promotions, coupons, discounts and clearance sales, to respond to market fluctuations and demand uncertainty.

When the demand seems to be higher than the available supply, most online retailers, such as Amazon, Walmart, and Apple, that sell non-perishable goods temporally increase prices to slow demand and counter the possibility of stock-outs. Baker et al. [12] mention that the internet allows for the dynamic pricing of products, without ignoring the conditions of companies and the behavior of customers. For a detailed review of the impact of the internet on pricing strategies, the reader is referred to Chan et al. [13].

A case study is conducted by a leading online retail sales research company to investigate the practice of a price increase on Amazon.com for real products (Sellics [14]). The case study indicates that if a seller on Amazon is low on inventory and stock levels will not hold out until the next shipment arrives, the seller can rather raise the price on the remaining products to slow down the demand. Then, going out of stock can be avoided or postponed, and product ranking will not drop sharply due to stock-outs. The study concludes that a slight increase in price could be legitimate, while a considerable increase can lower the rankings unexpectedly.

Upstream Commerce, a retail intelligence company that uses predictive and dynamic pricing solutions, analyzed the pricing behaviors of Amazon, Walmart, Best Buy, Macy's and Zappos over the holiday shopping seasons of 2011, 2012 and 2013 (Businesswire.com [15], Retailewire.com [16]). The company found that Amazon raised its prices for Cyber Monday when other retailers discounted theirs on Black Friday. Although faced by a surge in demand, Walmart's average prices were higher on both Black Friday and Cyber Monday (in 2011). The study notes that Amazon's behavior on Cyber Monday was similar to Walmart's. It raised prices of many less expensive products that are usually not restocked once they go out of stock.

The temporary price increase applies especially to seasonal and fad items, whose demand is often difficult to predict. For example, at Christmas, it is so difficult to accurately predict which toy will rank at the top of sales until the season begins. Because of long production times, it is not always possible to fulfill shortages quickly, which pushes retailers to increase the prices temporarily (Mochizuki [17]). For example, in the Christmas of 2006, respectively, Tickle-Me Elmo (a list price of $39.99) and Nintendo Wii (a list price of $539.93) were in such a short supply that sellers on eBay were asking $100 and $3000 for these items (Taylor [18]).

Chip prices more than doubled in the year before Apple announced its launching of the iPhone 8. The demand for this phone was expected to worsen the global squeeze on the supply chips. Clients for phone chip suppliers are accepting higher prices to make sure they get enough memory chips for their products (Lee [19]). The iPhone remains to be a critical source of demand as per its huge sales volume, and recent moves to increase storage capacity on the device. The surge in demand has affected the NAND market. Long [20] notes that a shortage of NAND in 2017 will boost the prices of SSD (solid-state drives), by about 9%, which are the standard for fast storage. Shortages of LCD screens, resulting from factories having difficulty in procuring essential raw material, required Apple to temporarily increase the prices of replacing them with iPhone models 5, 5C, and 5S ([21]). Similarly, Parallax Inc., a provider of electronic hardware and software such as RFID systems announced on July 14, 2014, that while they were resolving an agreement with the intellectual property legal company representative on a patent right issue, they temporarily increased the prices of RFID tags to reduce customer demand. This price increase decreased demand for RFID by about 70%. After resolving the royalty issues, Parallax reduced RFID tag prices by 70% (Gracey [22]).

Another example applies to the US car industry, where dealers started realizing shortages for some Japanese vehicles in the months following the 2011 earthquake in Japan. According to Boudette [23], to keep up with the supply shortage, Japanese vehicle prices have increased by an average of $400 per vehicle. In this regard, he wrote "new-car sales were running at a slower pace than before the quake..., but store profit was well above the previous year's level and almost at its target level".

The above examples are actual practices of temporary price increases adopted by many companies from various industries, e.g., retail, high tech, and automotive manufacturing, to name few. The literature on dynamic pricing has focused solely on varying prices dynamically in the context of markdowns, promotions, discounts, clearance, personalized pricing, in the face of fluctuating demand. To the best of our knowledge, the Operations Management literature shows no work on the practice of temporary price increase during the lead time to cut down on the lost sales caused by product shortages. Thus, this study is the first to address this topic.

We consider a continuous review inventory replenishment model for a single product with a fixed ordering cost and non-zero lead time. An order is triggered whenever inventory drops to a predetermined level. Shortages result in lost sales. During an order lead time, the firm increases the price temporarily to reduce the effect of an inventory shortage. After an order arrives, the firm charges back the original product price. This paper illustrates how a temporary price increase can be used to better match supply and demand in the case of supply shortages and would yield significant profit improvements. Our objective is to develop a model to determine the optimal inventory replenishment and pricing policy under a temporary price increase. We then compare the results to a fixed price policy to gain insights into the benefits that a temporary price increase brings to a company. Our results indicate that introducing a temporary price increase has negligible effects on the regular replenishment policy. Its significance becomes more pronounced when the reorder level is lower. Moreover, it yields a higher increase in profits for relatively slow-moving and price-sensitive products.

In our model, we define two price change trigger variables. The price is changed when the inventory during a replenishment lead time hits a trigger level in a specific time window to be determined. The rationale for a temporary price increase is to slow demand and buffer against supply shortage until a shipment is received. We formulate a long-run expected profit function for the continuous review problem with price increase applied during the lead time.

Analytical investigation of simultaneous optimization of the inventory replenishment and pricing decisions is difficult when it has four decision variables: the inventory replenishment decisions, when and how much to order, and the price change decisions, when to increase the price as a function of both the inventory level and time. Thus, we conducted extensive numerical analyses because it was difficult for us to provide an analytical solution due to intractable mathematics. We observe that a temporary price increase during the lead time, which leads to considerable profit improvements, has valuable managerial implications as this policy is easy to implement in practice. It consists of one price change only, as opposed to multiple price changes suggested in the literature. Despite its simplicity, it leads to significant profit improvements. Adopting a single price change in which price switches from regular to high, and back simplify operations, is easy to implement, and cheaper than more complex policies.

The authors believe that this paper is the first to introduce a single price increase as a hedge against supply shortage, and the first in the dynamic pricing literature, in the context of markdowns and promotions, to consider a fixed cost and non-zero lead time. Unlike the work on dynamic pricing in the literature, this paper assumes continuous review with a fixed cost and non-zero lead time. The presence of a non-zero lead time significantly complicates the model. In this regard, we believe the contribution of this paper is significant and enriches the relevant literature. We also offer a simple policy that is easy to implement in practice and demonstrate that it would have significant benefits if adopted. When compared to the conventional model, the approximate (exact) solution improved profits, on average, by 9% (13%).

This paper is organized as follows. The related literature is reviewed in Section 2. In Section 3, we develop the objective function by calculating the expected revenue, costs, and cycle time conditioned on the price change probability. Section 4 reports numerical results and analyses and discusses them. It also provides some managerial insights. Moreover, we show how simplified models can be utilized to obtain significant benefits with a price change. We present our conclusions and future directions in Section 5.

2. Literature review

Our study is related to two distinct streams of research, joint pricing and inventory replenishment, and emergency orders. The literature on the joint optimization of pricing and inventory decisions is quite rich and has been referred to repeatedly in studies on revenue or yield management (Gallego and Van Ryzin [24], Elmaghraby and Keskinocak [25]).

In the joint pricing and inventory replenishment research stream, the earliest work is that of Whitin [26], which models price-dependent demand in the inventory replenishment problem and determines the price endogenously. For replenishable products, pricing decisions are made either after every customer arrival or every replenishment period. Federgruen and Heching [27] and Chen and Simchi-Levi [28] model periodic review systems where a firm determines, in each period, its joint inventory and pricing policy before realizing demand. Chen and Simchi-Levi [29] and Chao and Zhou [30] investigate joint dynamic pricing and inventory policies for a continuous review system. They recommend altering the price after every demand arrival. Chen et al. [31] consider a continuous review inventory model that allows for multiple price changes at different inventory levels between two replenishments. The studies surveyed above assume the lead time is zero, assuming otherwise remains a challenging problem to solve (Chen and Simchi-Levi [29]). Our work contributes to the dynamic pricing literature in that it develops a model with a non-zero lead time. For complete reviews on the joint dynamic pricing and inventory control problems, see Elmaghraby and Keskinocak [25], Yano and Gilbert [32], and Gimpl-Heersink [33].

Although the research on joint control of inventory and pricing uncovers the benefits of dynamic pricing, it shows that few price changes are enough to attain the objective. Chen et al. [31] and Gayon et al. [34] show that two price changes are enough to reap the benefits of dynamic pricing. Gallego and van Ryzin [24] show that single price change policies asymptotically reach the optimal prices with an increase in sales. Similarly, we consider a single price change during replenishment lead time and investigate its benefits. Moreover, for a continuous review, zero lead time, and backlogging model, Chao and Zhou [30] show that as the inventory level decreases (as long as it is non-negative), the optimal price to charge increases.

Table 1
Notations.

Parameters	
K	Fixed ordering cost per order
c	Procurement cost per unit
h	Inventory holding cost per unit per unit-time
b	Shortage cost per unit demand lost
L	Order lead time
p_1	Regular unit selling price
p_2	Temporary unit selling price during the lead time, where $p_2 > p_1$
$\mu(p)$	Mean demand rate per unit-time as a function of price p
$D(p, t)$	Demand at time t for price p with a probability density function $f(\cdot, p, t)$, a cumulative distribution function $F(\cdot, p, t)$, and a mean $\mu(p)$
Variables	
Q	Order quantity
R	Reorder point
r	Inventory level that triggers a temporary price increase from p_1 to p_2, where $r < R$
T	Time window within which a price change can be made after an order release
Other Variables	
τ	Time interval during which the inventory level drops from R to r with probability density function $g(t)$ and cumulative distribution function $G(t)$
θ	Probability of utilizing a price change during a cycle, i.e. $\theta = P(\tau \leq T)$.

So, when a single price change is allowed, it usually results in a price surge. Similarly, we consider a price increase during the lead time, while the inventory level is decreasing.

Our study is also related to the research works that consider emergency shipments between subsequent regular replenishment orders. There are various methods to deal with emergency shipments in the literature. For example, last-minute emergency orders from a single (Moinzadeh and Nahmias [35]) or multiple suppliers for higher unit prices (Tomlin [36]), order splitting (Thomas and Tyworth [37]) and reduce the order lead time (faster production and delivery) at an additional cost (Duran et al. [38]). Moreover, when the regular replenishment from the manufacturer is not available to satisfy immediate demand, the transshipment of goods within the same echelon can be initiated, such as trading vehicles among dealers when the shipment from the factory is still due (Çömez et al. [39]).

This paper is closely related to a few earlier works that study changing demand patterns between replenishments when inventory is low. Xu et al. [40] assume that a retailer stops selling an item when its inventory level is critically low. They use this policy to prevent shortage costs. Cheung [41] models an inventory situation as a continuous review problem where a retailer offers a price discount to customers willing to backorder their demand until the next replenishment. Not accepting the offer results in a lost sale. DeCroix and Arreola-Risa [42] introduce offering compensation to convince customers to backorder in an (s, S) inventory model. Ding et al. [43] consider multiple customer classes, where each has its price discount for back-ordering demand. Using a deterministic EOQ model, Bhargava et al. [10] determine the lengths of the in-stock and stockout periods and the item's price. Drake and Pentico [44] modify an EOQ by making the percentage dependent on the size of the discount. They investigate the optimality of offering a discount and its size.

In this paper, we consider only one price change during a replenishment cycle. Chen et al. [31] show that a single price change (i.e. two prices) is enough to reach the most benefit. A firm may avoid emergency orders if it exercises a price change. Moreover, the period during which the firm needs an emergency action may not be long enough to accommodate multiple price changes. Excessive price changes may be costly depending on the sales channel. It may be, among many, updating the price tags and the associated web pages. Gayon et al. [34] discuss that frequent price changes are perceived negatively by customers.

3. Model

We study a continuous-review inventory replenishment model of a firm that is managed by the well-known (Q, R) policy, where R is the reorder point and Q the order size. Thus, whenever the inventory level hits R, an order of size Q is placed. The system goes through one replenishment cycle between the placement of two successive orders, where the inventory level R renews itself(goes from R back to R). The notations used are listed in Table 1 below.

In a continuous review inventory model, when the unmet demand is lost, the number of outstanding orders is the largest integer less than or equal to $(Q + R)/Q = 1 + R/Q$ (Tekin et al. [45]). Then, if we assume that $R < Q$, it is always true that there is only a single order outstanding, which is an assumption that has been extensively used in the literature; e.g., Hadley and Whitin [46], Archibald [47], Moinzadeh and Nahmias [35], Cheung [41], Johansen and Thorstenson [48], Tekin et al. [45], and Duran et al. [38].

During a replenishment cycle, the firm faces a price-sensitive, stochastic demand. Let $D(p, t)$ be the random demand at time t for price p. $D(p, t)$ has a probability density function $f(\cdot, p, t)$, cumulative distribution function $F(\cdot, p, t)$, and mean $\mu(p)$. We assume that demand in consecutive periods is independent. Each item kept in inventory for a unit of time incurs a unit cost of h. If demand at a point in time exceeds the inventory on-hand, then unmet demand is lost at a unit cost

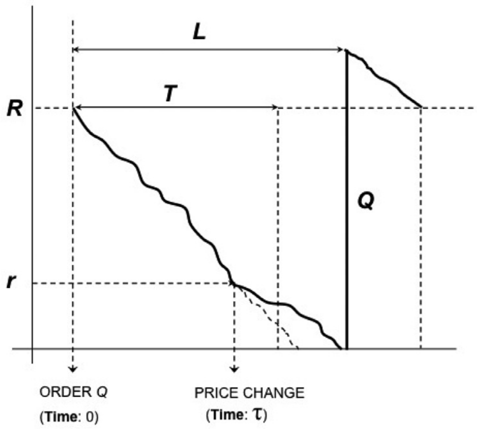

Fig. 1. Inventory dynamics with a temporary price increase from p_1 to p_2.

of b. The lost sales cost may include losing customer goodwill and future sales, and any compensation paid to unsatisfied customers (Petruzzi and Dada [49] and Serel [50]).

In the (Q, R) inventory model with at most one outstanding order at any given time, lost sales are realized during the order lead time L, which starts right after the inventory level hits R and until a lot of size Q is delivered. The firm charges a price p_1 when the inventory level is higher than R. During the lead time, if the inventory level reaches r, $0 \le r \le R$, the initial price p_1 is increased to p_2 until the end of the lead time, where r is the inventory level that triggers the price change. In our model, r is a decision variable to be determined along with the other decision variables R and Q. When the order is received at the end of the lead time period L, the regular market price p_1 is charged. For a price change to be rational, selling prices and corresponding demand rates should satisfy $p_1\mu(p_1) > p_2\mu(p_2)$ when $p_1 < p_2$. Otherwise, the firm would always charge p_2, i.e., there would be no incentive to charge a lower price p_1 (Feng and Gallego [51]). The inventory dynamics are illustrated in Fig. 1.

The time period during which a temporary price increase is exercised, is restricted to the first T time units after an order is released, T being a decision variable that satisfies $T \le L$. The reason is that if the inventory level hits r late during the lead time, then for the remaining time until the next order arrives, r units may be enough, which does not necessitate a price change. On the other hand, if the inventory level hits r earlier during the lead time, then the likelihood of realizing stock-outs later during the lead time is higher. Increasing the unit selling price to p_2, which slows demand, might prevent this situation from happening.

One could conjecture that the time point to change the price depends on both the inventory on-hand and the time remaining to receive the next order. In their finite horizon perishable inventory model, Feng and Gallego [51] show the optimality of time thresholds to find the best time for a single price change, either high to low or low to high. Cheung [41] uses a similar time limit for offering discounts to customers who will wait for delayed orders. In our infinite horizon model, such a time threshold policy occurs during the replenishment lead time.

Let the time when inventory level is R set as zero. Then let τ to denote the time when the inventory level hits r, $\tau \le T$. After an order is placed and during the interval [0, T], if the inventory level reaches r, price p_2 is charged for the remaining time period $L - \tau$. If $\tau > T$, no price change is made and the regular unit price p_1 is charged for the whole replenishment cycle. Since the following events are equivalent, $\{\tau \le t\} = \{D(p_1, t) \ge R - r\}$, the cumulative distribution function $G(t)$ and the probability density function $g(t)$ of τ can be written as follows

$$G(t) = P\{D(p_1, t) \ge R - r\} = \int_{R-r}^{\infty} f(x, p_1, t)dx,$$

$$g(t) = \frac{d}{dt}\left[\int_{R-r}^{\infty} f(x, p_1, t)dx\right].$$

The firm's objective is to maximize the long-run average profit per unit-time, with the decision variables being (Q, R, r, T). To obtain the long-run average profit, we next calculate the expected values of the revenue, the average number of units held, the number of lost sales, and the cycle time.

Expected Revenue Per Cycle

The expected revenue for a cycle, $E[RV]$, is obtained by conditioning on τ, the time when the inventory level hits r.

$$E[RV] = E[RV, \tau > T] + E[RV, \tau \le T]$$

If $\tau > T$, there will be no price change, and p_1 is charged throughout the cycle. In this case, the total revenue is as follows

$$E[RV, \tau > T] = \int_{T}^{\infty} p_1 Q g(t)dt. \tag{1}$$

If $\tau \le T$, p_2 is charged during the time interval $(\tau,\ L)$ while p_1 is charged for the remaining time in the cycle, i.e., between the reorder point (time zero) and τ and also between the order arrival and the next reorder point.

$$E[RV, \tau \le T] = \int_0^T E[RV|\tau = t]g(t)dt. \tag{2}$$

The conditional revenue $E[RV|\tau = t]$ in (2) depends on whether or not there is a stock-out until the next replenishment.

$$[RV|\tau = t] = \begin{cases} p_1Q + (p_2 - p_1)D(p_2, L - t), & \text{if } D(p_2, L - t) < r \\ p_1Q + (p_2 - p_1)r, & \text{if } D(p_2, L - t) \ge r. \end{cases}$$

Then, we get the following revenue expressions

$$E[RV|\tau = t] = p_1Q + \int_0^r (p_2 - p_1)xf(x, p_2, L - t)dx + \int_r^\infty (p_2 - p_1)rf(x, p_2, L - t)dx$$

$$E[RV, \tau \le T] = \int_0^T \left(p_1Q + (p_2 - p_1)\int_0^r xf(x, p_2, L - t)dx + (p_2 - p_1)r\bar{F}(r, p_2, L - t)\right)g(t)dt. \tag{3}$$

Summing up (1) and (3), the total expected revenue is obtained as

$$E[RV] = p_1Q + (p_2 - p_1)\int_0^T \left(\int_0^r xf(x, p_2, L - t)dx + r\bar{F}(r, p_2, L - t)\right)g(t)dt,$$

where $\bar{F} = 1 - F$ denotes the complement of the cumulative distribution function F.

Expected Lost Sales per Cycle

The expected number of lost sales per cycle, $E[LS]$ is also calculated by conditioning on the time τ.

$$E[LS] = E[LS, \tau > T] + E[LS, \tau \le T].$$

When there is no price change within a cycle, the event $\{\tau > T\}$ can be replaced by $\{D(p_1, T) < R - r\}$, as they are equivalent.

$$\begin{aligned} E[LS, \tau > T] = E[LS, D(p_1, T) < R - r] &= \int_0^{R-r} E[LS|D(p_1, T) = x]f(x, p_1, T)dx \\ &= \int_0^{R-r} E[D(p_1, L) - R|D(p_1, T) = x]^+ f(x, p_1, T)dx \\ &= \int_0^{R-r}\int_{R-x}^\infty (x + y - R)f(y, p_1, L - T)f(x, p_1, T)dydx. \end{aligned}$$

When the demand within period T and after the placement of an order is at least $R - r$, a temporary price change is exercised. In this case, the expected number of lost sales per cycle is

$$\begin{aligned} E[LS, \tau \le T] &= \int_0^T E[LS|\tau = t]g(t)dt = \int_0^T E[D(p_2, L - \tau) - r|\tau = t]^+ g(t)dt \\ &= \int_0^T\int_r^\infty (x - r)f(x, p_2, L - t)g(t)dxdt. \end{aligned}$$

Then the expected number of lost sales per cycle is obtained as follows

$$E[LS] = \int_0^{R-r}\int_{R-x}^\infty (x + y - R)f(y, p_1, L - T)f(x, p_1, T)dydx + \int_0^T\int_r^\infty (x - r)f(x, p_2, L - t)g(t)dxdt.$$

Expected Cycle Time

In a continuous review inventory model with random demand, the exact change in inventory levels is different in each cycle, whose length is a random variable. In a simple (Q, R) model with backorders, the expected cycle length is $Q/\mu(p)$, where Q is the total demand received per cycle and $\mu(p)$ is the expected demand per unit-time. When lost sales are considered, the expected cycle time is increased to $(Q + E[LS])/\mu(p)$, where $E[LS]$ is the expected lost sales per cycle, as the actual demand observed includes the lost part $E[LS]$.

In our model, the demand rate changes over a cycle if a price change is made. Thus, the expected cycle length is conditioned on the possibility of a price change occurring during a cycle.

$$E[CT] = E[CT, \tau > T] + E[CT, \tau \le T].$$

Noting that the event $\{\tau > T\}$ is equivalent to $\{D(p_1, T) < R - r\}$, the cycle time expression when there is no price change is obtained as follows

$$\begin{aligned}
&E[CT, D(p_1,T) < R-r] \\
&= \int_0^{R-r} E[CT|D(p_1,T)=x] f(x,p_1,T)dx \\
&= \int_0^{R-r} \left(L + \int_0^{R-x} \frac{Q-x-y}{\mu(p_1)} f(y,p_1,L-T)dy + \frac{Q-R}{\mu(p_1)} \bar{F}(R-x,p_1,L-T) \right) f(x,p_1,T)dx.
\end{aligned}$$

If a price change is made, then the expected cycle time is

$$E[CT, \tau \leq T] = \int_0^T \left(L + \int_0^r \frac{Q+r-x-R}{\mu(p_1)} f(x,p_2,L-t)dx + \frac{Q-R}{\mu(p_1)} \bar{F}(r,p_2,L-t) \right) g(t)dt.$$

Then the expected cycle time is written as follows

$$\begin{aligned}
E[CT] = L &+ \int_0^{R-r} \left(\int_0^{R-x} \frac{Q-x-y}{\mu(p_1)} f(y,p_1,L-T)dy + \frac{Q-R}{\mu(p_1)} \bar{F}(R-x,p_1,L-T) \right) f(x,p_1,T)dx \\
&+ \int_0^T \left(\int_0^r \frac{Q+r-x-R}{\mu(p_1)} f(x,p_2,L-t)dx + \frac{Q-R}{\mu(p_1)} \bar{F}(r,p_2,L-t) \right) g(t)dt.
\end{aligned}$$

Expected Average Inventory per Cycle

To calculate the expected on-hand inventory, following Hadley and Whitin [46], we assume that for a (Q, R) model, the stock-out probability is sufficiently small. Such an assumption is reasonable in our case since a temporary price increase helps prevent or decrease stock-outs. Moreover, Lau and Lau [52] show that the method that Hadley-Whitin used to compute average inventory is quite robust and often more accurate than alternative ones suggested in the literature. The exact computation of inventory costs is complicated, especially for a lost sales model relative to a backorder model. While the inventory position is uniformly distributed between R and $R+Q$ and independent of the current inventory state in a backorder model, it is not the case when demand is lost. Thus, the approximate calculation of Hadley and Whitin [46] is often preferred in the literature (Moinzadeh and Nahmias [35], Cheung [41], Johansen and Thorstenson [48], Tekin et al. [45], and Duran et al. [38]).

We calculate the expected average inventory by conditioning on the occurrence of a price change. First, we define the probability of a price change during a cycle, θ, such as

$$\theta = P(\tau \leq T) = P(D(p_1,T) \geq R-r) = \int_{R-r}^{\infty} f(x,p_1,T)dx,$$

where $(1-\theta)$ is the probability of no-price change within a replenishment cycle.

The average demand rate with price p_1 during lead time L can be calculated by conditioning on the given information that a price change is made or not. Define λ_1 as the average demand rate with price p_1 within a time period $(0, T)$ given that no price change is made during a cycle.

$$\begin{aligned}
\lambda_1 &= E[D(p_1,T)|D(p_1,T) < R-r])/T \\
&= E[D(p_1,T), D(p_1,T) < R-r]/(TP(D(p_1,T) < R-r)) \\
&= \int_0^{R-r} xf(x,p_1,T)dx/(T(1-\theta)).
\end{aligned}$$

When a price change is made, the average demand rate with price p_1 during the period T is denoted by λ_2 and obtained as follows.

$$\begin{aligned}
\lambda_2 &= E[D(p_1,T)|D(p_1,T) \geq R-r]/T \\
&= E[D(p_1,T), D(p_1,T) \geq R-r]/(T\theta) \\
&= \int_{R-r}^{\infty} xf(x,p_1,T)dx/(T\theta).
\end{aligned}$$

By definition, the following equality should also hold.

$$(1-\theta) * \lambda_1 + \theta * \lambda_2 = \mu(p_1).$$

Then, the expected average inventory can be written by calculating the areas under the expected inventory levels shown in Figs. 2 and 3. $E[OH|\tau > T]$ and $E[OH|\tau \leq T]$ are the expected average inventories when there is no price change and when there is price change, respectively.

$$E[OH|\tau > T] = RT - Q(L-T) + \lambda_1 T \left(\frac{1}{\mu(p_1)} \left(\frac{\lambda_1 T}{2} - Q - R \right) - \frac{T}{2} \right) + \frac{Q}{\mu(p_1)} \left(\frac{Q}{2} + R \right)$$

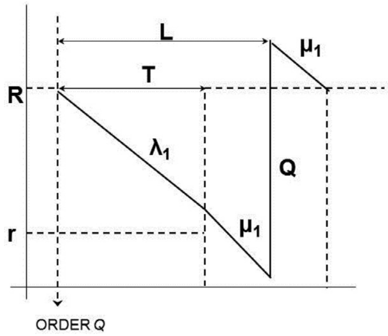

Fig. 2. Expected inventory levels when no price change is exercised.

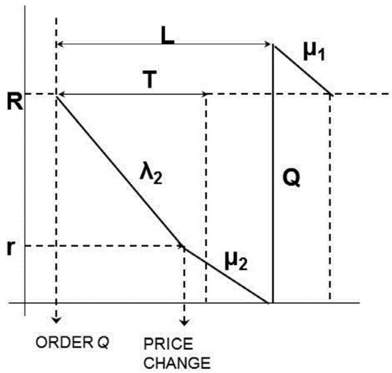

Fig. 3. Expected inventory levels when a price change is exercised.

$$E[OH|\tau \le T] = \frac{R^2 - r^2}{2}\left(\frac{1}{\lambda_2} - \frac{1}{\mu(p_1)}\right) + \frac{Q}{\mu(p_1)}\left(\frac{Q}{2} + r\right) + \frac{\mu(p_2)}{2}\left(L - \frac{R-r}{\lambda_2}\right)^2\left(\frac{\mu(p_2)}{\mu(p_1)} - 1\right)$$
$$+ \left(L - \frac{R-r}{\lambda_2}\right)\left(r - \frac{r\mu(p_2)}{\mu(p_1)} - \frac{Q\mu(p_2)}{\mu(p_1)}\right).$$

Note that during the calculation of expected inventory levels $E[OH|\tau > T]$ and $E[OH|\tau \le T]$ following Figs. 2 and 3, the inventory level right before an order arrives is not positively restricted, which can be correct when backordering is available. However, the inventory level is always non-negative when unmet demand is lost. Therefore, consistent with Hadley and Whitin [46], the expected inventory levels should be adjusted by adding the term $E[SL]E[CT]$. This term accounts for an increase in the inventory level resulting from losing a sale when back-ordering is not allowed. So, the unconditional expected amount of inventory held is obtained by conditioning on the probability of a price increase and also adjusting for the expected lost sales.

$$E[OH] = (1-\theta)E[OH|\tau > T] + \theta E[OH|\tau \le T] + E[SL]E[CT]. \tag{4}$$

With the calculation of the expected values of the revenue, the number of lost sales, and the number of average units held during a cycle. The expected profit per cycle is obtained such that

$$\pi(Q, R, r, T) = E[RV] - K - cQ - hE[OH] - bE[SL].$$

Given that the expected cycle time is also a function of the decision variables, the objective function is to maximize the expected profit per unit-time.

$$\max_{Q,R,r,T} \quad \Pi(Q, R, r, T) = \pi(Q, R, r, T)/E[CT]. \tag{5}$$

The analytical analysis of the objective function to derive closed-form expressions for the optimal values of the decision variables is not possible. Therefore, we focus next on numerically analyzing the model and gaining managerial insights. In Section 4, the optimal long-run profits and optimal decision variables are obtained through the complete enumeration of variables.

4. Numerical analysis

Numerical analyses are conducted to gain insights into the solution of the model, the sensitivity of the solution, and the simplified model. We compare the temporary price increase model to a replenishment model with no price change and illustrate the benefits our model brings; i.e., increasing the total expected profit and managerial implications and insights.

To conduct the numerical analyses, we need to define the demand function in more detail. $D(p, t)$ is demand and random during time interval t for price p, which is an independent variable. An additive form is used to model the random component of $D(p, t)$ as in Gimpl-Heersink [33], Jadidi et al. [53], Petruzzi and Dada [49], and Ray et al. [54]. Particularly, demand rate is defined as $D(p,1) = y(p) + \epsilon,$ where $y(p)$ is a decreasing function of p and ϵ is a non-negative random portion. Particularly, we use $y(p) = \alpha - \beta p,$ where $\alpha > 0$ and β is the price sensitivity coefficient. The price is restricted to the range $[c, \alpha/\beta]$ to prevent negative $y(p)$. Random demand portion of the demand ϵ is assumed to follow a Poisson distribution with mean μ, for ease of computation as in many studies such as Chen et al. [31] and Chen and Simchi-Levi [29]. Then, the demand during a time interval t, $D(p, t)$, is the sum of $(\alpha - \beta p)t$ and the random portion, which follows a Poisson distribution with mean μt. The density function of total demand during period t is

$$f(x,p,t) = \frac{e^{-\mu t}(\mu t)^{(x-(\alpha-\beta p)t)}}{(x-(\alpha-\beta p)t)!} \quad \text{for} \quad (x-(\alpha-\beta p)t) \in \{0,1,2,\dots\}.$$

Accordingly, the probability density function of τ, which is the time for the inventory level to drop from R to r, is as follows

$$g(t)e = \frac{((\alpha-\beta p)t+\mu)e^{-\mu t}(\mu t)^{R-r-(\alpha-\beta p)t-1}}{(R-r-(\alpha-\beta p)t-1)!}.$$

The correct choice of p_1 and p_2 is fundamental for the evaluation of the results. For a given value of p_1, if the introduction of an arbitrary price p_2, $p_2 > p_1$, leads to a higher total profit over the model with no price change, then this increase cannot be solely be associated with the use of a temporary price increase. Rather, p_2 can be a better choice than p_1, even for a constant price (Q, R) model. Thus, for each problem instance, we first obtain the optimal price for a model with no price change denoted by p', where a single optimal price p' is charged throughout the horizon. The price p' is obtained by modeling and solving a joint inventory and fixed pricing problem such that the long-run total profit is maximized over (Q, R, p) by benefiting from the optimality properties shown in Çömez and Kiessling [55]. Then, p_1 is set equal to the p' while solving the temporary price change model.

The optimal profit (5) is evaluated by first setting $p_2 = p_1,$ i.e. no price change is allowed. The resulting optimal order quantity, reorder point, and expected profit per unit-time are denoted respectively by Q^0, R^0, and Π^0. When there is no price change, the price change trigger-level and the price change window, respectively r and T, are irrelevant. Next, (5) is evaluated for a price increase model with the increased price p_2 set to various feasible values for testing. Given that α/β is the upper bound for the chargeable price, for each problem instance, we test for $p_2 \in \{1.05p_1, 1.1p_1, 1.15p_1, \dots, \alpha/\beta\}$.

To determine the problem parameters, we benefit from the values used by Chen et al. [31] and Chen and Simchi-Levi [29]. The base problem setting has $K = 55,$ $c = 10,$ $L = 1,$ $h = 1.5,$ and $b = 30$. The base demand setting is $\alpha = 40,$ $\beta = 2.25,$ and $\mu = 5$. By changing each parameter at a time while keeping the others fixed, a total of 29 problem instances are generated. Each problem instance is run with various p_2 values, where $p_2 \in \{1.05p_1, 1.1p_1, 1.15p_1, \dots, \alpha/\beta\}$. Thus, a total of 78 problem instances are reported in Tables 2–4. In each problem setting, only the parameter that is changed with respect to the base setting is reported.

First, using the developed model, we show that a temporary price increase increases the expected profit and investigate its sensitivity to changes in the system's parameters. Second, to demonstrate the usefulness of a simplified version of the model, we set the price increase period to L, thus reducing the set of decision variables to (Q, R, r). Third, we propose another simplification by solving a two-stage problem. In stage 1, we obtain Q and R for no price change. In stage 2, we use Q and R to determine the price change trigger-level r. Fourth and last, we report the results of extensive numerical analyses run with randomly generated problem settings.

4.1. Benefits of temporary price change over a fixed price system

In this section, the magnitude of the change in the expected profit as a result of a temporary price increase policy and the sensitivity of the model to system parameters are evaluated through 78 problem instances defined above. For each problem instance, first p_1 is obtained by solving a joint inventory and single price problem (Q, R, p), then the optimal profit (5) is evaluated by setting $p_2 = p_1$ and resulting optimal decision variables are Q^0, R^0, and Π^0. Next, (5) is evaluated for a price increase model for $p_2 \in \{1.05p_1, 1.1p_1, 1.15p_1, \dots, \alpha/\beta\}$ where α/β is the upper bound for the chargeable price, for each problem instance. According to this range, for each problem setting, the number of tested p_2 values ranges between two and four. For example, in Table 2, for the first problem set, $p_1 = 16.12$ and p_2 has three feasible values according to our setting, where the highest value of p_2 tested is $\alpha/\beta = 40/2.25 = 17.78$. For the second problem set, $p_1 = 15.81$ and p_2 has set to two different values, where its highest value is $\alpha/\beta = 38/2.25 = 16.89$. The optimal decision variables are denoted by Q^*, R^*, r^*, and T^* with the resulting expected profit per unit-time Π^*. Optimality search is done over integer values for Q, R, and r. For T, the search is done in increments of 0.1. The percent increase in the expected profit by utilizing a price change is denoted by $\Delta\Pi^* = (\Pi^*(Q^*, R^*, r^*, T^*) - \Pi^0(Q^0, R^0))/\Pi^0(Q^0, R^0) * 100$. Table 2 reports the results.

Table 2
Benefit of temporary price change over a fixed price system. Base setting has $\alpha = 40$, $\beta = 2.25$, $\mu = 5$, $K = 55$, $c = 10$, $L = 1$, $h = 1.5$, and $b = 30$.

p_1	(Q^0, R^0)	Changed Parameter	p_2	(Q^*, R^*, r^*, T^*)	$\Delta\Pi^*$ (%)	p_1	(Q^0, R^0)	Changed Parameter	p_2	(Q^*, R^*, r^*, T^*)	$\Delta\Pi^*$ (%)
16.12	(27,11)	–	16.93	(26,10,5,0.6)	7.1	16.01	(25,11)	$K = 45$	16.81	(24,11,5,0.6)	4.3
			17.74	(26,10,3,0.8)	13.1				17.61	(24,10,3,0.8)	8.3
			17.78	(26,10,2,0.9)	13.5				17.78	(24,10,3,0.8)	9.5
15.81	(25,9)	$\alpha = 38$	16.60	(25,8,5,1)	29.1	16.27	(28,11)	$K = 65$	17.09	(28,9,6,0.8)	12.3
			16.89	(24,8,5,1)	40.9				17.78	(28,9,3,0.8)	20.7
16.50	(28,12)	$\alpha = 42$	17.32	(28,12,4,0.7)	3.4	16.38	(30,10)	$K = 75$	17.20	(30,9,5,1)	20.3
			18.15	(28,11,4,0.7)	6.4				17.78	(29,9,3,0.8)	31.6
			18.67	(28,11,2,0.9)	8.3	14.99	(30,14)	$c = 8$	15.74	(30,13,7,0.5)	2.1
16.89	(29,14)	$\alpha = 44$	17.73	(29,13,7,0.5)	2.4				16.49	(30,13,3,0.8)	3.8
			18.58	(30,12,4,0.7)	4.1				17.24	(30,13,3,0.8)	4.5
			19.42	(30,12,2,0.9)	5.3				17.78	(30,12,2,0.9)	5.5
			19.56	(30,12,2,0.9)	5.7	15.55	(29,12)	$c = 9$	16.33	(28,12,5,0.6)	3.3
17.29	(31,15)	$\alpha = 46$	18.15	(31,14,6,0.6)	1.8				17.10	(28,11,4,0.7)	5.7
			19.02	(31,14,4,0.8)	2.9				17.78	(28,11,2,0.9)	7.7
			19.88	(31,13,3,0.8)	3.5	16.77	(25,9)	$c = 11$	17.61	(24,8,5,1)	47.1
			20.44	(31,13,2,0.9)	4.3				17.78	(24,8,5,1)	56.4
16.55	(27,12)	$\beta = 2.15$	17.37	(28,11,4,0.7)	4.2	17.42	(21,8)	$c = 12$	17.78	(22,7,0,1)	5.7
			18.20	(27,11,3,0.8)	7.0	15.89	(33,12)	$h = 1$	16.69	(33,11,6,0.5)	2.3
			18.60	(27,11,2,0.9)	8.5				17.48	(33,11,3,0.8)	4.2
16.33	(27,11)	$\beta = 2.20$	17.14	(27,11,5,0.6)	5.0				17.78	(33,11,2,0.9)	4.9
			17.96	(27,10,3,0.8)	9.3	16.01	(30,11)	$h = 1.25$	16.81	(29,11,5,0.6)	3.6
			18.18	(27,10,3,0.8)	10.8				17.61	(29,10,3,0.8)	6.8
15.98	(26,10)	$\beta = 2.30$	16.77	(26,9,6,0.9)	11.8				17.78	(29,10,3,0.8)	7.7
			17.39	(26,9,3,0.8)	19.2	16.29	(24,11)	$h = 1.75$	17.10	(24,10,5,0.6)	15.9
15.80	(25,10)	$\beta = 2.35$	16.59	(25,9,6,1)	15.2				17.78	(24,9,3,0.8)	27.5
			17.02	(25,9,3,0.7)	27.3	16.40	(22,10)	$h = 2$	17.22	(22,9,5,1)	54.3
15.77	(25,9)	$\mu = 3$	16.56	(24,8,4,0.7)	22.6				17.78	(22,9,3,0.8)	86.4
			17.35	(24,8,2,0.9)	37.4	16.14	(27,10)	$b = 20$	16.95	(27,9,5,1)	6.4
			17.78	(24,8,1,0.9)	48.6				17.75	(26,9,2,0.9)	11.3
15.97	(25,10)	$\mu = 4$	16.76	(25,9,4,1)	10.7				17.78	(26,9,2,0.9)	11.6
			17.56	(25,9,2,0.8)	18.1	16.15	(26,11)	$b = 25$	16.96	(26,10,5,0.6)	7.0
			17.78	(25,9,2,0.8)	21.2				17.77	(26,10,2,0.9)	12.1
16.32	(27,12)	$\mu = 6$	17.14	(27,11,6,1)	4.6				17.78	(26,10,2,0.9)	12.2
			17.78	(27,11,4,0.7)	8.4	16.13	(27,11)	$b = 35$	16.94	(26,10,6,1)	7.6
16.52	(28,13)	$\mu = 7$	17.35	(28,12,4,0.8)	3.1				17.74	(26,10,3,0.8)	14.5
			17.78	(28,12,3,0.8)	5.6				17.78	(26,10,3,0.8)	14.9
15.88	(22,12)	$K = 35$	16.68	(22,11,6,0.5)	3.7	16.14	(27,11)	$b = 40$	16.94	(26,11,5,0.6)	8.4
			17.47	(22,11,3,0.8)	6.9				17.75	(26,10,3,0.8)	15.9
			17.78	(22,11,2,0.9)	8.2				17.78	(26,10,3,0.8)	16.3
Average $\Delta\Pi^*$: 13.71%											

From Table 2, in all problem instances, we observe an increase in the expected profit when a price increase is allowed. The profit increases if p_2 increases. Increasing p_2 decreases r and increases T. This shows that the increase in the altered price is balanced by lowering the price change trigger level to limit the high price to a lower amount of product sales. It is observed that when a price change is allowed, the reorder level is slightly lowered ($R^0 \geq R^*$), but the change in order quantity can be in either direction ($Q^0 \geq Q^*$ or $Q^0 \leq Q^*$), while all observed changes are mostly negligible. The effect of each altered parameter has the same direction on inventory replenishment variables Q and R both in the model without a price change and with the price change. So, in the below discussion, we use notations Q and R, respectively, to refer to the optimal order size and the reorder level both in no price change and price change models.

Our sensitivity analysis indicates the following insights. It is observed that the benefit of a price change, measured by $\Delta\Pi^*$, in general increases with an increase in β, K, c, h, or b and with a decrease in α or μ. Particularly, when α or μ is low, or K, c, or h is high, the expected profit under no price change decreases significantly, resulting in a higher percentage increase by the price change.

$\Delta\Pi^*$ increases when the optimal reorder level R decreases with the change in the parameter. A valuable insight is that the price change becomes more crucial when lower levels of reorder level, i.e. lower inventory, are maintained and used more effectively. For example, both procurement c and holding cost per unit h lead to a higher p_1 to increase profit margin, resulting in lower Q and R because of decreased demand rate, and so an increase in $\Delta\Pi^*$. Accordingly, $\Delta\Pi^*$ decreases while the optimal reorder level increases, making it less likely to benefit from a price change, as the possibility of inventory level hitting r decreases.

Except for K and b, an altered parameter affects Q and R in the same direction. With a higher fixed ordering cost K, while the order quantity Q increases, the reorder point R decreases. The increase in $\Delta\Pi^*$ is again because of the better use of a reduced inventory level during the lead time. Our results on the effects of K are consistent with those in Chen et al.

Table 3
Benefit of temporary price change over a fixed price system with a fixed price change window $T = L$. Base setting has $\alpha = 40$, $\beta = 2.25$, $\mu = 5$, $K = 55$, $c = 10$, $L = 1$, $h = 1.5$, and $b = 30$.

p_1	(Q^0, R^0)	Changed Parameter	p_2	(Q^T, R^T, r^T)	$\Delta\Pi^T$ (%)	p_1	(Q^0, R^0)	Changed Parameter	p_2	(Q^T, R^T, r^T)	$\Delta\Pi^T$ (%)
16.12	(27,11)	–	16.93	(26,10,6)	6.9	16.01	(25,11)	$K = 45$	16.81	(25,10,6)	3.3
			17.74	(26,10,2)	12.0				17.61	(24,11,0)	6.7
			17.78	(26,10,2)	12.5				17.78	(24,10,2)	8.2
15.81	(25,9)	$\alpha = 38$	16.60	(25,8,5)	29.1	16.27	(28,11)	$K = 65$	17.09	(28,9,5)	12.3
			16.89	(24,8,5)	40.9				17.78	(28,9,3)	20.2
16.50	(28,12)	$\alpha = 42$	17.32	(28,11,6)	3.2	16.38	(30,10)	$K = 75$	17.20	(30,9,5)	20.3
			18.15	(28,11,3)	5.1				17.78	(29,9,3)	30.8
			18.67	(28,11,2)	7.5	14.99	(30,14)	$c = 8$	15.74	(30,13,6)	1.9
16.89	(29,14)	$\alpha = 44$	17.73	(29,13,5)	1.8				16.49	(30,13,3)	3.1
			18.58	(30,12,4)	3.3				17.24	(30,13,2)	4.1
			19.42	(30,12,2)	4.9				17.78	(30,13,1)	5.2
			19.56	(30,12,2)	5.3	15.55	(29,12)	$c = 9$	16.33	(28,12,6)	2.6
17.29	(31,15)	$\alpha = 46$	18.15	(31,14,6)	1.3				17.10	(28,11,4)	4.8
			19.02	(31,14,3)	2.4				17.78	(28,11,2)	7.3
			19.88	(31,14,1)	3.2	16.77	(25,9)	$c = 11$	17.61	(24,8,5)	47.1
			20.44	(31,13,1)	4.0				17.78	(24,8,5)	56.4
16.55	(27,12)	$\beta = 2.15$	17.37	(27,11,6)	4.0	17.42	(21,8)	$c = 12$	17.78	(22,7,0)	5.7
			18.20	(27,11,2)	6.4	15.89	(33,12)	$h = 1$	16.69	(33,11,6)	2.0
			18.60	(27,11,2)	8.0				17.48	(33,11,2)	3.8
16.33	(27,11)	$\beta = 2.20$	17.14	(27,10,5)	4.8				17.78	(33,11,1)	4.7
			17.96	(27,10,3)	8.5	16.01	(30,11)	$h = 1.25$	16.81	(29,11,0)	2.7
			18.18	(27,10,2)	10.2				17.61	(29,11,0)	5.5
15.98	(26,10)	$\beta = 2.30$	16.77	(26,9,5)	11.6				17.78	(29,10,2)	6.7
			17.39	(26,9,3)	18.7	16.29	(24,11)	$h = 1.75$	17.10	(24,9,5)	15.8
15.80	(25,10)	$\beta = 2.35$	16.59	(25,9,6)	15.2				17.78	(24,9,3)	27.0
			17.02	(25,9,3)	22.5	16.40	(22,10)	$h = 2$	17.22	(22,9,5)	54.3
15.77	(25,9)	$\mu = 3$	16.56	(25,8,3)	21.6				17.78	(22,9,3)	84.9
			17.35	(24,8,2)	36.2	16.14	(27,10)	$b = 20$	16.95	(27,9,5)	6.4
			17.78	(24,8,1)	48.1				17.75	(26,9,2)	10.6
15.97	(25,10)	$\mu = 4$	16.76	(25,9,4)	10.7				17.78	(26,9,2)	10.9
			17.56	(25,9,2)	17.6	16.15	(26,11)	$b = 25$	16.96	(26,10,5)	6.5
			17.78	(25,9,1)	20.5				17.77	(26,9,3)	11.4
16.32	(27,12)	$\mu = 6$	17.14	(27,11,6)	4.6				17.78	(26,9,3)	11.5
			17.78	(27,11,2)	7.7	16.13	(27,11)	$b = 35$	16.94	(26,10,6)	7.6
16.52	(28,13)	$\mu = 7$	17.35	(28,11,8)	3.0				17.74	(26,10,2)	13.3
			17.78	(28,12,3)	4.2				17.78	(26,10,2)	13.8
15.88	(22,12)	$K = 35$	16.68	(22,11,5)	3.5	16.14	(27,11)	$b = 40$	16.94	(26,10,6)	8.3
			17.47	(22,11,2)	6.3				17.75	(26,10,3)	14.5
			17.78	(22,11,1)	7.7				17.78	(26,10,2)	15.0
Average $\Delta\Pi^T$: 13.10%											

[56] that study joint pricing and replenishment problem for periodic-review systems. It is surprising to observe that the optimal replenishment policy is almost insensitive to changes in the shortage cost b with a slight increase in R^*. So, as b increases, the lead time inventory level R and accordingly expected amount of prevented loss sales do not change much. On the other hand, the total value of prevented loss sales increases, so the benefit of exercising a price increase during lead time is more pronounced.

An increase in α or μ increases the expected demand rate, which increases p_1, the order quantity, and the reorder point. In such a case, the improvement in profits expected from a price change becomes less apparent. With a higher R, the probability of experiencing a stock-out situation decreases, with no need for a price change. On the other hand, β, the price sensitivity coefficient, negatively affects p_1, Q, and R, and positively $\Delta\Pi^*$. Managerially, these results suggest that a price change could be more effective in increasing profits for systems dealing with relatively slow-moving and price-sensitive products. Besides, from a practical point, a price change is somewhat easier to implement when demand arrivals are more dispersed in time.

As the new price p_2 increases, while we observe a higher improvement in profit, the price change trigger-level r^* decreases. Moreover, the optimal price change window T^* mostly increases with p_2, and in all instances $T^* \geq 0.5$. This result motivates us to introduce a simpler model, where the price change time window is exogenously set to the lead time and evaluated in the next section.

4.2. Benefits of temporary price change with a fixed time window T

In Table 2, we observe that in 27 out of 29 problem settings, $T^* \geq 0.8$ for the highest p_2 tested, where $L = 1$. Relying on this observation, we propose a simplified and practical way to solve the model with price change by setting the price change

Table 4
Benefits of two-stage solution of temporary price change model over a fixed price system with $T = L$. Base setting has $\alpha = 40$, $\beta = 2.25$, $\mu = 5$, $K = 55$, $c = 10$, $L = 1$, $h = 1.5$, and $b = 30$.

Changed Parameter	p_2	(Q^0, R^0, r^S)	$\Delta\Pi^S$ (%)	Changed Parameter	p_2	(Q^0, R^0, r^S)	$\Delta\Pi^S$ (%)	Changed Parameter	p_2	(Q^0, R^0, r^S)	$\Delta\Pi^S$ (%)
–	16.93	(27,11,0)	3.5	$\mu = 3$	16.56	(25,9,2)	17.1	$c = 11$	17.61	(25,9,1)	30.4
	17.74	(27,11,1)	8.1		17.35	(25,9,1)	27.3		17.78	(25,9,1)	40.7
	17.78	(27,11,1)	8.3		17.78	(25,9,1)	33.7	$c = 12$	17.78	(21,8,0)	3.2
$\alpha = 38$	16.60	(25,9,2)	21.6	$\mu = 4$	16.76	(25,10,3)	6.2	$h = 1$	16.69	(33,12,1)	1.5
	16.89	(25,9,2)	34.6		17.56	(25,10,1)	12.7		17.48	(33,12,1)	2.7
$\alpha = 42$	17.32	(28,12,0)	2.2		17.78	(25,10,1)	14.5		17.78	(33,12,1)	3.2
	18.15	(28,12,1)	4.6	$\mu = 6$	17.14	(27,12,1)	2.5	$h = 1.25$	16.81	(30,11,0)	2.7
	18.67	(28,12,1)	6.2		17.78	(27,12,1)	5.0		17.61	(30,11,0)	5.4
$\alpha = 44$	17.73	(29,14,0)	0.7	$\mu = 7$	17.35	(28,13,0)	1.7		17.78	(30,11,1)	6.0
	18.58	(29,14,0)	1.4		17.78	(28,13,0)	2.6	$h = 1.75$	17.10	(24,11,1)	5.3
	19.42	(29,14,0)	2.1	$K = 35$	16.68	(22,12,1)	2.4		17.78	(24,11,1)	11.9
	19.56	(29,14,0)	2.2		17.47	(22,12,1)	4.6	$h = 2$	17.22	(22,10,1)	31.6
$\alpha = 46$	18.15	(31,15,1)	0.7		17.78	(22,12,1)	5.4		17.78	(22,10,1)	63.3
	19.02	(31,15,1)	1.3	$K = 45$	16.81	(25,11,0)	3.3	$b = 20$	16.95	(27,10,5)	5.2
	19.88	(31,15,1)	1.9		17.61	(25,11,0)	6.6		17.75	(27,10,1)	9.4
	20.44	(31,15,1)	2.2		17.78	(25,11,1)	7.3		17.78	(27,10,1)	9.5
$\beta = 2.15$	17.37	(27,12,2)	2.1	$K = 65$	17.09	(28,11,1)	3.8	$b = 25$	16.96	(26,11,0)	2.7
	18.20	(27,12,1)	4.1		17.78	(28,11,1)	8.5		17.77	(26,11,1)	6.4
	18.60	(27,12,1)	5.1	$K = 75$	17.20	(30,10,1)	11.4		17.78	(26,11,1)	6.4
$\beta = 2.20$	17.14	(27,11,5)	4.4		17.78	(30,10,1)	22.3	$b = 35$	16.94	(27,11,2)	4.6
	17.96	(27,11,1)	7.7	$c = 8$	15.74	(30,14,2)	1.2		17.74	(27,11,1)	9.9
	18.18	(27,11,1)	8.9		16.49	(30,14,2)	2.0		17.78	(27,11,1)	10.2
$\beta = 2.30$	16.77	(26,10,5)	9.7		17.24	(30,14,1)	2.8	$b = 40$	16.94	(27,11,3)	5.9
	17.39	(26,10,1)	16.6		17.78	(30,14,1)	3.3		17.75	(27,11,2)	12.0
$\beta = 2.35$	16.59	(25,10,0)	10.2	$c = 9$	16.33	(29,12,5)	2.4		17.78	(27,11,2)	12.2
	17.02	(25,10,0)	16.4		17.10	(29,12,2)	4.2				
					17.78	(29,12,1)	6.1				
Average $\Delta\Pi^S$: 9.10%											

window equal to the lead time ($T = L$). So, the problem decision variables reduce to (Q, R, r). We then rerun the problem instances reported in Table 2, with results reported in Table 3. $\Delta\Pi^T = (\Pi^T(Q^T, R^T, r^T) - \Pi^0(Q^0, R^0))/\Pi^0(Q^0, R^0) * 100$ denotes the percentage improvement provided by a price change optimization model under a fixed price change window over a no-price change model.

Setting the price change window equal to the lead time reduces the complexity of the model. It is observed to be a very good approximation. It still generates significant benefits compared to the model with no price change. For the 78 problem instances tested, the average improvement over the no-price change system is 13.10% with $T = L$, down from 13.71% when T was set to be a decision variable. In only 3 out 78 instances, the optimal order quantity Q^T is different from Q^*, though very slightly. In general, for higher values of p_2, r^T is lower than r^*. The result is intuitive, because if the price change is allowed during the whole replenishment lead time, then it can be triggered later during the lead time with a lower trigger-level.

4.3. Benefits of temporary price change under a two-stage solution

Table 2 shows that when a price change is allowed, the reorder level is slightly lowered ($R^0 \geq R^*$), and the order quantity can either increase or decrease ($Q^0 \geq Q^*$ or $Q^0 \leq Q^*$), with other observed changes negligible. Thus, a simultaneous calculation of replenishment decisions with the price change decisions, which is computationally very challenging, is no longer necessary. Therefore, we also test the sequential evaluation of the price change model by first obtaining (Q^0, R^0) under no price change and then the best price change trigger-level r^S with the suboptimal replenishment decisions (Q^0, R^0). As reported in Table 3, setting the price change interval to the lead time ($T = L$) is a very good approximation of the model, which is practical and easy to implement. Thus, we also set $T = L$ for this sequential optimization solution. In summary, instead of a simultaneous optimization of the (Q, R, r) model, two models are solved sequentially. The first obtains the replenishment decisions (Q^0, R^0), while the second obtains the price change decision r^S, with results reported in Table 4. $\Delta\Pi^S = (\Pi^S(Q^0, R^0, r^S) - \Pi^0(Q^0, R^0))/\Pi^0(Q^0, R^0) * 100$ denotes the percent improvement provided by a sequential price change optimization model under a fixed price change window over a no-price change model. Note that p_1 is already reported in Tables 2 and 3, so it is not repeated in Table 4.

When the replenishment decisions are obtained sequentially, and the price change is allowed, we observe that $r^S \leq r^*$. The rationale is to correct for the effects of suboptimal replenishment decisions and the price change window. The price change trigger-level is lowered, postponing a change in price. However, the resulting expected profits are still good enough compared to the fixed price policy. The average profit gain is 9.1% for the tested instances in Table 4. Thus, even with a sequentially solved replenishment and price change problem, the resulting decisions provide significant improvements over a conventional single price policy.

Table 5
Distributions of Randomly Generated Parameters.

α	U(38, 46)	μ	U(3, 7)	K	U(35, 75)	c	U(8, 12)
β	U(2.1, 2.4)	h	U(1, 2)	b	U(5, 50)		

4.4. Results from randomly generated problems

Next, we generate 100 additional problem instances to repeat the above analyses. Each parameter is randomly generated for each problem instance from Uniform distributions with ranges listed in Table 5, with L set to 1. The range of distribution is wide enough to test the robustness of the results. Note that, like in the above analyses, and for each problem setting, the system performance is tested or different values of price p_2 ranging between $1.05p_1$ and α/β with multiples of 5% increment on p_1. A total of 309 instances with a price change are tested, corresponding to 100 problem instances with no price change.

For a total of 309 problems instances with a price change, our results demonstrate the price change policy registered an average profit improvement of $\Delta\Pi^*$=8.11% when optimized over variables (Q, R, r, T). For these tested problems, the standard deviation of $\Delta\Pi^*$ is 13.78, the minimum being 0.79% and the maximum 97.68%. When the price change window T is set to L, the optimization of the (Q, R, r) model results in an average profit improvement of $\Delta\Pi^T$=7.64% over the single price policy with a standard deviation of 13.71%, a minimum of 0.33%, and a maximum of 96.32%. Moreover, the performance of sequential optimization of price change policy is also tested with these randomly generated problems. When the price change trigger-level r is optimized, the resulting policy provides an average increase of $\Delta\Pi^S$=5.15% in expected profits with a standard deviation of 10.2%, a minimum of 0.21%, and a maximum of 81.25%.

In summary, the numerical analyses indicate a high potential to improve the expected profits by making only a single price change during the replenishment lead time. Although it is theoretically optimal to make the inventory replenishment and price decisions simultaneously, the proposed simplified policies demonstrate considerable improvements over the conventional single price policy. First, the price change decision can be limited to a trigger inventory level without being constrained to a time window, resulting in a three decision variable problem (Q, R, r). Second, this three-decision problem can be solved sequentially, by first obtaining the replenishment policy (Q, R) and then solving for the trigger inventory level r. Such a sequential solution is simple and easy to compute and results in significant profit improvement. In a sequential optimization, the optimal price change trigger-level is lower than the one in simultaneous optimization. Even with a low trigger-level, the expected profit can still be improved, due to the better use of the remaining inventory and the ability to manage demand to avoid shortages.

5. Concluding remarks

In this study, we consider a continuous review inventory model with a fixed cost and non-zero lead time and introduce a temporary price increase exercised over a particular time window during the lead time. This business practice is common among retailers, such as Amazon, Walmart, and Apple. It helps in better managing demand and increases revenues. This problem has not been addressed before in the dynamic pricing literature. Furthermore, none of the work on dynamic pricing considered a continuous review model with a fixed cost has assumed a non-zero lead time. We solve for the continuous review inventory replenishment and temporary pricing decisions. The price change policy is defined such that, if the inventory level in a time window within the replenishment lead time drops to a critical level, the regular market price is increased. The corresponding trigger inventory level and time window are two decision variables that define the price change policy. The resulting optimization problem includes two replenishment decisions and two price change decisions.

We develop the long-run expected profit function. As the analytical tractability of the problem is highly challenging, we conduct extensive numerical and sensitivity analyses to analyze the temporary price change policy. The results indicate that a model with joint replenishment and price change decisions provides significant improvements in expected profits (13% on average) over a conventional single price model. We further simplify the proposed model by allowing price changes to occur over the entire lead time and still observe significant improvements in profit. We then propose a sequential optimization approach to find the optimal values of the the decision variables as a heuristic to the original model. Numerical analyses demonstrate that proposed two-stage optimization approach results in considerable profit improvements (9% on average), thus it can be easily implemented in practice. Systems of complex structures, including price optimization with more than on price change, can be built on the premise of improvements provided by this simple policy.

Our results indicate that a temporary price increase is most beneficial when the unit purchasing cost, fixed order cost, unit holding cost, or the unit lost sale cost is high. The price increase policy yields a better use of the available inventory when the optimal reorder level and sizes of orders are low with the increased costs. Moreover, exercising a temporary price increase is more attractive when the demand rate is low such as for slow-moving items and also for seasonal products experiencing a surge in demand.

This study is the first research in the operations management literature to investigate, in particular, temporary price increase policies. The time to change the price could depend on the on-hand inventory and the time until receiving the next order. Besides, a firm could exercise multiple price changes during the lead time. However, each added decision would complicate the model and its practical applicability. Thus, we could extend this work in light of these observations.

Acknowledgment

M.Y. Jaber thanks the Natural Sciences and Engineering Research Council of Canada (NSERC) for supporting his research and the FEA Dean and the MIE Chair at Ryerson University for partially funding his travel. He also thanks and the American University of Beirut for the in-kind support.

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