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# Capital regulation and banking bubbles☆

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ABSTRACT

This paper develops a dynamic general equilibrium model in infinite horizon with a regulated banking sector. We borrow the methodology of Miao and Wang (2015) to analyse how Basel capital requirement recommendations may generate and affect banking bubbles and macroeconomic key variables. We show that when banks face capital requirements based on credit risk, as in Basel I, bubbles cannot exist. Alternatively, under a regulatory framework where capital requirements are based on Value-at-Risk, as in Basel II and III, two different equilibria emerge and can coexist: the bubbleless and the bubbly equilibria. Bubbles can be positive or negative, depending on the tightness of capital requirements based on Value-at-Risk. We find a maximum value of capital requirement below which bubbles are positive and provide a larger welfare compared to the bubbleless equilibrium. Our results also suggest that a change in banking policies might lead to a crisis without external shocks.


## 1. Introduction

The Great Recession of 2007–2009 has highlighted the importance of the banking sector in the economy and its role in the propagation of the crisis. Both academics and policy makers acknowledged the failure of banking regulation. The U.S. Federal Reserve chairman, Ben Bernanke, in his 2008 speech on the financial crisis, suggested that inadequate financial regulation had contributed to the severity of the crisis.[1] In particular, banking regulation failed to prevent valuation and liquidity problems in the U.S. banking system, which have contributed to the propagation of the financial crisis (Miao and Wang, 2015). Banks' values were tied to the real estate pricing, which sharply decreased during the bursting of the U.S. housing bubble. Banks experienced large losses, leading to concerns about banks' solvency, reducing investors' confidence and banks' profits further (Miao and Wang, 2015). This has highlighted the importance of understanding the role of banking regulation on financial crises, in particular its role in banks stock price changes that are not due to economic fundamentals, i.e. the emergence and existence of banking bubbles.

Most countries rely on the Basel banking regulation framework to regulate their banking system. The first Basel Accord goal was to prevent international banks from growing without adequate capital.[2] Therefore, the committee imposed minimum capital requirements which were calculated based on credit risk weighted assets. Credit risk weights take into account possible losses on the asset side of a bank's balance sheet. The idea is that banks holding riskier assets had to hold more capital than other banks in order to ensure solvency. This approach has been criticised by researchers and regulatory agencies because it only considers credit risk and does not encompass market risk.[3] Market risk refers to the risk of losses from changes in market prices, which increases banks' default risk. The Basel committee has recognised this problem and released the Basel II Capital Accord.[4] This new accord considers market values into the banking regulation framework in order to take into account market risk of the trading book. It allows banks to use an internal model based

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1 See Bernanke et al. (2008).

2 The first Basel Accord was released in 1988. For more details, see Basel Committee overview, https://www.bis.org/bcbs/.

3 For example, Dimson and Marsh (1995) analyse the relationship between economic risk and capital requirements using trading book positions of UK securities firms. They find that the Basel I approach leads only to modest correlation between capital requirements and total risk.

4 See Basel Committee on Banking Supervision (2004).

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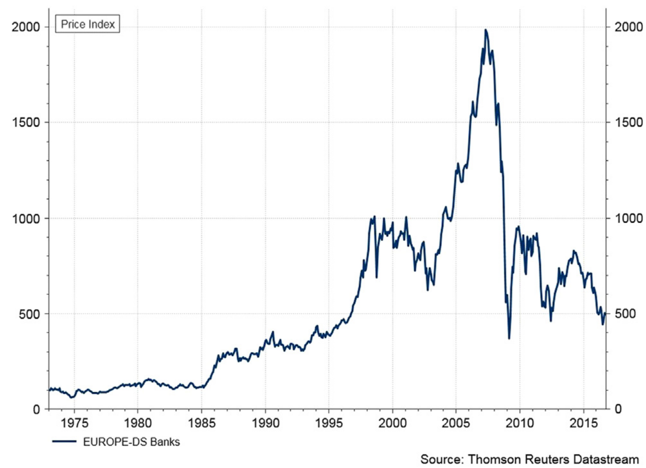


**Fig. 1.** Bank real stock price index.

on *Value-at-Risk* to quantify their minimum capital requirements. The idea of capital requirements based on Value-at-Risk is to impose a solvency condition for banks which requires that the maximum amount of debt that banks can hold does not exceed the market value of banks' assets in the worst case scenario. Value-at-Risk capital requirements remained in Basel III.[5]

Fig. 1 shows sharp movements in the bank real stock price index for the period from 1973 to 2016. This index includes 168 banks listed in Europe. The theory on asset prices argues that stock price changes are hardly explained by fundamentals alone. Several authors suggest that changes in stock prices can be due to the existence of bubbles. For instance, Kocherlakota (1992) and Miao and Wang (2015) show that bubbles may emerge when agents' portfolios are constrained. As the stock price index boom in 2004 coincides with the release of Basel II capital requirements, and although there might be other explanations, we investigate if Basel capital requirements based on Value-at-Risk (Basel II and III) might help to explain the increase in prices observed from 2004, which can be partially interpreted as a bubble by the theory on asset prices.

This paper combines the economic theory of banking regulation with the one of asset price bubbles. It aims at determining if Basel regulatory recommendations may generate and affect banking bubbles. It provides policy implications of banking regulation by evaluating their effect on banking bubbles, and as a consequence on the economy. A bubble on a bank stock price is defined as a temporary deviation of the bank stock price from its fundamental value. The bank fundamental value is the value of the bank value without a bubble. Positive bubbles are defined as banks stock prices being above their fundamental values. In contrast, negative bubbles are defined as banks stock prices being undervalued. In addition, as in Blanchard and Watson (1982) and Weil (1987), bubbles have a constant probability of bursting.

We develop a dynamic general equilibrium model with three types of infinitely lived agents, banks, households, and firms, as well as a regulatory authority. Banks raise funds by accumulating net worth and demanding deposits (supplied by households) to provide loans to firms. Firms produce the consumption goods using capital. The regulatory authority imposes two banking regulations. The first requires that banks keep a fraction of deposits as reserves. These reserves cannot be used to invest in loans (risky assets). The second measure is the capital requirement based on Value-at-Risk. It ties banks' deposits to banks' market value, as recommended in Basel II and III accords.

We show that bubbles emerge if agents believe that they exist. Thus, expectations of agents are self-fulfilling. Results suggest that when banks face capital requirements based on Value-at-Risk, two different equilibria emerge and can coexist: the bubbleless and the bubbly equilibria. In contrast, under a regulatory framework where capital requirements are based on credit risk only, as in Basel I, banking bubbles are explosive and, as a consequence cannot exist. The bubbly equilibrium is characterised by positive or negative bubbles depending on the tightness of capital requirements. We find a maximum value of the capital requirement based on Value-at-Risk below which bubbles are positive. Below this value and until the bubble bursts, the bubbly equilibrium provides larger welfare than the bubbleless equilibrium. The intuition is that, when agents consider that a bubble exists, lower capital requirements lead to optimistic beliefs about bank valuation. Banks demand more deposits and make more loans. This effect reduces the lending rate and provides higher welfare. Thus, profits of banks rise which increases the value of banks. As a consequence, initial beliefs about the value of banks are realised. In contrast, above this maximum capital requirement, bubbles are negative leading to a credit crunch and thus, reduce welfare. Therefore, our model shows that a change in regulation might lead to a crisis, by shifting the economy from higher to lower welfare. This can explain the existence of crises without external shocks. We also show that the equilibrium with positive (negative) bubbles exists if the probability that bubbles collapse is small (large). This is consistent with Weil (1987) and Miao and Wang (2015).

This paper is related to two strands of literature. First, it is related to the literature on banking regulation. As in Dangl and Lehar (2004) and Tomura et al. (2014), we study the impact of banking regulation on the economy. Dangl and Lehar (2004) compare the effect of capital regulation based on Basel I and the Value-at-Risk internal model approach. They find that the latter reduces risk in the economy. Tomura et al. (2014) introduce asset illiquidity in a dynamic stochastic general equilibrium model and show that capital requirements based on Value-at-Risk can lead banks to adopt a macro-prudential behaviour. We contribute to this literature by showing how capital requirements may generate and affect banking bubbles.

Second, this study is related to the literature on the existence and the effect of rational bubbles in infinite horizon. For instance, the literature on the existence of bubbles in general equilibrium models with infinitely lived agents is scarce and marked with few important contributions (Miao, 2014). Tirole (1982) shows that bubbles under rational expectations with infinitely lived agents cannot exist. In addition, Blanchard and Watson (1982) argue that "the only reason to hold an asset whose price is above its fundamental value is to resell it at some time and to realise the expected capital gain. But if all agents intend to sell in finite time, nobody will be holding the asset thereafter, and this cannot be an equilibrium". Such behaviour implies that agents over save so that they do not consume everything they could. This cannot be an equilibrium since agents would deviate to increase their consumption levels and, thus, the so called *transversality condition* (TVC) would not be satisfied. In contrast, Kocherlakota (1992) demonstrates that bubbles may exist in an infinite horizon general equilibrium model with borrowing or wealth constraints. These constraints limit the agent arbitrage opportunities by introducing some portfolio constraints. Santos and Woodford (1997) show that bubbles do not exist in standard infinite horizon settings. Nevertheless, they prove that they might emerge under

[5] Basel III, released in 2011, also proposes to use the Value-at-Risk to measure the minimum capital requirement. The difference with Basel II is that it is amended to include a Stressed-Value-at-Risk (SVaR). It aims at reducing procyclicality of the market risk approach and insures that banks hold enough capital to survive long periods of stress.

some specific conditions. The setting presented in this paper is a close version of the example 4.5 in Santos and Woodford (1997) which illustrates the existence of bubbles in incomplete markets, following the terminology of Giménez (2003) and Santos (2006). We contribute to this literature by showing that banking bubbles may emerge with banking regulation based on Value-at-Risk in an infinite horizon general equilibrium framework.

This paper is mostly related to Miao and Wang (2015). They insert an endogenous borrowing constraint and show that bubbles can emerge in an infinitely lived general equilibrium framework without uncertainty. Our model contrasts with Miao and Wang (2015) regarding three major characteristics. First, we introduce the aspect of banking regulation to analyse how it may generate and affect banking bubbles. We provide policy implications by evaluating the role of regulatory parameters on banking bubbles, and as a consequence on the economy. Second, Miao and Wang (2015) consider an agency problem to justify a minimum dividend policy that links dividends to net worth. A dividend policy constraint is not assumed in our model. Third, negative bubbles as well as positive bubbles can arise in this model, while Miao and Wang (2015) study positive bubbles. There are few authors that study negative bubbles in the literature. For instance, Weil (1990) shows that negative bubbles defined as persistent undervaluations of asset prices may arise. In particular, as in our model, pessimistic beliefs might lead to persistent undervaluations. Allen and Gale (2004) define negative bubbles as "asset prices that fall below their fundamentals". They show that they can occur when banking crises lead banks to simultaneously liquidate their assets. They provide the 1990 stock prices collapse in Japan as an example of such phenomenon. Acharya and Naqvi (2019) use the same definition and show that negative bubbles on safe assets can arise when the monetary policy is loose and intermediaries' liquidity is abundant.

The present paper is organised as follows. Section 2 presents the model. Sections 3 and 4 analyse, respectively, the bubbleless and the bubbly general equilibrium. Section 5 analyses the impact of banking capital requirements on the emergence of bubbles and on the economy. Section 6 presents a numerical example and the local dynamics around the equilibria. Finally, Section 7 concludes.

## 2. Model

We consider an economy with three types of infinitely lived agents, banks, households, and firms, as well as a regulatory authority. In this model, banking bubbles can arise. They emerge only if agents believe that banks stock prices contain bubbles. Bubbles are, thus, self-fulfilling. Banks, households, and firms are respectively represented by a continuum of homogeneous agents of mass one. Households are shareholders of banks and owners of firms. It is assumed that banks have the necessary technology and knowledge to engage in lending activity while households do not. Thus, the latter do not lend directly to non-financial firms and have recourse to banks. At the end of each period, banks raise funds internally, using net worth, and externally, by taking deposits from households. Using raised funds, they lend to firms which produce consumption goods. In the model, a bubble is introduced through the bank problem, as in Miao and Wang (2015). Moreover, we consider a bubble with an exogenous probability of burst, i.e., a *bubble* as in Blanchard and Watson (1982). Although a bubble can only arise if agents believe in its existence, it is not an agent's choice. Agents are "bubble takers". The optimisation problem of each agent is presented in this section.

### 2.1. Households

Households are represented by a continuum of homogeneous agents of unit mass. Each household starts with an initial endowment of stocks $s_0$ and deposits $D_0$. At each period $t$, the representative household receives net profits $\pi_t$ generated by firms, chooses its optimal consumption $c_t$, amount of stocks $s_{t+1}$, and deposits $D_{t+1}$ for the next period. It also receives dividends $d_t$ from the shares $s_t$ it owns, sells its shares at price $p_t$ and obtains an interest rate $r_t$ on the amount deposited $D_t$ in the previous period. There is no uncertainty on savings and thus $r_t$ is the risk-free interest rate. We assume that preferences of households are represented by a linear utility function in consumption. Given their budget constraint (1), each household chooses the optimal amount of shares, deposits and consumption $\{s_{t+1}, D_{t+1}, c_t\}_{t=0}^{\infty}$ that maximises its lifetime linear utility. Each household optimisation problem is defined as follows:

$$Max_{\{s_{t+1},D_{t+1},c_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t c_t,$$

subject to

$$D_t (1 + r_t) + s_t (p_t + d_t) + \pi_t = D_{t+1} + c_t + s_{t+1}p_t, \tag{1}$$

$$c_t, D_t, s_{t+1} \geq 0 \text{ for all } t,$$

where $\beta \in ]0, 1[$ is the household discount factor.

The first order conditions with respect to $D_{t+1}$ and $s_{t+1}$, are given by

$$\beta (1 + r_{t+1}) = 1, \tag{2}$$

$$p_t = \beta (d_{t+1} + p_{t+1}). \tag{3}$$

The combination of (2) and (3) gives the households no arbitrage condition, $(d_{t+1} + p_{t+1})/p_t = 1 + r_{t+1}$. This last condition states that the return on stocks is equal to the return on deposits. If it is met, households are indifferent between both types of assets and both are held in the portfolio of agents. However, if this condition is not satisfied, the optimal solution of households yields to a corner solution, thus, only stocks or only deposits are held, depending on which has the highest return.

Since the optimisation problem has an infinite horizon, consider also the transversality condition

$$\lim_{T\to\infty} \beta^T p_T s_T = 0. \tag{4}$$

Condition (4) ensures that the household spends all its budget and thus, does not hold positive wealth when $T \to \infty$. It is a necessary condition for an optimum choice of the household. Tirole (1982) shows that bubbles under rational expectations with infinitely lived agents cannot exist since the transversality cannot be satisfied. However, in our framework, we will show that banking bubbles may satisfy this condition and therefore, may exist.

### 2.2. Firms

Firms are represented by a continuum of homogeneous producers of unit mass. Each firm starts with an amount of loans $L_0$ to buy its initial capital $K_0$. In each period $t$, firms produce $y_t$ using capital $K_t$ bought in the last period with the loans money and reimburse their loans with interests $r_t^l$ such that the total reimbursement is $L_t \left(1 + r_t^l\right)$. The parameter $A$ is the total factor productivity. Then, they distribute net profits to households and choose their optimal amount of total loans and capital for the next period $\{L_{t+1}, K_{t+1}\}_{t=0}^{\infty}$ to maximise their future discounted profits subject to their budget constraint (5), the capital constraint (6), and the investment constraint (7). Note that, for expositional reasons, we consider capital that fully depreciates such that the new stock of capital $K_{t+1}$ is equal to the investment $I_t$, which is equal to the amount of new loans $L_{t+1}$.[6] Each firm optimisation problem is defined as follows

$$Max_{\{L_{t+1},K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \pi_t,$$

subject to

$$\pi_t = y_t - I_t + L_{t+1} - L_t\left(1 + r_t^l\right), \tag{5}$$

$$y_t = AK_t^{\psi},$$

$$K_{t+1} = I_t, \tag{6}$$

$$I_t = L_{t+1}, \tag{7}$$

$$\pi_t \geq 0 \text{ and } L_t, K_t > 0 \text{ for all } t, \tag{8}$$

where $\psi \in ]0, 1[$ is the output elasticity of capital. Using the Lagrange method, the interior solution of the first order condition with respect to $L_{t+1}$ is given by

$$\psi A L_{t+1}^{\psi-1} = 1 + r_{t+1}^l. \tag{9}$$

In the optimum, (9) shows that the marginal product of capital is equal to the marginal cost of loans.

### 2.3. Banks

The banking sector is represented by a continuum of homogeneous banks of unit mass. To provide loans $L_{t+1}$ to firms, banks raise funds by accumulating net worth $N_{t+1}$ and demanding deposits $D_{t+1}$. The regulatory authority imposes that banks keep a fraction $\phi \in [0, 1[$ of deposits as reserves[7]

$$R_t \equiv \phi D_t. \tag{10}$$

Each bank has a balance sheet composed of deposits $D_t$ and net worth $N_t$ on the liability side and of loans $L_t$ and reserves $R_t$ on the asset side such that

$$R_t + L_t = N_t + D_t. \tag{11}$$

Thus, at the end of each period $t$, each bank accumulates net worth using profits from assets earned in $t$, net of deposit repayments and dividends. Let $r_t^l$ be the lending rate earned in $t$ and $r_t$ the risk-free interest rate paid in $t$, so that

$$N_{t+1} = \left(1 + r_t^l\right) L_t + R_t - D_t\left(1 + r_t\right) - d_t - \mathbb{C}_t, \tag{12}$$

where $\mathbb{C}_t = \tau N_t$ represents the cost of holding net worth (capital) and the parameter $\tau \in ]0, 1]$ is the opportunity cost of net worth. The motivation for this cost comes from an agency problem between shareholders and managers of the bank when their interests diverge, making debt less expensive than net worth. To model this agency problem we use the approach of Jensen (1986). The author argues that substituting net worth with debt can reduce agency costs of free cash flow. Managers with excess cash flow may invest the money in a less profitable way instead of paying out dividends. Jensen (1986) shows that there is a benefit of debt in reducing agency costs. As in Estrella (2004) and Blum (2008), we model this debt benefit with the cost of net worth $\mathbb{C}_t$, which is proportional to the level of net worth of a bank. As stated by Estrella (2004), this cost may represent the difference between the cost of net worth funding and funding through other means such as debt.[8] A richer micro founded model could be considered to model the divergence of interest between shareholders and managers. However, this would not add insights on the results and complicate the paper. Miao and Wang (2015) also consider a similar agency problem between shareholders and bankers. They explain that bankers prefer to oversave instead of paying out dividends, as long as the lending rate $r_t^l$ is higher than the deposit rate $r_t$. To limit this behaviour, it is assumed that bankers pay a fraction of the bank's net worth as dividends.

We will show later that the cost of net worth plays two key roles in banks' decisions. First, it leads banks to have a preference for deposits over net worth. Thus, banks take deposits up to their regulatory limit. Second, we will show that it allows bubbles to exist.

Banks are also subject to capital requirements based on Value-at-Risk, as recommended by the Basel committee in Basel II and III.[9] This regulation imposes that banks hold a minimum level of capital, which is calculated with the aim of avoiding banks becoming insolvent. The objective of the regulator is to preserve a safety buffer, such that the market value of banks' assets $\text{VA}_t$ is sufficient to repay depositors. Therefore, the regulator imposes a solvency condition which requires that the maximum amount of deposits banks can hold does not exceed the market value of banks' assets in the worst case scenario such that

$$D_t \leq (1 - \mu)\,\text{VA}_t,$$

where $\mu \in [0, 1[$ is a regulatory parameter which captures the loss in market value of assets in the worst case scenario, as motivated by the Value-at-Risk (VaR) regulation. This parameter depends on the risk faced by banks and on the severity of the regulation. As in Miao and Wang (2015, 2018), the market value of assets $\text{VA}_t$ is the going-concern value and not the liquidation value as in Kiyotaki and Moore (1997). Miao and Wang (2015, 2018) argue that the going-concern value may contain a bubble component because it is priced in the stock market. Thus, since the market value of assets $\text{VA}_t$ is calculated as the sum of the market value of equity $V_t(N_t)$ and the book value of debt $D_t$, and since the market value of equity $V_t(N_t)$ is determined in the stock market, a bubble might exist.[10]

Using the above definition of the market value of assets, the VaR regulation may be written as

$$D_t \leq \eta V_t(N_t), \tag{13}$$

where the market value of equity $V_t(N_t)$ may contain a bubble component and $\eta = (1-\mu)/\mu > 0$ is the Value-at-Risk regulation parameter. It represents the maximum allowed leverage ratio in market value. Altman (1968) derives the same capital regulation and shows that when debt is above a fraction of the market value of equity, companies become insolvent. Note that the VaR regulation (13) is equivalent to the borrowing constraint used by Miao and Wang (2015, 2018).

From now on, we consider that banks finance loans with deposits and hold net worth only for regulatory purposes. In particular, it is assumed that banks' marginal benefits from holding

6 This assumption can be easily relaxed but this would complicate the model without adding much to the intuition derived from the model.

7 Note that the reserve requirement $\phi$ is not crucial for the model nor for the bubble existence. However, it is of interest as it allows the derivation of additional policy implications.

8 This cost can also be interpreted as operational costs paid by banks such as accounting and legal fees, bank charges and management costs. Banks often use a third party such as large business service companies (KPMG, Deloitte) to prepare the legal and accounting side of public offerings and to monitor activities.

9 See the BIS publication, the First Pillar Minimum Capital Requirements, http://www.bis.org/publ/bcbs107.html.

10 Altman (1968), Keeley (1990) and Dangl and Lehar (2004), among others, define the market value of assets as the sum of the market value of equity $V_t(N_t)$ and the book value of debt $D_t$, but do not study the existence of bubbles.

deposits exceed their marginal costs so that banks always want more deposits. We show in Appendix A that this is equivalent to assuming

$$\tau\beta(1-\phi) > \phi(1-\beta). \quad (14)$$

Note that if the opportunity cost of net worth $\tau$ is null, condition (14) cannot be satisfied and thus, banks do not take deposits up to their regulatory limit.

Since we assume that (14) always holds, the VaR capital regulation always binds and becomes

$$D_t = \eta V_t(N_t). \quad (15)$$

For low values of $\eta$, the regulation is severe. Indeed, the amount of authorised deposits that banks can hold compared to banks' values is low. However, for high $\eta$, the regulation is lenient.

The aim of this paper is to determine the regulatory conditions under which banking bubbles may exist and evaluate their macroeconomic consequences. A bubble on a bank stock price is defined as a temporary deviation of the bank stock price from its fundamental value (above or below). The framework follows Blanchard and Watson (1982), Weil (1987) and Miao and Wang (2015). All agents have the same beliefs and have two choices regarding their beliefs. They may believe a bubble exists or not. If agents do not believe a banking bubble exists in period $t$, a bubble can never emerge. As a benchmark exercise, we first present the problem of banks when agents do not believe a bubble exists. We then present the problem of banks when agents believe that it exists and show the conditions under which bubbles are not explosive and thus may exist.

*Bubbleless path*

At the end of period $t$, each bank chooses its optimal net worth to accumulate for next period $\{N_{t+1}\}$ in order to maximise its current dividends and the present value of future dividends subject to the reserve requirement (10), the balance sheet (11), the budget constraint (12) and the capital requirement (15). If agents do not believe a bubble exists, the value of the bank in period $t$ is denoted $V_t^*(N_t)$. The bank problem can be summarised by the following Bellman equation

$$V_t^*(N_t) = Max_{\{N_{t+1}\}}\left\{d_t + \beta V_{t+1}^*(N_{t+1})\right\},$$

subject to

$$d_t = \left(1+r_t^l\right)N_t + D_t\left[r_t^l(1-\phi) - r_t\right] - \tau N_t - N_{t+1}, \quad (16)$$

$$D_t = \eta V_t^*(N_t), \quad (17)$$

$$N_{t+1}, D_t \geq 0 \quad \text{for all } t. \quad (18)$$

We show in Appendix B that the solution of the above maximisation problem gives the following form for the value function

$$V_t^*(N_t) = q_t^* N_t, \quad (19)$$

where $q_t^* \geq 0$ is the marginal value of net worth. It can also be interpreted as the Tobin's Q (Tobin, 1969). Define the bank stock price in $t$ by

$$p_t = \beta V_{t+1}^*(N_{t+1}).$$

**Proposition 1.** *When agents do not believe a bubble exists, the solution of each bank maximisation problem is given by the following system of equations*

$$q_{t+1}^* = \frac{1}{\beta}, \quad (20)$$

$$q_t^* = \left(1 + r_t^l - \tau\right) + \eta q_t^*\left[r_t^l(1-\phi) - r_t\right]. \quad (21)$$

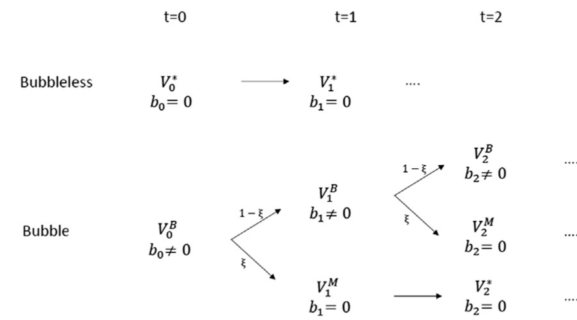


**Fig. 2.** Timeline of events.

Proof of Proposition 1 is presented in Appendix B.

When agents do not believe a bubble exists, the marginal value of net worth given by (20) is constant. This is because the bank is risk-neutral. Thus, by increasing one unit of net worth today, the bank gets the discounted marginal value of net worth. Eq. (21) shows that an additional unit of net worth today gives the discounted return due to the increase in loans minus the opportunity cost of net worth. It also allows the bank to relax the constraint by taking $\eta$ units of additional deposits (see Eq. (15)). Then, the bank earns an additional return of $\eta\left[r_t^l(1-\phi) - r_t\right]$. Using (20) and (21), results show that the lending rate is also constant, which is consistent with the risk neutrality assumption.

*Bubbly path*

When agents believe that a bubble exists in period $t$, the bank's value $V_t^B(N_t)$ contains a bubble $b_t \neq 0$. In this section, we discuss two cases of asset price bubbles, positive bubbles ($b_t > 0$) and negative bubbles ($b_t < 0$). When the bubble is positive, the bank stock price is over-valued. In contrast, when a negative bubble arises, the bank stock price is below its fundamental value. Most of the related literature only focuses on positive bubble price. There are few authors that study negative bubbles. For instance, Weil (1990), Allen and Gale (2004) and Acharya and Naqvi (2019) also define negative bubbles as asset prices that fall below their fundamentals.

As in Blanchard and Watson (1982), there exists a probability $\xi \in ]0,1[$ that the bubble bursts in $t+1$ such that $b_{t+1} = 0$ and thus, that the bank's value becomes $V_{t+1}^M(N_{t+1})$. Note that following Blanchard and Watson (1982), we assume that once the bubble bursts, it never reappears. Therefore, the bank's value can take two different possible values in $t+1$: $V_{t+1}^B(N_{t+1})$ or $V_{t+1}^M(N_{t+1})$, which occur, respectively, with a probability $(1-\xi)$ and $\xi$. The timeline of events of the bubble and the value function are summarised in Fig. 2.

When a banking bubble exists in $t$, each bank chooses the optimal net worth $\{N_{t+1}\}$ in order to maximise its current dividends and expected present value of future dividends subject to the reserve requirement (10), the balance sheet (11), the budget constraint (12) and capital requirements (15), such as

$$V_t^B(N_t) = Max_{\{N_{t+1}\}}\left\{d_t + \beta\left[(1-\xi)V_{t+1}^B(N_{t+1}) + \xi V_{t+1}^M(N_{t+1})\right]\right\}, \quad (22)$$

subject to

$$d_t = \left(1+r_t^l\right)N_t + D_t\left[r_t^l(1-\phi) - r_t\right] - \tau N_t - N_{t+1}, \quad (23)$$

$$D_t = \eta V_t^B(N_t), \quad (24)$$

$$N_t, D_t \geq 0 \quad \text{for all } t, \quad (25)$$

where $V_{t+1}^M(N_{t+1})$ is the value of the bank if the bubble bursts in $t+1$ and is defined by $V_{t+1}^*(N_{t+1})$, where their difference lies in their initial values of net worth.

We show in Appendix C that the solution of the bank maximisation problem with a bubble gives the following value function, until the bubble bursts

$$V_t^B(N_t) = q_t^B N_t + b_t, \tag{26}$$

where $q_t^B \geq 0$ is the marginal value of net worth when there is a bubble and $b_t \neq 0$ is the bubble term on the bank's value. Variables $q_t^B$ and $b_t$ are to be endogenously determined. As it will become clear later, the bubble term is a self-fulfilling component that can be increasing, decreasing or explosive. Note that (26) is the same as in Miao et al. (2015). Define the stock price in $t$ when agents believe a bubble exists and before the bubble bursts by

$$p_t = \beta\left[(1-\xi)V_{t+1}^B(N_{t+1}) + \xi V_{t+1}^M(N_{t+1})\right].$$

**Proposition 2.** *When agents believe a bubble exists in t, until the bubble bursts, the solution of each bank maximisation problem is given by the following system of equations*

$$q_{t+1}^B = \frac{1-\xi\beta q_{t+1}^M}{\beta(1-\xi)}, \tag{27}$$

$$q_t^B = \left(1 + r_t^l - \tau\right) + \eta q_t^B\left[r_t^l(1-\phi) - r_t\right], \tag{28}$$

$$(1-\xi)\beta b_{t+1} = b_t\left\{1 - \eta\left[r_t^l(1-\phi) - r_t\right]\right\}. \tag{29}$$

*From the regulation based on Value-at-Risk (15), if $b_{t+1} = 0$, the value of $q_{t+1}^M$ is given by*

$$q_{t+1}^M = \frac{1}{\eta}\frac{D_{t+1}}{N_{t+1}}. \tag{30}$$

Proof of Proposition 2 is presented in Appendix C.

Eq. (27) shows that, by increasing one unit of net worth today, the bank gets the discounted marginal value of net worth if the bubble lasts plus the discounted marginal value of net worth if the bubble bursts. The probability of a burst introduces a price distortion because it changes inter-temporal arbitrage conditions. An increase in the marginal value of net worth if the bubble bursts $q_{t+1}^M$, decreases the marginal value of net worth if the bubble stays $q_{t+1}^B$. Therefore, the bank's incentive to accumulate net worth if the bubble remains is reduced, and then, the bank distributes more dividends compared with when $b_t = 0$ for all $t$. Eq. (28) has the same intuition than in the case where $b_t = 0$ for all $t$. However, here, the lending rate is not constant anymore and is positively correlated with the marginal value of net worth. The intuition is that the larger the lending rate is, the larger the incentive for banks to accumulate net worth is. Eq. (29) exists if and only if agents believe in the bubble such that $b_t \neq 0$. It represents the bubble growth rate. It grows faster with $\xi < 1$ to compensate for the probability of bursting.

**Proposition 3.** *If*

$$\frac{\left\{1 - \eta\left[r_t^l(1-\phi) - r_t\right]\right\}}{\beta(1-\xi)} < 1/\beta, \tag{31}$$

*the transversality condition of the household (4) is always satisfied and therefore, there exists a bubbly equilibrium.*

Proof of Proposition 3 is presented in Appendix D.

Proposition 3 states that the transversality condition (TVC) is satisfied, i.e. bubbles are not ruled out, if the growth rate of the bubble does not exceed the rate of time preference of households. The transversality condition insures that individuals do not hold positive wealth when $t \to \infty$.

An important point to highlight here, is that without the capital requirement constraint, the bubble growth is given by $b_{t+1}/b_t = 1/\beta(1-\xi)$, which is ruled out by the TVC. The bubble cannot exist. Thus, it is straightforward that under regulation based on book values as in Basel I, instead of on market values such that with the Value-at-Risk regulation, bubbles cannot exist.[11] In addition, the combination of (27), (28) and (29) yields $b_{t+1}/b_t = \left(1 + r_t^l - \tau\right)/\left(1 - \beta\xi q_t^M\right)$. The opportunity cost of net worth ($\tau > 0$) reduces the growth rate of net worth by limiting bankers' ability to accumulate assets. Then, by no arbitrage, the growth rate of the bubble is reduced. Therefore the bubble is no longer explosive and is not ruled out. Analogously, Miao and Wang (2015) reduce the growth of net worth by assuming a minimum dividend policy as a function of net worth.

The bubble return can be written as

$$b_t\left(\frac{1}{\beta} - 1\right) = \underbrace{\frac{1}{\beta(1-\xi)}\left\{\eta\left[r_t^l(1-\phi) - r_t\right] - \xi\right\}b_t}_{\text{dividend yield}} + \underbrace{b_{t+1} - b_t}_{\text{capital gain}}. \tag{32}$$

This equation shows that the return on the bubble is equal to a capital gain $b_{t+1} - b_t$ plus a dividend yield. The dividend yield in the infinite horizon model guarantees that the transversality condition does not rule out the bubble. For positive bubbles, the intuition is that a liquidity premium is provided by the bubble. An additional unit of the positive bubble allows the bank to relax the borrowing constraint by $\eta$ units, and hence to increase deposits by $\eta$ units. These additional units of deposits allow the bank to make $\eta$ more units of loans, thereby raising bank net worth by $\eta\left[r_t^l(1-\phi) - r_t\right]$ units. As long as there is a dividend yield term, the growth rate of the bubble is lower than the household rate of time preference.

The negative bubble can be interpreted in much the same way as the positive one, albeit in reverse. The existence of a negative bubble implies that the underlying asset is undervalued. The gain for the buyer is a lower purchase price (the negative term on the left-hand side of (32)) and the perspective of a capital gain (the second term on the right-hand side). This opportunity of having a lower price however comes with a cost. The lower valuation of the bubble reinforces the effects of the regulatory constraint by reducing the maximum amount of deposits that banks can demand and implies a negative "dividend yield", corresponding to the lost profit margin (first term on the right-hand side). Without this cost, the bubble could not persist as the perspective of capital gains would induce arbitrage operations that would eliminate the bubble and bring the asset price back to its fundamental value. We will show later that, provided the probability the bubble bursts $\xi$ is sufficiently high, the perspective of the capital gain is large enough to compensate for the lost profit margin, and an equilibrium with an undervalued asset is possible.

The setting presented in this paper is a close version of the example 4.5 in Santos and Woodford (1997) which illustrates the existence of bubbles in incomplete markets. Following the terminology of Giménez (2003) and Santos (2006), incomplete financial markets are characterised by less securities than states of nature. In this model, we have three securities that transfer wealth across time, deposits, loans and net worth. Because there exist two bidding constraints on these securities (the VaR regulation and the balance sheet), the model can be summarised by one security. Moreover, if the bubble exists, there are two states of nature at each period, the bubble bursts or does not (see Fig. 2). Thus, there are not enough securities to transfer wealth across periods and a bubble might emerge. Note also that Santos and Woodford (1997) show that bubbles exist in incomplete markets

11 The Basel ratio Tier 1 is based on book values and takes the following form: $N_t = \chi D_t$ where $\chi > 0$ is a regulation parameter.

if the asset is in positive net supply and the economy is not sufficiently productive. In this model, these two conditions are satisfied since we assume asset net supply of one at all times and because the opportunity cost of net worth $\tau$ reduces the bubble and the economy growth rates, the economy is not sufficiently productive.

## 3. Bubbleless general equilibrium

This section defines and analyses the bubbleless general equilibrium where variables are denoted $x_t^*$.

**Definition 4.** A competitive general bubbleless equilibrium with $b_t = 0$ for all $t$, is defined as sequences of allocations and prices

$$\mathcal{E}_t^* = \left\{d_t^*, N_{t+1}^*, K_{t+1}^*, L_{t+1}^*, D_{t+1}^*, \pi_t^*, y_t^*, c_t^*, s_{t+1,}^* q_t^*, r_t, r_t^{l*}, p_t^*\right\} \forall t,$$

such that taking prices as given, all agents maximise their future payoffs subject to their constraints and the transversality condition is satisfied. Finally, the market for loans, deposits, and stocks ($s_{t+1}^* = 1$) clear. The equilibrium consumption is given by the combination of the three budget constraints (1), (5), and (12), such that

$$c_t^* = y_t^* - L_{t+1}^* - (R_{t+1}^* - R_t^*) - \tau N_t^*. \tag{33}$$

Eq. (33) is the condition on the goods market. Households consumption is equal to output net of investment, variation in reserves, and the cost of net worth.

*Bubbleless stationary equilibrium*

Here, we analyse a stationary bubbleless equilibrium when variables are constant over time such that $\mathcal{E}_0^* = \cdots = \mathcal{E}_t^* = \mathcal{E}^*$ for all $t$. The equilibrium deposit rate is given by (2) such that $r = 1/\beta - 1$. The marginal value of net worth in (20) is $q^* = 1/\beta$. From the regulation based on Value-at-Risk in (15) and the value function (19),

$$\frac{D^*}{N^*} = \frac{\eta}{\beta}. \tag{34}$$

From (21), the lending rate is

$$r^{l*} = \frac{r\left(\eta + \beta\right) + \beta\tau}{\beta + \eta\left(1 - \phi\right)}. \tag{35}$$

**Proposition 5.** *The lending rate $r^{l*}$ in a bubbleless stationary equilibrium increases with the reserves $\phi$ and the opportunity cost of net worth $\tau$. In contrast, it decreases with the Value-at-Risk regulation parameter $\eta$.*

Proof of Proposition 5 is presented in Appendix E. The intuition is that larger opportunity costs of net worth and reserves reduce the supply of loans, and as a consequence increase the lending rate. In contrast, a larger Value-at-Risk regulation parameter $\eta$ allows banks to raise money using cheaply acquired funds, i.e. deposits. This effect raises banks' size and reduces the lending rate.

For more insights, we also look at the interest rate spread, which is given by

$$\beta\left(r^{l*} - r\right) = \frac{\left(1 - \beta\right)\eta}{\beta + \eta\left(1 - \phi\right)}\phi + \frac{\beta^2}{\beta + \eta\left(1 - \phi\right)}\tau.$$

The above equation shows that the discounted interest rate spread increases with the opportunity cost $\tau$ and the fraction of reserves $\phi$. For $\phi = 0$, the interest spread is only a function of the opportunity cost. When there are no costs for the bank such that $\phi = \tau = 0$, the lending rate falls to the safe rate $r$.

The stationary level of loans is given by the first order condition (9) so that $L^* = \left[\left(1 + r^{l*}\right)/\psi\right]^{1/(\psi - 1)}$. From the balance sheet constraint (11) and (34), $N^* = L^*/[1 + (1 - \phi)(\eta/\beta)]$. Thus, the equilibrium consumption is given by $c^* = (L^*)^{\psi} - L^* - \tau L^*/[1 + (1 - \phi)(\eta/\beta)]$. Denote $W^*$ the welfare in a bubbleless stationary equilibrium. Therefore, $W^* = c^*$. Appendix F shows that $W^*$ and $L^*$ are decreasing in the lending rate $r^{l*}$.

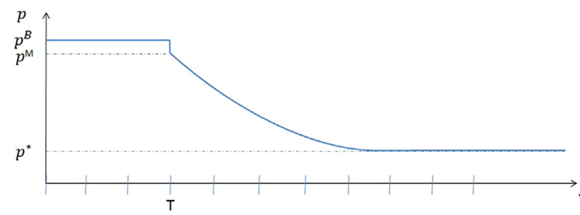


**Fig. 3.** Stock price dynamic when the positive bubble bursts.

## 4. Bubbly general equilibrium

This section defines and analyses the bubbly general equilibrium where variables before and after the bubble bursts at $t = T$ are, respectively, denoted by $x_t^B$ and $x_t^M$.

**Definition 6.** If a bubble exists in $t$ such that $b_t \neq 0$, until the bubble bursts in $T$, a competitive bubbly general equilibrium is defined as

$$\mathcal{E}_t^B = \{d_t^B, N_{t+1}^B, K_{t+1}^B, L_{t+1}^B, D_{t+1}^B, \pi_t^B, y_t^B, c_t^B, b_t, s_{t+1,}^B q_t^B, q_t^M, r_t, r_t^{lB}, p_t^B\} \forall t < T,$$

such that taking prices as given, all agents maximise their future payoffs subject to their constraints and the transversality condition is satisfied.[12] Finally, the market for loans, deposits, and stocks ($s_{t+1}^B = 1$) clear. At $t = T$, the bubble crashes such that $b_t = 0\ \forall t \geq T$, a competitive bubbly general equilibrium $\mathcal{E}_t^M$ is defined as $\mathcal{E}_t^*\ \forall t \geq T$ with $N_T^M = N_T^B$, such that taking prices as given, all agents maximise their future expected payoffs subject to their constraints and the transversality condition is satisfied. Finally, the market for loans, deposits, and stocks ($s_{t+1}^M = 1$) clear. As in the bubbleless equilibrium, the condition on the goods market is given by (33), where variables correspond to the ones from the bubbly general equilibrium.

For simplicity, as in Weil (1987) and Miao and Wang (2015), we study a bubbly equilibrium with the following properties. The equilibrium is constant until the bubble collapses at $t = T$, such that $\mathcal{E}_0^B = \cdots = \mathcal{E}_{T-1}^B = \mathcal{E}^B$ with $b_0 = \cdots = b_{T-1} = b \neq 0$. We call it a semi-stationary equilibrium. At $t = T$, the banking bubble collapses such that $b_T = 0$ and the equilibrium is denoted by $\mathcal{E}_T^M$. Then, for all $t > T$, the equilibrium $\mathcal{E}_T^M$ converges to the bubbleless stationary equilibrium $\mathcal{E}^*$. Fig. 3 shows the dynamic of the price when a positive banking bubble exists and then bursts.

At $t = T$, the bubble bursts such that $b_t = 0$ and stays at this value for all $t \geq T$. The price $p_t^B$ falls to $p_T^M$. Then, the bank maximises dividends and discounted future dividends such that the bubble is over and will never reappear. Therefore, the price converges to $p^*$ for all $t > T$.

The semi-stationary equilibrium, i.e. until the bubble bursts, is characterised by the following values. The deposit rate is given by (2) $r = 1/\beta - 1$. The lending rate before the bubble collapses is defined by (29) such that

$$r^{lB} = \frac{r(\beta + \eta) + \beta\xi}{(1 - \phi)\eta}. \tag{36}$$

[12] Note that the bank marginal value of net worth $q_t^B$ until the bubble bursts is a function of the marginal value of net worth after the bubble collapses $q_t^M$.

**Proposition 7.** *In a semi-stationary bubbly equilibrium, the lending rate increases with the fraction of reserves $\phi$ and the probability of burst $\xi$. In contrast, it decreases with the Value-at-Risk regulation parameter $\eta$.*

Compared to the bubbleless lending rate given by (35), the lending rate is independent of the opportunity cost $\tau$. This characteristic will be explained later.

The interest rate spread between the lending rate and the risk-free deposit rate, until the bubble collapses, is

$$\beta\left(r^{lB}-r\right)=\frac{1-\beta}{(1-\phi)}\phi+\frac{\beta(1-\beta)}{(1-\phi)}\frac{1}{\eta}+\frac{\beta^2}{(1-\phi)}\frac{\xi}{\eta}.$$

Hence, the spread is a function of the bank's costs. It is increasing with a large probability of burst to compensate for the risk and with a high fraction of reserves $\phi$. In contrast, it decreases with less stringent capital requirement, which is represented by a high $\eta$. If $\xi = \phi = 0$, then the interest rate spread is equal to $\beta(1-\beta)/\eta$, which is proportional to the tightness of the regulatory constraint.

The marginal value of net worth while the bubble lasts and when the bubble collapses are, given by (27) and (28)

$$q^B=\frac{1-\beta\xi q^M}{\beta(1-\xi)}=\frac{1-\tau+r^{lB}}{\beta(1-\xi)}, \tag{37}$$

and

$$q^M=\frac{\tau-r^{lB}}{\beta\xi}. \tag{38}$$

From (30), the leverage ratio is

$$\frac{D^B}{N^B}=\eta q^M. \tag{39}$$

From the first order condition of firms (9), we obtain the equilibrium quantity of loans

$$L^B=\left[\frac{1}{\psi}\left(1+r^{lB}\right)\right]^{\frac{1}{\psi-1}}. \tag{40}$$

From (10), (11), (39) and (40),

$$N^B=\frac{L^B}{1+(1-\phi)\,\eta q^M}.$$

It can be shown that $N^B$ is strictly positive if and only if $q^M>0$, which is equivalent to

$$\eta>\eta^c, \tag{41}$$

where $\eta^c = (\xi+r)/[r-\tau(1-\phi)]$. Equivalently, this can be rewritten as $\xi<\xi^c$, where $\xi^c=[\tau(1-\phi)\eta-r(\beta+\eta)]/\beta$.

In the remainder of the paper, we consider that condition (41) always holds. From the regulation (15) and the value function when the bubble exists (26),

$$b=\frac{D^B}{\eta}-q^BN^B. \tag{42}$$

Using (37), (39) and (42), the bubble term can be re-written as

$$\begin{aligned} b &= \left(q^M-q^B\right)N^B \\ &= \left[\frac{\eta(\tau-\xi)(1-\phi)-r(\eta+\beta)+\beta\xi}{\beta\xi(1-\xi)(1-\phi)\eta}\right]N^B. \end{aligned} \tag{43}$$

The equation above shows that the bubble increases with a large opportunity cost of net worth. An increase in the opportunity cost of net worth $\tau$ should, without bubble, raise the lending rate. However, in the presence of a bubble, the increase in $\tau$ enlarges the bubble, which relaxes the capital requirement constraint. Thus, the loan supply increases, cancelling out the effect of $\tau$ on the lending rate. Depending on the parameters in Eq. (43), the bubble might be positive or negative. Note that the bank's value $V^B$ is always positive even in the presence of a negative bubble. The proof is presented in Appendix G. From (1), and (15), the equilibrium consumption is $c^B = \left(L^B\right)^{\psi} - L^B-\tau L^B/\left[1+(1-\phi)D^B/N^B\right]$. Finally, we define the bubbly semi-stationary welfare as $W^B = c^B$. Compared to the bubbleless stationary equilibria, the welfare has the same form. However, it now depends on the bubble. Indeed, the bubble modifies the value of lending rate by affecting the capital requirement constraint and thus, the equilibrium quantity of loans. Appendix H shows that the welfare $W^B$ is decreasing in the lending rate $r^{lB}$.

## 5. Banking regulation policy implications

This section analyses the impact of capital requirements banking regulation on the emergence of banking bubbles and on the economy.

As shown in Section 2.3, bubbles cannot emerge under Basel I regulation. However, Basel II and III capital requirements based on VaR allow bubbles to exist.

Moreover, the strength of the VaR regulation, measured by the VaR regulation parameter, determines the existence and the properties of bubbles. Using (43), we can show that a bubble might exist for all values the VaR regulation parameter in $]\eta^c,\infty[$ except for $\eta\neq\bar{\eta}$, where

$$\bar{\eta}=\frac{1-\beta(1-\xi)}{(\tau-\xi)(1-\phi)-r}.$$

**Proposition 8.** *Under (14), (31), (41) and $\eta\neq\bar{\eta}$, a semi-stationary bubbly equilibrium exists ($b\neq 0$). For $\eta>\bar{\eta}$, the bubble is positive. In contrast, for $\eta^c<\eta<\bar{\eta}$, it is negative.*

Proposition 8 suggests that the semi-stationary equilibrium with a bubble exists if the regulation parameter based on Value-at-Risk is such that $\eta \neq \bar{\eta}$ and $\eta > \eta^c$. Indeed, under the conditions described in Proposition 8, the transversality condition is satisfied. The bubble is positive only for large values of the regulation parameter, i.e. more lenient banking regulation. In contrast, for stringent capital regulation, the bubble is negative, i.e. the bank stock price is below its fundamental value. As shown in Appendix G, the bank value is always positive at equilibrium, even in the presence of a negative bubble. Therefore, a negative bank stock price is excluded. Note that for too restrictive regulation ($\eta < \eta^c$), negative bubbles cannot exist. Indeed, a low value of the regulation parameter implies a larger loan rate $r^{lB}$ and makes the cost of holding an undervalued asset too large.

In our model, the presence of positive or negative bubbles are conditioned on (1) the beliefs of agents (bubbles arise only if agents believe a bubble exists); (2) the stance of the economy (regulatory parameters and probability of burst of the bubble).

A change in the VaR regulation parameter $\eta$ might modify the sign of the bubble as well as its existence. Indeed, if the regulatory authority changes the VaR regulation parameter, the maximisation problems are reconsidered by the agents and their beliefs about bubbles can change.[13] Another important policy implication, here, is that the reserve requirement parameter $\phi$ affects negatively the threshold $\bar{\eta}$. As a consequence, when $\phi$ is large, the regulation parameter $\eta$ should be even greater for the economy to be in the positive bubbly semi-stationary equilibrium. The model results could be empirically tested as the regulation has been modified (in particular the tightness of the constraint) several times in the past decades.

13 A bubble can start at any moment if there is a change in a parameter of the model. However, as explained in Section 2.3, when a bubble bursts, it will never reappear if the parameters are the same.

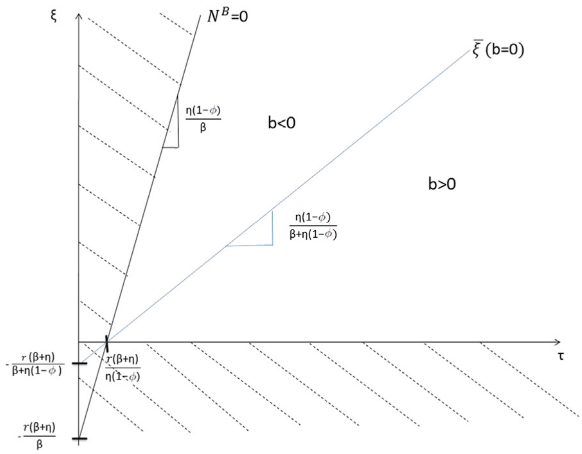


**Fig. 4.** Bubble value in the parameter space.

Alternatively, the condition under which a semi-stationary bubbly equilibrium exists, for $\xi \in ]0, \xi^c[$, can be rewritten as $\xi \neq \bar{\xi}$, where

$$\bar{\xi} = \frac{\eta\left[(1-\phi)\tau - r\right] - (1-\beta)}{\beta + \eta(1-\phi)}. \tag{44}$$

Therefore, the semi-stationary bubbly equilibrium exists for a probability of burst $\xi \in ]0, \xi^c[$ and $\xi \neq \bar{\xi}$. It can be shown that a positive bubble exists for small values of the probability of burst, $\xi < \bar{\xi}$. This is consistent with Weil (1987) and Miao and Wang (2015) who also find that positive bubbles exist only for small values of the bursting probability. In contrast, negative bubbles might arise when the probability of burst $\xi$ is high ($\xi^c > \xi > \bar{\xi}$). Indeed, when the probability that the bubble bursts $\xi$ is sufficiently high, the perspective of the capital gain is large enough to compensate for the lost profit margin, and a semi-stationary equilibrium with an undervalued asset is possible. However, the probability that the bubble bursts should not be too large. A larger value of $\xi$ implies a larger loan rate $r^{lB}$ and makes the cost of holding the undervalued asset (the negative dividend yield) even larger. Note that a change in beliefs concerning the probability of burst might modify the equilibrium, for instance, from a positive semi-stationary bubbly equilibrium to a bubbleless stationary equilibrium.

Fig. 4 displays the bubble value in the parameter space $(\xi, \tau)$, for a given $\eta$ and $\phi$. At $\xi = \bar{\xi}$, the bubble term is zero. For $\xi < \bar{\xi}$ (resp. $\xi^c > \xi > \bar{\xi}$), the bubble is positive (resp. negative). The slope of the line $\bar{\xi}$ increases with large values of the Value-at-Risk regulation parameter $\eta$. Thus, the parameter space for the positive bubble widens. As the regulator becomes more lenient such that $\eta$ is high, the economy can enter a state in which bubbles are positive.

**Proposition 9.** *If $\eta \in ]\eta^c, \infty[$ and $\eta \neq \bar{\eta}$ both equilibria with and without a bubble on stock prices coexist.*

**Proposition 10.** *If $\eta > \bar{\eta}$, the bubbly equilibrium lending rate before the bubble collapses is lower than the bubbleless lending rate. Thus, welfare is larger with a positive bubble. In contrast, a negative bubble ($\eta^c < \eta < \bar{\eta}$) reduces welfare.*

Proof of Proposition 10 is in Appendix I.

Proposition 9 states that for all VaR regulation parameters in $]\eta^c, \infty[$, except for the value of $\eta \neq \bar{\eta}$, both bubbly and bubbleless equilibria coexist. The existence of a bubble depends on agents beliefs. The threshold $\bar{\eta}$ can be viewed as a point of reversal at which you may move from a positive bubbly to a negative bubbly semi-stationary equilibrium. At this reversal point, the equilibrium can move from higher to lower welfare. For $\eta > \bar{\eta}$, the capital requirement based on Value-at-Risk is less stringent. In that case, the semi-stationary bubbly equilibrium provides larger welfare than the bubbleless equilibrium. The intuition is that, when agents consider that a bubble exists, a lower capital requirement leads to optimistic beliefs on banks' values. The positive bubble allows banks to relax the capital requirement constraint, and thus banks demand more deposits, which raises their leverage, and make more loans. This effect reduces the lending rate and provides better welfare. In contrast, for more stringent capital requirements $\eta^c < \eta < \bar{\eta}$, when agents consider that a bubble exists, tighter capital requirements leads to pessimistic beliefs on banks' values. Banks demand less deposits and household deposit less because they believe that profits of banks are undervalued, compared to their fundamentals. Since banks have less deposits, it leads to a credit crunch. Then, the lending rate increases which reduces the equilibrium welfare. An important point to highlight here is that a change in banking regulation may modify the equilibrium and lead to crises, by reducing welfare levels. This effect can explain the occurrence of crises without any external shocks. In addition, using (44), results also show that a change in beliefs about the probability of burst may also lead to a crisis, as in Miao and Wang (2015).

Basel II and III introduce successively more stringent capital requirements to banks. The model predicts that in presence of a positive bubble, as capital requirements become tighter, the welfare decreases. Note that in this case, the welfare is still higher than the bubbleless welfare. The proof is presented in Appendix H. The model also shows that, as capital requirements based on Value-at-Risk become tighter ($\eta$ small), the space of parameters that lead to negative bubbles becomes larger, increasing the likelihood of negative bubble emergence and the likelihood of a crisis.

Table 1 summarises and compares the main results discussed in this section. It shows that, when agents believe a bubble exists, a positive bubble arises for lenient regulatory Value-at-Risk constraints, $\eta > \bar{\eta}$. It leads to the highest equilibrium welfare level, highest equilibrium quantity of loans and leverage levels. On the opposite, a negative bubble arises when capital requirement based on Value-at-Risk are more stringent. The negative bubbly semi-stationary equilibrium is characterised by the lowest equilibrium level of welfare, credit and leverage.

**Table 1**
Policy implication.

| Variables | $\eta > \bar{\eta}$ | $\eta^c < \eta < \bar{\eta}$ |
|---|---|---|
| $b$ | $b > 0$ | $b < 0$ |
| $r^l$ | $r^{lB} < r^{l*}$ | $r^{lB} > r^{l*}$ |
| $L$ | $L^B > L^*$ | $L^B < L^*$ |
| $\frac{D}{N}$ | $\frac{D^B}{N^B} > \frac{D^*}{N^*}$ | $\frac{D^B}{N^B} < \frac{D^*}{N^*}$ |
| $W$ | $W^B > W^*$ | $W^B < W^*$ |

## 6. Equilibrium dynamics

This section studies the local dynamics around the bubbleless stationary equilibrium and the semi-stationary bubbly equilibrium. To analyse the stability of the system, we present a numerical example.

We calibrate the parameters and we report the implied values for variables in the bubbleless stationary and bubbly semi-stationary equilibria. The discount factor is calibrated to $\beta = 0.99$, the capital share to $\psi = 0.33$, the probability of burst to $\xi =$

**Table 2**
Bubbleless and bubbly equilibria.

| Variables | Bubbly > 0 | Bubbleless |
|---|---|---|
| $N$ | 0.0132024 | 0.0166121 |
| $D$ | 0.173818 | 0.169664 |
| $d$ | 0.000171925 | 0.000167799 |
| $L$ | 0.185282 | 0.184579 |
| $p$ | 0.0170206 | 0.0166121 |
| $q$ | 0.977657 | 1.0101 |
| $r^l$ | 0.0210922 | 0.0236939 |
| $b$ | 0.00428355 | 0 |
| $W$ | 0.386042 | 0.385514 |

0.1. The regulatory parameter is $\mu = 0.09$, which implies that $\eta = 10.11$. This calibration for $\mu$ allows us to have a tier 1 ratio around 8% as recommended by the Basel committee.[14] This ratio is 8.99% for the bubbleless stationary equilibrium and 7.12% for the semi-stationary bubbly equilibrium. The reserve parameter $\phi = 0.01$ is set as required by the European Central Bank.[15] Finally, we set the opportunity cost of net worth $\tau = 0.15$.[16] Under these values of parameters, Propositions 9 and 10 show that the bubbly and the bubbleless stationary equilibria, until the bubble bursts coexist and that the bubbly semi-stationary equilibria has a positive bubble ($\eta > \bar{\eta}$). Moreover, under this calibration, the marginal value of net worth in $T$, once the bubble has burst is $q_T^M = 1.3021$.

Table 2 confirms results summarised in Table 1. Compared to the bubbleless steady state, the quantity of loans supplied by banks is larger in the bubbly semi-stationary equilibrium. This gives a lower lending rate $r^l$, leading to a higher welfare $W$.

To analyse the stability and uniqueness properties of the system, we log-linearise the system around the stationary and the semi-stationary equilibria. This results in a system of linear difference equations. When agents do not believe a bubble exists, $b_t = 0$ for all $t$, as well as when agents believe a bubble exists, $b_t > 0$ for $t = 0, \ldots, T$, until the bubble bursts, the eigenvalues associated with the linearised system around, respectively, the stationary bubbleless and the semi-stationary bubbly equilibria, show that the number of unstable eigenvalues (eigenvalues that lie outside the unit circle) is equal to the number of forward looking variables.[17] Thus, under this calibration, the system of equations when $b_t = 0$ for all $t$ and when $b_t > 0$ for all $t < T$, is determined and both the bubbleless and the bubbly equilibria are stable and unique. This implies that given an initial value of $N_t^*$ in the neighbourhood of the stationary bubbleless equilibrium, there exists a unique value of $q_t^*$ such that the system of linear difference equations converges to the unique stationary bubbleless equilibrium along a unique saddle path (see Blanchard and Kahn, 1980). Similarly, given an initial value of $N_t^B$ in the neighbourhood of the semi-stationary bubbly equilibrium, there exists a unique value of $q_t^B$ such that the system of linear difference equations converges to the unique semi-stationary bubbly equilibrium along a unique saddle path, for all $t < T$.

## 7. Conclusion

In this paper, we develop a general equilibrium model in infinite horizon with a regulated banking sector where a banking bubble may arise endogenously. This paper combines the economic theory of banking regulation with the one of asset price bubbles. We show that a bubble emerges if agents believe that it exists (i.e. expectations of agents are self-fulfilling) and when banks face capital requirements based on Value-at-Risk. Two different equilibria emerge and can coexist: the bubbleless and the bubbly equilibria. Capital requirements based on Value-at-Risk allow the bubble to exist. Alternatively, under a regulatory framework where capital requirements are based on credit risk only as specified in Basel I, a bubble is explosive and as a consequence cannot exist. The bubbly equilibrium is characterised by a positive or a negative bubble depending on capital requirements based on Value-at-Risk. We find a maximum capital requirement below which the bubble is positive. Below this threshold, the bubbly equilibrium provides larger welfare than the bubbleless equilibrium. Therefore, this result suggests that a change in banking policies might lead to a crisis. This can explain the existence of crises without any external shocks. We also show that a semi-stationary equilibrium with a positive (resp. negative) bubble exists if the probability that the bubble collapses is small (resp. high). Consequently, a change in beliefs about the bubble probability of burst also modifies the equilibrium, from a higher to a lower welfare.

For future research, it could be interesting to empirically test the model implications. For instance, one can test if bubbles have emerged following the introduction of VaR regulation. Furthermore, one could analyse the effect of a change in the tightness of the regulation parameter on bank stock prices. It could also be interesting to extend our model by the addition of different elements. Risk aversion of households and endogenous labour choice can be considered. However, endogenous labour choice will complicate the model without changing our main results. Risk aversion can be introduced by a quadratic utility function for households and thus, the emergence of bubbles can be studied in this context. One can also add a probability of default on loans repayments in order to model credit risk in the economy and analyse its impact on key macroeconomic variables.

## Appendix A

Here, we show that without capital requirement, each bank chooses to hold the maximum amount of deposits.

Each bank maximisation problem without capital requirement is given by

$$V_t\left(N_t, D_t\right) = Max_{\{N_{t+1}, D_{t+1}\}}\left[d_t + \beta V_{t+1}(N_{t+1}, D_{t+1})\right],$$

subject to

$$d_t = \left(1 + r_t^l\right) N_t + D_t\left[r_t^l(1-\phi) - r_t\right] - \tau N_t - N_{t+1},$$

$N_{t+1} \geq 0$ and $D_{t+1} > 0$ for all $t$.

From the problem described above,

$$\begin{aligned} &V_t(N_t, D_t) = \\ &Max_{\{N_{t+1}, D_{t+1}\}}\left\{\left(1 + r_t^l\right) N_t + D_t\left[r_t^l\left(1-\phi\right) - r_t\right] - \tau N_t \right. \\ &\qquad \left. - N_{t+1} + \beta V_{t+1}(N_{t+1}, D_{t+1})\right\}. \end{aligned} \tag{45}$$

The marginal value from an increase in net worth and deposits are given by

$$\frac{dV_t\left(N_t, D_t\right)}{dN_{t+1}} = -1 + \beta\frac{dV_{t+1}(N_{t+1}, D_{t+1})}{dN_{t+1}}, \tag{46}$$

and

$$\frac{dV_t\left(N_t, D_t\right)}{dD_{t+1}} = \beta\frac{dV_{t+1}(N_{t+1}, D_{t+1})}{dD_{t+1}}.$$

Using the envelop theorem,

$$\frac{dV_t\left(N_t, D_t\right)}{dN_t} = 1 + r_t^l - \tau,$$

[14] This ratio is defined as total net worth over risky assets.

[15] See https://www.ecb.europa.eu/mopo/implement/mr/html/calc.en.html.

[16] For simplicity, the full depreciation of capital assumption is kept for the numerical example.

[17] Eigenvalues are reported in Appendix J.

and

$$\frac{dV_t\left(N_t, D_t\right)}{dD_t} = r_t^l\left(1-\phi\right) - r_t.$$

Banks decide to hold a maximum amount of deposits if $dV_t\left(N_t, D_t\right)/dD_{t+1} > 0$, which is equivalent to

$$r_t^l\left(1-\phi\right) - r_t > 0. \quad (47)$$

The interior solution for the net worth is given by $dV_t\left(N_t, D_t\right)/dN_{t+1} = 0$. Eq. (46) becomes

$$r_t^l = \frac{1}{\beta} - 1 + \tau. \quad (48)$$

From the household problem and putting (48) in (47), we get the following condition

$$\tau\beta\left(1-\phi\right) > \phi\left(1-\beta\right).$$

If the above condition holds, banks always choose the maximum amount of deposits, and consequently the capital requirement regulation always binds. Note that the constraint $D_{t+1} > 0$ is always verified since the marginal value from deposits is positive ($dV_t\left(N_t, D_t\right)/dD_{t+1} > 0$) and the problem is linear.

## Appendix B

This appendix presents the proof of Proposition 1. From the bank bubbleless maximisation problem,

$$V_t^*\left(N_t\right) = Max_{\{N_{t+1}\}}\left\{d_t + \beta V_{t+1}^*\left(N_{t+1}\right)\right\},$$

subject to

$$d_t = \left(1+r_t^l\right)N_t + D_t\left[r_t^l\left(1-\phi\right) - r_t\right] - \tau N_t - N_{t+1},$$

$$D_t = \eta V_t^*\left(N_t\right),$$

$$N_t, D_t \geq 0 \quad \text{for all } t.$$

The Bellman equation becomes

$$V_t^*\left(N_t\right) = Max_{\{N_{t+1}\}}\left(1+r_t^l-\tau\right)N_t + \eta V_t^*\left(N_t\right)\left[r_t^l\left(1-\phi\right)-r_t\right] - N_{t+1} + \beta V_{t+1}^*\left(N_{t+1}\right).$$

The marginal value from a net worth increase is given by

$$\frac{dV_t^*\left(N_t\right)}{dN_{t+1}} = -1 + \beta\frac{dV_{t+1}^*\left(N_{t+1}\right)}{dN_{t+1}}.$$

By the envelop theorem,

$$\frac{dV_t^*\left(N_t\right)}{dN_t} = \left(1+r_t^l-\tau\right) + \eta\frac{dV_t^*\left(N_t\right)}{dN_t}\left[r_t^l\left(1-\phi\right)-r_t\right].$$

The interior solution for the net worth is given by $dV_t^*\left(N_t\right)/dN_{t+1} = 0$. Therefore,

$$\frac{dV_{t+1}^*\left(N_{t+1}\right)}{dN_{t+1}} = \frac{1}{\beta}.$$

Since the problem is linear in $N$, we get

$$V_t^*\left(N_t\right) = q_t^* N_t. \quad (49)$$

Replacing (49) in the maximisation problem, the solution is given by the following system:

$$\begin{aligned} q_{t+1}^* &= \frac{1}{\beta}, \\ q_t^* &= \left(1+r_t^l-\tau\right) + \eta q_t\left[r_t^l\left(1-\phi\right)-r_t\right]. \end{aligned}$$

## Appendix C

This appendix proves Proposition 2. From the bank maximisation problem when agents believe in a bubble such that $b_t \neq 0$, we have

$$V_t^B\left(N_t\right) = Max_{\{N_{t+1}\}}\left\{d_t + \beta\left[(1-\xi)V_{t+1}^B\left(N_{t+1}\right) + \xi V_{t+1}^M\left(N_{t+1}\right)\right]\right\},$$

subject to

$$d_t = \left(1+r_t^l\right)N_t + D_t\left[r_t^l\left(1-\phi\right) - r_t\right] - \tau N_t - N_{t+1},$$

$$D_t = \eta V_t^B\left(N_t\right),$$

$$N_t, D_t \geq 0 \quad \text{for all } t,$$

where $V_{t+1}^M(N_{t+1})$ is the value of the bank if the bubble bursts in $t+1$ and is defined as $V_{t+1}^*\left(N_{t+1}\right)$ for the bubbleless maximisation problem.

The Bellman equation becomes

$$V_t^B\left(N_t\right) = Max_{\{N_{t+1}\}}\left(1+r_t^l-\tau\right)N_t + \eta V_t^B\left(N_t\right)\left[r_t^l\left(1-\phi\right)-r_t\right] - N_{t+1} + \beta\left[(1-\xi)V_{t+1}^B\left(N_{t+1}\right) + \xi V_{t+1}^M\left(N_{t+1}\right)\right]$$

The marginal value from a net worth increase is given by

$$\frac{dV_t^B\left(N_t\right)}{dN_{t+1}} = -1 + \beta(1-\xi)\frac{dV_{t+1}^B\left(N_{t+1}\right)}{dN_{t+1}} + \beta\xi\frac{dV_{t+1}^M\left(N_{t+1}\right)}{dN_{t+1}}.$$

By the envelop theorem,

$$\frac{dV_t^B\left(N_t\right)}{dN_t} = \left(1+r_t^l-\tau\right) + \eta\frac{dV_t^B\left(N_t\right)}{dN_t}\left[r_t^l\left(1-\phi\right)-r_t\right].$$

The interior solution for the net worth is given by $dV_t^B\left(N_t\right)/dN_{t+1} = 0$. Therefore,

$$\frac{dV_{t+1}^B\left(N_{t+1}\right)}{dN_{t+1}} = \frac{1-\xi\beta\frac{dV_{t+1}^M\left(N_{t+1}\right)}{dN_{t+1}}}{\left(1-\xi\right)\beta}.$$

Since the problem is linear in $N$, we get

$$V_t^B\left(N_t\right) = q_t^B N_t + b_t. \quad (50)$$

Replacing (50) in the maximisation problem, the solution is given by the following system:

$$\begin{aligned} q_{t+1}^B &= \frac{1-\xi\beta q_{t+1}^M}{\beta\left(1-\xi\right)}, \\ q_t^B &= \left(1+r_t^l-\tau\right) + \eta q_t^B\left[r_t^l\left(1-\phi\right)-r_t\right], \\ (1-\xi)\beta b_{t+1} &= b_t\left\{1-\eta\left[r_t^l\left(1-\phi\right)-r_t\right]\right\}. \end{aligned}$$

## Appendix D

This appendix presents the proof of Proposition 3. We show the condition to ensure that the bubbly equilibrium, until the bubble bursts, satisfies the transversality condition. The following transversality condition is required

$$\lim_{t\to\infty}\beta^t p_t = \lim_{t\to\infty}\beta^t\left[\xi\left(q_t^M N_t\right) + \left(1-\xi\right)\left(q_t^B N_t + b_t\right)\right] = 0.$$

It is satisfied if

$$\lim_{t\to\infty}\beta^t\left[\xi q_t^M N_t + \left(1-\xi\right)q_t^B N_t\right] = \lim_{t\to\infty}\left(1-\xi\right)b_t\beta^t = 0.$$

Using Eq. (27), the transversality condition can be rewritten as follows

$$\lim_{t\to\infty}\beta^t q_t^* N_t = \lim_{t\to\infty}\left(1-\xi\right)b_t\beta^t = 0.$$

Since the bubble growth rate is

$$\frac{b_{t+1}}{b_t} = \frac{1}{\beta(1-\xi)}\left\{1-\eta\left[r_t^l(1-\phi)-r_t\right]\right\},$$

the TVC requires that

$$\frac{1}{\beta(1-\xi)}\left\{1-\eta\left[r_t^l(1-\phi)-r_t\right]\right\} < \frac{1}{\beta}.$$

Thus, the condition to allow a bubble to exist is

$$\eta\left[r_t^l(1-\phi)-r_t\right] > \xi.$$

## Appendix E

This appendix proves Proposition 5. Here, we prove that the interest rate of loans in the bubbleless stationary equilibrium is negatively correlated with the Value-at-Risk regulation parameter $\eta$. Using (35), we have that

$$r^{l*} = \frac{r(\eta+\beta)+\beta\tau}{\beta+\eta(1-\phi)}.$$

Therefore,

$$\frac{\partial r^{l*}}{\partial\eta} = \frac{(1-\beta)-[1-\beta(1-\tau)](1-\phi)}{[\beta+\eta(1-\phi)]^2} < 0.$$

The numerator is negative if and only if $\tau\beta(1-\phi) > \phi(1-\beta)$, which is always satisfied (see Appendix A).

## Appendix F

Appendix F shows that $W^*$ and $L^*$ are decreasing in the lending rate $r^{l*}$. From Section 3, the stationary bubbleless steady state welfare is given by the consumption such that

$$W^* = L^{*\psi} - \left(1+\frac{\tau}{1+(1-\phi)\frac{D^*}{N^*}}\right)L^*.$$

Therefore, the marginal impact of the lending rate on welfare is

$$\frac{dW^*}{dr^{l*}} = \psi\left(L^*\right)^{\psi-1}\frac{dL^*}{dr^{l*}} - \frac{dL^*}{dr^{l*}}\left(1+\frac{\tau}{1+(1-\phi)\frac{D^*}{N^*}}\right).$$

Thus, $\frac{dW}{dr^l} < 0$ if and only if

$$\psi\left(L^*\right)^{\psi-1} > \left(1+\frac{\tau}{1+(1-\phi)\frac{D^*}{N^*}}\right). \tag{51}$$

Since $L^* = \left[(1+r^l)/\psi\right]^{\frac{1}{\psi-1}}$,

$$r^{l*} > \frac{\tau}{1+(1-\phi)\frac{D^*}{N^*}}. \tag{52}$$

From Eqs. (34) and (35), condition (52) becomes

$$r^{l*} = \frac{r(\eta+\beta)+\beta\tau}{\beta+(1-\phi)\eta} > \frac{\tau\beta}{\beta+(1-\phi)\eta}.$$

It is equivalent to

$$r(\eta+\beta) > 0.$$

which is always verified.

## Appendix G

This appendix proves that the bank value is always positive, even in the bubbly equilibrium with a negative bubble.

In the bubbly equilibrium, the bank value is $V^B\left(N^B\right) = q^B N^B + b$. From Eq. (43), the bubble term is equal to $b = \left(q^M - q^B\right)N^B$. Thus, $V^B\left(N^B\right) = q^M N^B$ is always positive if condition (41) holds. Note that we consider that this condition is always satisfied.

## Appendix H

Here, we show that the welfare $W^B$ is decreasing in the lending rate $r^{lB}$. From Section 4, the semi-stationary bubbly steady state welfare is given by the consumption such that

$$W^B = L^{B\psi} - \left(1+\frac{\tau}{1+(1-\phi)\frac{D^B}{N^B}}\right)L^B.$$

From Eq. (39), $D^B/N^B = \eta(\tau-r^l)/\beta\xi$. Therefore, the marginal impact of the lending rate on welfare is

$$\frac{dW^B}{dr^{lB}} = \psi\left(L^B\right)^{\psi-1}\frac{dL^B}{dr^{lB}} - \frac{dL^B}{dr^{lB}}\left(1+\frac{\tau}{1+(1-\phi)\frac{D^B}{N^B}}\right) - L^B\left(\frac{\tau\beta\xi(1-\phi)\eta}{\left[\beta\xi+(1-\phi)\eta\left(\tau-r^{lB}\right)\right]^2}\right).$$

Thus, $\frac{dW}{dr^l} < 0$ if and only if

$$\psi\left(L^B\right)^{\psi-1}\frac{dL^B}{dr^{lB}} - \frac{dL^B}{dr^{lB}}\left(1+\frac{\tau}{1+(1-\phi)\frac{\eta\left(\tau-r^{lB}\right)}{\beta\xi}}\right) - L^B\left(\frac{\tau\beta\xi(1-\phi)\eta}{\left[\beta\xi+(1-\phi)\eta\left(\tau-r^{lB}\right)\right]^2}\right) < 0. \tag{53}$$

The above condition holds if the first two terms are negative as well as the last term. Since the last term $-\tau\beta\xi(1-\phi)\eta/\left[\beta\xi+(1-\phi)\eta\left(\tau-r^{lB}\right)\right]^2$ is always negative, one only needs to show that the sum of the first two terms is also negative. This implies to verify that the following holds

$$r^{lB} > \frac{\tau}{1+(1-\phi)\frac{\eta\left(\tau-r^{lB}\right)}{\beta\xi}}. \tag{54}$$

From Eq. (29), the lending rate is $r^{lB} = \frac{r(\eta+\beta)+\beta}{\eta(1-\phi)}$. Therefore, condition (54) becomes

$$r^{lB} = \frac{r(\eta+\beta)+\beta\xi}{(1-\phi)\eta} > \frac{\tau}{1+(1-\phi)\eta\frac{\left[\tau-\left(\frac{r(\eta+\beta)+\beta\xi}{(1-\phi)\eta}\right)\right]}{\beta\xi}}.$$

It is equivalent to

$$\tau > \frac{r(\eta+\beta)+\beta\xi}{(1-\phi)\eta}.$$

which is equal to condition (41). Since this last condition is assumed to always hold, the first two terms are always negative. Thus, welfare decreases with the lending rate.

## Appendix I

Here, we display the proof of Proposition 10. The spread between the bubbly and the bubbleless lending rate is

$$r^{lB} - r^{l*} = \frac{r\eta+1-\beta(1-\xi)}{\eta(1-\phi)} - \frac{1-\beta(1-\tau)+\eta r}{\beta+\eta(1-\phi)}.$$

Therefore, $r^{lB} - r^{l*} > 0$ if

$$\eta < \frac{1-\beta(1-\xi)}{(\tau-\xi)(1-\phi)-r} = \bar{\eta}.$$

Hence, the bubbly lending rate is higher than the bubbleless lending rate if and only if a negative bubble exists. For a positive bubble, we have $r^{lB} - r^{l*} < 0$.

As a consequence, it can be shown that the welfare is higher in the presence of a positive bubble. In contrast, it is lower with a negative bubble.

**Table 3**
Eigenvalue of the bubbly and bubbleless equilibria.

| Bubbly ($b_t > 0$) | Bubbleless ($b_t = 0$) |
|---|---|
| Values | Values |
| 0 | 0 |
| 0 | 2.236e−55 |
| 0 | 3.012e−36 |
| 0 | 3.452e−36 |
| 1.456e−19 | 4.408e−19 |
| 9.661e−18 | 1.321e−17 |
| 9.161e−17 | 1.472e−17 |
| 0.95 | 0.95 |
| 1.01 | 1.01 |
| 1.038 | 1.915e+39 |
| Inf | Inf |
| Inf | Inf |
| Inf | |

## Appendix J

Table 3 displays eigenvalues associated with the linearised system around the stationary bubbleless and the semi-stationary bubbly equilibrium.

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