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Review

Arabic classical traditions in the history of the exact sciences: The case of Ibn al-Haytham*

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Abstract. This paper examines selected *leitmotifs* from the Arabic classical traditions in the history of the exact sciences while taking into account their adaptive assimilation and expansion of ancient Greek scientific knowledge within the pre-modern Islamicate intellectual *milieu*, and the subsequent transmission and reception of the Greco-Arabic sciences within the European mediaeval and Renaissance circles of scholarship. To give a more concrete focus to this thematic orientation I will address the legacy in the science of optics of the 11th century Arab polymath al-Ḥasan Ibn al-Haytham (known in Latin renditions of his name as Alhazen; died ca. 1041 CE), as encompassed primarily in his *Kitāb al-manāẓir* (*Book of Optics, De aspectibus or Perspectiva*). This opus contained his explications of the nature and comportment of light and direct vision, including studies in catoptrics and dioptrics (respectively, the optical sciences that investigate the mathematical and physical principles of the reflection and refraction of light, along with their associated scientific instruments). This inquiry is situated within the wider epistemic setting of Ibn al-Haytham's scientific experimental method of controlled testing that is underpinned by an isomorphic composition of geometric modelling with physics in studying natural phenomena.

1 Introduction

Ibn al-Haytham's contributions to the theory of visual perception in optics resolved ancient disputes between the exponents of the emission theory of light amongst the mathematicians in the Euclidean Ptolemaic traditions on one side, and those who advocated an Aristotelian intromission thesis in natural philosophy (*qua* ancient physics) on the other side. This gives a framework for grasping the extent of the influence of Ibn al-Haytham's optics and geometry on the unfolding of the *perspectiva* traditions in the European *milieu* within the mediaeval history of the natural sciences and its subsequent adapted integration within the pictorial and architectural spatial arts of the Renaissance, and up to the 17th century.

Prior to focusing on Ibn al-Haytham's scientific legacy *per se*, I will briefly consider his biography within its intercultural *milieu*.

We know that Ibn al-Haytham might have been born in Basra in Iraq around the year 965 CE, and died in Cairo around 1041 CE. Two principal biographical-bibliographical sources on his life and oeuvres come two centuries after his death as embodied in the Ta'rīkh al-Ḥukamā' (Historiography on the Philosophers) that is authored by Ibn al-Qifṭī (d. 1248 CE), and the *Ṭabaqāt al-aṭibbā'* (On the Ranks of Physicians) that is composed by Ibn Abī Uṣaybi'a (d. 1270 CE) [1,2]. Despite the significance of these two mediaeval historiographers, their accounts concerning Ibn al-Haytham's life were not always commensurable with each other, and they rather tended to embellish their narrative reconstruction of his biography. Based on their chronicles, Ibn al-Haytham started his early career as a civil servant in the secretariat of the Būyid governorate in Basra, which operated under the suzerainty of the 'Abbasid Caliphate in Baghdad. Ibn al-Haytham nurtured a penchant for science during his youth that began to conflict with the demands of his secretarial office as his knowledge expanded. He eventually became frustrated with the constraints of his bureaucratic job, and resigned from his administrative position to immerse himself in the pursuit of scientific inquiry. Having excelled in architectural-engineering, and cultivated a reputable record as a technologist, the news of his expertise reached the Fatimid Imam and Caliph al-Ḥākim bi-Amr'illāh in Cairo. It was reported to the Fatimid Caliph that Ibn al-Haytham

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may have figured out a potential solution to control the ebbs and flows of the Nile. Upon hearing this, al-Ḥākim extended an invitation to Ibn al-Haytham to settle in Egypt. Ibn al-Haytham relinquished his service to the Būyid chancellery under the ʻAbbasids, and accepted the patronage of their dynastic rivals in Fatimid Egypt. After his arrival to Cairo, he was appointed as a chief-engineer to the Fatimid court, and headed an expeditionary surveying mission to Upper Egypt (*ṣaʻīd miṣr*) to inspect the topography of potential sites on the banks of the Nile that can possibly act as locales for the construction of a dam. After his surveys, he identified an optimal potential site in a gorge known as the Ganādil Cataract, which is situated in the vicinities of Aswan (the locality of the current modern Nile barrier). Completing his survey of this location, and taking into account the magnificence of what the Ancient Egyptians were able to achieve with their architectural ingenuities, Ibn al-Haytham became convinced that any proposed design of a barrier that can potentially control the Nile was unfeasible in his epoch, and that this is due to the topographic conditions of the riverbanks, the specifications of workmanship, and the capabilities in the building techniques.

Following this explorative field-mission, Ibn al-Haytham returned to Cairo and confessed to his patron that the design was unachievable. Disappointed with his chief-engineer, al-Ḥākim decommissioned Ibn al-Haytham from his engineering offices, and apportioned him a symbolic administrative role to maintain a stipend before pushing him into an early retirement. Becoming concerned about his personal safety, and worried from the impulsiveness of his patron, Ibn al-Haytham feigned madness, and remained confined to his apartment in the vicinity of the Azhar Mosque until the Caliph's death in 1021 CE. After this episode of internment, Ibn al-Haytham resurfaced into the public domain, and his main breakthroughs in science may have been accomplished in that period, especially between 1028 CE and 1038 CE. During this phase, he earned his income from sizeable remunerations he received for copying manuscripts of principal classical scientific texts, including Euclid's *Elements* in geometry and Ptolemy's *Almagest* in astronomy. For further accounts concerning Ibn al-Haytham's biography and oeuvres, refer to [3,4] and [5].

Besides his research in optics, Ibn al-Haytham's corpus encompassed a vast constellation of epistles and treatises on the various branches of mathematics, in addition to his critical analytic commentaries on the works of his predecessors in the Greco-Arabic legacies in the exact sciences. For instance, he reviewed Apollonius of Perga's *Conica* (conics opus) in the *Maqāla fī tamām kitāb al-makhrūṭāt* (On the Completeness of the Conics). In his epistle on *Shakl Banū Mūsā* (The Proposition of Banū Mūsā), Ibn al-Haytham also reassessed the mathematics of the erudite sons of Mūsā ibn Shākir who flourished in Baghdad in the 9th century. His geometric *Lemmas* (*al-Muqaddimāt*), became known in the 17th century European mathematical circles as the formulation of "Alhazen's Problem", and these constituted a set of propositions that aimed at offering a solution to the question: "How, from any two points opposite a reflecting surface [plane, spherical or cylindrical], can we find a point on that surface at which the light from one of these two points reflects unto the other?"

Like his contemporaries amongst specialized Arabic mathematicians, Ibn al-Haytham attempted to solve the problem of parallelism in Euclid's fifth postulate, and composed an exegetical commentary on the premises of Euclid's *Elements* (*Sharḥ muṣādarāt kitāb Uqlīdis*), along with the investigation of "The Quadrature of Crescent Figures or Lunes" (*al-Ashkāl al-hilāliyya*). Moreover, in response to the mathematical and epistemic needs of his epoch, and in investigating geometric transformations (*al-naql*) in the Arabic Archimedean-Apollonian tradition, especially in research on conic sections, Ibn al-Haytham studied the kinematic application of motion to geometric definitions, propositions, proofs, and demonstrations. He did this in his mathematical treatise *Fī al-Maʻlūmāt* (On Knowable [Entities]) and in his tract *Fī al-Taḥlīl wa'l-tarkīb* (On mathematical Analysis and Synthesis), venturing therein into novel territories beyond Greek mathematics. Moreover, Ibn al-Haytham systematized the development of a rudimentary form of analytic geometry, by deploying algebra in geometric constructions, and laying down the early systematization of infinitesimal mathematics (*al-Riyāḍiyyāt al-taḥlīliyya*), which is distinct from the early modern infinitesimal calculus, in terms of its own internal coherence; even though it contributed to founding a prehistory of calculus.

Ibn al-Haytham aimed at reinventing the methods of proof, and specifically in response to the Archimedean problem of calculating areas and volumes, including the exhaustion technique, and the reliance on numerical computation instead of merely following a theory of proportion. He also studied number theory and built in this on the works of the 9th century polymath Thābit ibn Qurra (d. 901 CE), and on the findings of the latter's grandson, Ibrāhim ibn Sinān (d. 946 CE), by investigating amicable and perfect numbers, ratios and proportions.

Besides his research in pure mathematics, Ibn al-Haytham was also an astronomer. In his *Shukūk ʻalā Baṭlāmiyūs* (*Dubitationes in Ptolemaeum*; Doubts Concerning Ptolemy), he critiqued key propositions in Ptolemy's *Almagest, Planetary Hypotheses, Astronomical Optics*, and judged the use of the equant as being unsatisfactory in accounting for uniform circular motion, while disapproving the correlation of actual physical motion with imaginary mathematical points in astronomy. He also offered a *Model for the Motions of the Seven Planets* in a geometrized planetary geocentric system that had epicycles without equant. The unfolding of such critical inquiry was beneficial for later astronomers, such as the Persian polymath Naṣīr al-Dīn al-Ṭūsī (d. 1274 CE), who proposed a geometric reciprocation mechanism or rolling device (the "Tusi-couple")[1] in his *Taḥrīr al-Majisṭī* (Editing the *Almagest*) as a substitute for Ptolemy's

[1] The rolling device consists of having a small circle rotating inside a large circle that has a diameter that is twice the size of the diameter of the small circle, whereby a point on the circumference of the smaller circle moves on the vertical diameter of the larger circle.

equant, which also carried geometrical resemblances to what figured in the 16th century in Copernicus's heliocentric model [6,7]. Ibn al-Haytham also investigated optical astronomy, like in his *R'uyat al-kawākib* (On Seeing the Stars) and in his *Aḍwā'al-kawākib* (On the Lights of the Stars), and by way of studying the characteristics of the eclipse (*Maqāla fī ṣūrat al-kusūf*), and the nature of the twilight and moonlight (*Risāla fī ḍaw' al-qamar*).

Ibn al-Haytham's most impactful accomplishments were *grosso modo* embodied in his research in optics as principally set in his *Kitāb al-manāẓir* (*Book of Optics; De aspectibus or Perspectiva*) [8–10], which he composed in Cairo in the second quarter of the 11th century (most probably in the period of 1028 CE to 1038 CE). This compendium was divided into seven books that were grouped under three principal divisions: Books I–III dealt with rectilinear direct vision, including the analysis of the errors in visual perception, Books IV–VI studied the principles of catoptrics and reflection of light, and Book VII focused on dioptrics and the refraction of light.

Ibn al-Haytham's *Optics* was translated into Latin in the 12th century, possibly starting with renderings within the circle of Gerard of Cremona in Toledo. Several Latin renditions of this text were established under the titles *De aspectibus* (visible aspects) or *Perspectiva* (perspective) perhaps in reflection of two channels of transmission into Europe, one through Sicily, then to Salerno, Montpelier, Paris, and Basel, and the second via Andalusia, then to the Severn Valley in England to reach the Franciscan college at Oxford.

The Latin translation (*Perspectiva*) acted as the basis for the Italian version of the text in the 14th century that carried the title: *Prospettiva* (perspective). An edited version of the Latin translation of Ibn al-Haytham's *Kitāb al-manāẓir* was eventually printed in Basel in 1572, under the editorship of Friedrich Risner, and was entitled: *Opticae Thesaurus Alhazeni* [11,12]. Many historians continue to speculate to date about the circumstances and extent in which this Risner edition may have been consulted by scientists such as Kepler, Descartes, Huygens, Sevelius, or possibly even by Newton.

The transmission of the Arabic manuscripts of Ibn al-Haytham's *Optics* to Europe, and their Latin translations, all had an influence on the development and maturation of the perspectivist traditions in mediaeval Franciscan optical workshops, such as in Oxford, and subsequently impacting the perspectival praxis in Renaissance art and architecture. Ibn al-Haytham's *Optics* influenced the studies of 13th century opticians, such as Roger Bacon and Witelo, and had an impact on the investigations of Theodoric of Freiburg in the 14th century, as well as being studied in Padua, with an impress that they left on Biagio Pelacani da Parma, who in his turn inspired early Renaissance theorists of perspective [13–15].

Ibn al-Haytham's optics was ultimately assimilated within the applied forms of pictorial representation in Renaissance art and spatial design in architecture. The geometrical structures that described the properties of the *perspectiva naturalis* (natural perspective), which were studied in optics, grounded the invention of the *perspectiva artificialis* (artificial perspective) as a projective geometric method of constructing pictorial representations of visible spatial depths. Such developments were embodied in the works of notable Renaissance architects like Leon Battista Alberti, as set in his *De Pictura* (On Painting), and Lorenzo Ghiberti in his *Commentario terzo* (Third Commentary), which rested on the theories of Ibn al-Haytham and Witelo by appealing to the Italian rendition (*Prospettiva*) of the Latin text (*De aspectibus/Perspectiva*) of *Kitāb al-manāẓir* (Book of Optics) and of Witelo's references to it.

While Ibn al-Haytham's *Optics* was widely disseminated and integrated within European scholarship, up to the times of the projective perspectivism of Girard Desargues in the 17th century, its circulation in the pre-modern Islamicate context was less intense. The principal adaptation of Ibn al-Haytham's theories of vision and light in the context of the history of scientific ideas in the classical Islamic cultures took its most influential turn with the *Tanqīḥ al-manāẓir* (Revision of the Optics [16]) of the 14th century Persian optician Kamāl al-Dīn al-Fārisī (d. 1320 CE) who completed his studies under the tutorship of the student of the polymath Naṣīr al-Dīn al-Ṭūsī (d. 1274 CE), namely, the astronomer Quṭb al-Dīn al-Shirāzī (d. 1311 CE) at the Maragha observatory.

Kamāl al-Dīn al-Fārisī expanded Ibn al-Haytham's findings in the *Optics* in terms of advancing a novel theory of the colouration of the rainbow (*qaws quzaḥ*) based on analysing the geometric structure of double refractions and reflections of light rays through rain droplets. This natural phenomenon was simulated by Kamāl al-Dīn al-Fārisī in an experimental model that consisted of a large transparent spherical glass vessel that was filled with water, to represent a single rain droplet, which was then subjected to controlled light beams passing through selected apertures, while being situated within a *camera obscura* as an experimental space. Kamāl al-Dīn al-Fārisī analysed the comportment of light and its trajectory as it passed through this vessel, while registering the colour spectrum that emerged from it. He was able to offer a new understanding of the ontological nature of colour that rectified Ibn al-Haytham's existential separation of illumination from colouration; since Ibn al-Haytham took colour to be possibly distinct from light even though it followed the same physical laws of propagation and always accompanied secondary lights that are emitted from the lit and visible surfaces of opaque objects. Kamāl al-Dīn al-Fārisī removed Ibn al-Haytham's doubts by showing that colour is a form of light that appears due to specific refraction and reflection conditions. In this he came close to explaining what was later systematically explicated via Newton's theory of the decomposition and composition of light. However, al-Fārisī was not the only one to offer this explanation in an experimental context, given that his contemporary, Theodoric of Freiburg, also arrived at similar conclusions while being also based on studying Ibn al-Haytham's optics [17]. These two 14th century scientists worked on separate manuscripts of Ibn al-Haytham's *Book*

of Optics, respectively in Arabic and Latin, without being in touch with one another. The precision in their proofs can be partly credited to the accuracy of Ibn al-Haytham's original description of his measurable experimental procedures and observational data, which facilitated the reforming of his own theory of colour by these two opticians. The next chapter in the unfolding of this line of research in optics within the Islamicate setting is perhaps embodied in the 16th century commentary on Kamāl al-Dīn al-Fārisī's oeuvre by Taqī al-Dīn Muḥammad ibn Ma'rūf (d. 1585 CE), the Syrian astronomer at the Ottoman court [18,19].

Ibn al-Haytham's optics resolved the ancient disputes over the nature of vision and the comportment of light, and aimed at completing the work initiated by Ptolemy through novel methods of experimental inquiry in the context of mathematizing the science of physics. Ibn al-Haytham reworked the explications offered by the Aristotelian physicists (*al-Ṭabī'iyyūn*), who advocated intromission theories of vision, and according to which the form of a visible object is introduced into the eye as abstracted from its matter when the transparent medium, such as the air between the eyes of the observer and the objects of vision, is actualized by physical light. Ibn al-Haytham also adapted the geometric models of mathematicians (*aṣḥāb al-ta'ālīm*) of Euclidean and Ptolemaic orientations, who upheld emission theories of light, and according to which visible objects are seen by way of light rays that consist of non-consuming gentle irradiating fires, which are emitted from each of the eyes in the shape of a cone or a pyramid of vision. Critically reassessing these incommensurable theories, Ibn al-Haytham rejected the claim of the mathematicians that vision occurs by way of the emission of a light ray from the eye, while revealing the geometric principles that underpin the intromission theory, which were obscured by the ambiguities of natural philosophers. Ibn al-Haytham demonstrated that sight resulted from the introduction into the eyes of light rays that are emitted from the lit and visible surfaces of the objects of vision, while taking into account that these light rays propagate rectilinearly across a given single homogeneous transparent medium (such as air), and that they enter the eye in the shape of a virtual geometric cone of vision (*makhrūṭ al-shu'ā'*), with its vertex at the centre of the eye and its base intersecting with the lit and visible surfaces of the seen objects [20].

Ibn al-Haytham distinguished the conditions of sight from those of light, and showed that the rays of light irradiate spherically in every direction and from every point on the lit visible surfaces of the objects of vision. He also demonstrated that they travel across a given homogeneous transparent medium in straight lines, if unobstructed by opaque bodies, or if they did not pass through distinct transparent media that differed in their indices of refraction.

Ibn al-Haytham supported his explication of vision through anatomical examinations of the eye [21–23]. He ultimately explained the conditions of binocular vision and the errors in direct sight, and grounded his theory of visual perception, which combined geometry with physics, on physiology, neurology, and psychology. He drew a careful distinction between the immediate mode of visual perception by glancing and the contemplative aspect of vision, which is accompanied by prior knowledge. He also examined the visual perception of particular visible properties (*al-ma'ānī al-mubṣara; intentiones visibiles*), of which he enumerated twenty-two aspects, including phenomena such as distance, position, solidity, shape, size, opacity, beauty, and so forth. Moreover, he distinguished pure sensation (*mujarrad al-ḥiss*), which only perceives light *qua* light and colour *qua* colour, from the psychological workings of recognition (*ma'rifa*), judging discernment (*tamyīz*), and comparative inferential measure (*qiyās*), as they all get aided by imagination (*takhayyul*), memory (*dhikr*), and at times by prior knowledge. He argued that sensation in connection with vision was ultimately effected by what he referred to as "the last sentient" (*al-ḥāss al-akhīr; sentiens ultimum*), which according to his analysis was located in the anterior part of the brain (*muqaddam al-dimāgh*). Vision was therefore a physiological and neurological nested cluster of physical phenomena that pertained to the effect of light on the anatomy of healthy eyes, and through them in generating sensations that are transmitted via the functioning optical nerves, common nerve, and last sentient in the brain. Besides the physical reception of light in the eyes, the proper functioning of the optical nerves and the anterior part of the brain, vision also depended on psychological healthiness in the mental faculties since there is no vision without the cognitive aspects of recognition, discerning judgment, comparative measure, imagining, and recollecting. Vision was not therefore merely a phenomenon that resulted from the ocular functioning of the eyes as photo-receptors.

Ibn al-Haytham's explanations also touched upon the phenomena of binary vision and parallax by way of showing that two images of a single form of a given visible object are received in each of the crystalline of the eyes, passing then via the vitreous to the hollow nerves as sensations, and ultimately being unified in the cavity of the common nerve, by virtue of which they reach the last sentient as an ordered single neural sensation that results in the psychological recognition of a unified form of a single visible object.

Ibn al-Haytham's optics served his scientific empirical methodology that relied on reporting observational data in controlled repeated tests, besides being guided by geometric modelling. His experimental method, known in Arabic by the appellation: *al-i'tibār* (literally to derive a lesson by way of experiential evidence) [24–26], necessitated the design and use of scientific instruments, including installations such as *al-qumra al-muẓlima* (the *camera obscura*; darkroom [11]). Ibn al-Haytham aimed at setting criteria to ground the reliance on the veridical data that can be derived from visual observation in an experimental setting. For example, he stressed that the viewed object must be bright enough and positioned at a moderate distance from the perceiver's eyes in a clear and homogeneous transparent medium (*shafīf*). Such visible object must be in a plane shared with the eyes, and its body should have a proper

volume, not too small and not too large when viewed from a given distance. The object should also have some opacity or translucency, given that a highly transparent body is virtually invisible. In addition, the observer should have sufficient time to see the object with two healthy eyes that are able to perform effective concentrations in scrutiny and contemplation, and to be repeated within experiments to gradually isolate the variables. He also held that when sight perceives individuals of the same species repeatedly and continually, a universal form (*ṣūra kulliya; forma universalis*) of that species takes shape in the imagination and gets recollected by recognition, while consequently assisting in grasping the quiddity (*māhiyya*, essence) of the corresponding visible object and its inspected seen properties; hence resulting in prior knowledge that aids imagination and memory. Ibn al-Haytham also presented detailed phenomenological observations in experimental contexts regarding the role of embodied experiencing and cognition in vision, whereby the observer's proper own body contributes to estimating the visible distances and sizes of visible objects in the perceptual field when sharing a common terrain with the things being perceived in space.

Moreover, Ibn al-Haytham's theory of light complemented his theory of visual perception in geometric optics. He held that light consists of fiery physical minute particles moving at an extremely high speed that is undetected by the eye, and that they propagate rectilinearly via homogeneous transparent media, reflect off polished surfaces in mechanical ways, and get refracted by way of a reduction in their velocity as they pass between transparent media of varying densities. He explored catoptrics and the reflection of light by using polished surfaces, including parabolic, cylindrical and spherical mirrors, and studied the dioptrical effects of similar solids when made out of glass and at times filled also with water to study the trajectories of the refraction of light. His studies in dioptrics rested on his analysis of the findings of Abū al-'Alā' Ibn Sahl, who flourished in 10th century Baghdad, and who is credited as being the first to discover a principle that is akin to the 17th century "Snell's law of refraction" (namely a principle that determines the refractive index of a transparent medium in connection with a given geometric shape, which can act as a basis for designing lenses [27,28]). Ibn al-Haytham's optical theories were also associated with meteorological explorations of the refraction and reflection of light in accounting for phenomena such as the halo and the rainbow (*Maqāla fīal-hāla wa qaws quzaḥ*), and in explicating the "Moon illusion" (namely when the Moon appears larger at the horizon than at the culmination or zenith).

Ibn al-Haytham's research in optics was closely tied to his reform in geometry. The 10th century studies in Arabic mathematics were marked by an unprecedented level of research on geometrical transformations (similitude, translation, homothety, affinity), which were conducted on various forms of investigating conics and their anaclastic properties as models of lenses, in addition to projective studies in the field of spherics and stereography. This involved the introduction of movement in geometry, which opened up novel pathways that were underexplored by the ancient Greeks. In response to these developments, Ibn al-Haytham aimed at establishing an abstract domain for geometry, in view of reorganising its notions to accommodate the introduction of motion in geometrical definitions, demonstrations, and constructions. He also focused on grounding his own investigations in mathematical analysis and synthesis (*al-taḥlīl wa'l-tarkīb*) and on knowable entities (*al-ma'lūmāt*) in response to the need to rethink the foundations of geometry, and to reinforce his methodological procedures that combined mathematics with physics in serving his endeavour to geometrise the notions of natural philosophy. In response to these directives, he aimed at critically reassessing the dominant conception of place (*qua* space) in his age, which was burdened by philosophical disputes over the Aristotelian definition of *topos* (place). Ibn al-Haytham eventually advanced a thorough geometrical critique of the conception of *topos* as presented in *Book Delta* (IV) of Aristotle's *Physics* [29].

Using geometrical demonstrations, Ibn al-Haytham rejected the Aristotelian definition of place as "a two-dimensional boundary of a containing body that is at rest and is in contact with what it contains". Ibn al-Haytham demonstrated that place (*al-makān*) is an imagined qua postulated three-dimensional void (*khalā' mutakhayyal*) between the inner surfaces of the containing body. Among the various geometrical demonstrations and proofs he presented, he showed that the sphere is the largest in comparison with other isepiphanic geometric solids, namely of entities that have equal surface-area (and by also showing this in terms of examining isoperimetric figures, namely those that had equal perimeters). So, based on the Aristotelian definition of *topos*, if a sphere has a surface-area equal to that of a cylinder, then these two entities occupy places that are of equal magnitudes; however, following Ibn al-Haytham's geometric demonstration, the sphere will be (volumetrically) larger than the cylinder if they are isepiphanic (namely as having equal surface areas), consequently, and based on Aristotle's conception of *topos*, two solids that are unequal in size would occupy places of equal magnitudes, which is geometrically untenable. Ibn al-Haytham also demonstrated that when a given parallelepiped (cuboid; *mutawāzī al-aḍlā'*) is divided, it increases in surface-area while staying the same in volumetric magnitude. Based on Aristotle's definition of *topos*, this partitioned parallelepiped will occupy a larger place without changing in volume; hence two solids of a similar size occupy places that differ in magnitude, which is absurd from a geometric standpoint. Moreover, if a given parallelepiped is carved, its surface-area increases while its volume decreases. Following Aristotle's conception of *topos*, this carved parallelepiped occupies larger places as it diminishes in size, which is inadmissible from the standpoint of a geometry of measure [30–32].

Ibn al-Haytham posited place as being analogous to a room or an extended three-dimensional metric space, in a manner that echoed the corollaries of John Philoponus (from the 6th century), albeit that was derived through a pure mathematical *bijection* function (as a mode of one-to-one correspondence between sets) rather than being determined via natural philosophy, or simply by intuitive common sense, or as a mere procedure of abstraction.

Ibn al-Haytham's geometrization of place and its conception as *space* carried resonances with later developments in the 17th century conceptions of *extensio* by Descartes, Leibniz's *analysis situs*, and Girard Desargues' spatiality in perspectival projective geometry [33,34].

Ibn al-Haytham's geometrical conception of *al-makān* (place) was also commensurable with his experimental affirmation of the visibility of depth (*al-'umq*) and spatial distance (*al-bu'd*) as demonstrated in his *Book of Optics*, with observations that are akin to what much later is disclosed with greater precision through the phenomenon of the Ames Room and its optical illusion [35]. The affirmation of the visibility of spatial depth carried tacit phenomenological bearings in surpassing the epistemic limitations of the 8th century immaterialism of the philosopher George Berkeley who negated that space is visible [36]. This question, which was central to the visual depiction of spatial depth, and the rendering of its pictorial visibility in Renaissance perspectives, was revisited in response to Berkeley and Immanuel Kant by the French phenomenologist Maurice Merleau-Ponty in the 20th century, and specifically in the context of rethinking the problem of spatial depth (*la profondeur*) and embodied subjectivity in the *Phénoménologie de la Perception* (Phenomenology of Perception) [37–39].

The recognition of Ibn al-Haytham's legacy went beyond optics, perspectivism, and geometry, to reach other sciences such as the study of the physics of the Moon, as illustrated on the frontispiece of the *Selenographia* of Johannes Sevelius in 1647 [40], wherein a figure representing Ibn al-Haytham (Alhazen) stands on the pedestal of *ratione* (reason) to the left, and a figure representing Galileo stands on the pedestal of *sensu* (sense perception) to the right. This image was also displayed in recent times in 2011 at the "Arabick Roots" exhibition of The Royal Society in London in recognition of the Arabic roots of science, and in paying homage to Ibn al-Haytham [41].

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