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Production lot sizing with quality screening and rework

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ABSTRACT

Most production systems produce items which are of imperfect quality. Handling of the defective items varies by industry and product types. For example, defective items may be sold at discount in the apparel industry, or reworked in the automobile industry where the final product is very expensive. For simplicity, a common assumption in the literature is that items are lumped into two groups, non-defective and to-be-reworked products, ignoring the inspection time needed to identify the repairable items. Our paper explicitly integrates the inspection time into the economic production model with rework, and demonstrates the significant effect that the inspection time has on the results. We consider a manufacturing process with random supply and a screening process conducted during and at the end of production. We analyze two scenarios for dealing with the defective items produced: selling at a discount, and reworking. For each scenario, the demand during production is met using non-defective items only. The expected profit functions are developed using the renewal reward theory, and closed form expressions for the optimal production lot size are derived. Numerical analysis is performed to study the sensitivity of the expected profit and optimal lot size to various system parameters.

1. Introduction

We consider a machine producing a single item, with the possibility of producing a random proportion of defective items. To identify the defective items, screening is conducted at the end of the production period. Once identified, the defective items are reworked at a constant rate before they are returned to the inventory. A common assumption in the inventory literature with defectives is that the rework of a defective item is followed immediately after it is identified (Hayek and Salameh [1], Jamal et al. [2], Sarker et al. [3]). This assumption of continuous screening during production complicates the analysis and is not practical for most production systems, especially when the fraction of defective items is low and the production rate is high, which makes continuous screening during production very expensive. To simplify the analysis and the computation of the average inventory, it was assumed in the literature that the products are lumped into two groups, good products and to be reworked products. Our paper, on the other hand, avoids this assumption by integrating the screening process into the production lot sizing model with rework. During production, demand is met from non-defective items only. This implies that the system screens the items as they are being sold, i.e. the screening rate is the same as the demand rate before production ends. After production ends, a screening process is initiated for all the remaining products. In practice, decisions of when and where to place the screening stations are of special importance. The issues of comparing the screening strategies (after-production versus during production

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screening), and locating the screening stations, (whether the screening should be carried out after each production stage in a serial production line or at the end of the last stage), have been addressed in numerous research studies (Gurnani et al. [4]; Goyal et al. [5]; Tang [6]; Giri and Dohi [7]). Similarly to this paper, several research works in the literature have assumed screening at the end of production, for example, Ma, Gong, and Lin [8]. In this work, we analyze two models: In the first model, defective items identified at the end of the screening period are carried in inventory and sold at a discount as a single batch at the end of the production cycle. In the second model, defective items identified are reworked at a constant rate. We assume that no shortages are allowed.

For the model with rework, we assume that defective items are repairable, and no items are scrapped. This is true especially for expensive products which are not usually scrapped but reworked. Sarker et al. [3] discuss examples of such products, such as the metal book-shelves and defective filing cabinets. Another example is the wafer probe operation of wafer fabrication in the semiconductor manufacturing, where many types of deformations of the perfect integrated circuit can take place, as a result of the process disturbances (Lee [9]). Also, in automobile industries, the defective alignment of steering wheels is mostly repaired. Other examples include the air conditioning units, and ceiling fans components (Sarker et al. [3]). Scrapping such products would be very expensive, so companies collect defective items and repair them at a cost.

The models presented in this paper account for onsite and manual inspection as newly produced items are being consumed before the screening process begins. Obviously, applications for such models include systems where strict quality measures are enforced. Real world systems with strict quality control measures include the food industry. For example, a food safety law enforced in the United Kingdom (U.K.) resulted in retailers no longer being protected from liability by warranties or guarantees from suppliers, but they are required to make sure that the food they sell is of good quality (Bredahl et al. [10]). In such a case, the retailer inspects the newly received items at the same rate as they are being shelved for customer consumption before committing to a thorough inspection process of the entire production batch. Non-conforming food items cannot be reworked and have to be scrapped as represented by Model 1 or can be returned to the supplier which serves as a motivation for Model 2. On construction sites where the quality of material as well as on time delivery are critical factors, a project manager inspects the material to be consumed manually before accepting the order to identify damaged material (reports damaged items using check sheets) (Kerzner [11]). In such a case, material which is damaged is identified by the project manager at the same rate at which it is consumed where the damaged material can be sent back to the supplier production cycle for rework (Model 2).

Our models offer three practical aspects: (i) Screening starts right when production stops, due to a fast production process that makes screening during production difficult in practice, (ii) demand during production is met from non-defective items only, and (iii) once identified, defective items are carried in inventory and salvaged at the end of the production cycle, or reworked at a constant rate.

The paper is organized as follows: The literature is surveyed in Section 2. In Section 3, we present the mathematical formulation. First, the model with defective items sold at discount is presented in Section 3.1, and closed-form expression for the optimal lot size is obtained. Then the model with rework is analyzed in Section 3.2. Numerical and sensitivity analysis are conducted in Section 4 to investigate the effects of various model parameters on the expected profit and optimal lot size. A conclusion is provided in Section 5.

2. Literature review

Inventory models with imperfect quality items have received significant attention in the literature. For a survey on these models, we refer the reader to Yano and Lee [12]. One of the earliest work is Porteus [13] who incorporates the effect of defective items into the economic order quantity (EOQ) model, and analyzes the relationship between quality and lot size. Zhang and Gerchak [14] study a joint lot sizing and inspection policy for an EOQ model with defectives. Goyal and Cardenas-Barron [15] develop a practical approach for an economic production quantity (EPQ) model with defective items. Paknejad et al. [16] assume the number of defective items to be a random variable, and further assume stochastic demand and constant lead time. They show the optimality of a modified (s, Q) policy and obtain results for the case of exponential and uniform demand distributions. Salameh and Jaber [17] consider the realistic case where defective items are detected by a screening process and sold at discount at the end of the cycle. Quality control is obviously critical to any inventory system and has significant managerial implications which include customer satisfaction and demand among other implications. As a result, the impact of incorporating screening and quality control on the behavior of inventory systems is gaining interest in the recent literature. For this reason, the model presented in Salameh and Jaber [17] has received attention in the literature due to the wide applicability of screening and quality control in inventory systems. The authors in Khan et al. [18] extend the model of Salameh and Jaber [17] to include the case where defective items are either sold or replaced. The authors consider a model similar to Salameh and Jaber [17] where items salvaged as a single batch at the end of screening. The model accounts for Type 1 and Type 2 errors due to misclassification of items at the screening stage. Jaber et al. [19] extends Salameh and Jaber [17] by assuming that a shipment is coming from a distant supplier, so it is not possible to replace the imperfect items by placing a new order to the same supplier. Thus, they consider two models: The first assumes that imperfect items are sent to a repair shop and the second model assumes that imperfect items are replaced by good ones from a local supplier at a higher cost. Eroglu and Ozdemir [20] extend the model of Salameh and Jaber [17] to allow for shortage backordering. They assume the defective rate to be a random variable and classify defective items as scraps and imperfect. Liu and Cetinkaya [21] find the optimal lot sizes of a manufacturer and a buyer in an integrated supplier-buyer setting, with imperfect manufacturing process and quantity discounts. In their model, the supplier produces more than the buyer's order quantity to reduce the supplier's setup cost. They develop a model to determine the optimal lot sizes, retail price, mark-up rate, and the number of shipments per production run from the supplier to the buyer. Sana [22] develops a model to determine the optimal product reliability and production rate with the objective being to maximize the expected total profit. Sana [23] incorporates preventive maintenance into imperfect production systems. The recent work of Wee et al. [24] considers an economic production model with imperfect quality items, shortage and a screening constraint. Using the production time and time to eliminate backorders as decision variables, they develop an inventory policy for imperfect quality items.

None of the existing work considers the realistic case where screening occurs onsite which results in defective items being inspected at the same rate as demand prior to completing the production cycle. The manual inspection of items is addressed in Konstantaras et al. [25] where an EOQ model takes into account the effect of learning on the inspection process. Learning implies that the inspection process is not perfect but improves for subsequent ordering cycles which motivates considering finite and infinite time horizons. Our work assumes that the inspection process is perfect throughout, and the proportion of defective items is not sensitive to the learning process. The EOQ model in Konstantaras et al. [25] assumes that the inspection rate is faster than the demand in rate. Our EPQ model considers the case where manual inspection occurs as items are being consumed during the production cycle which results in a manual inspection rate being equal to the demand rate. A more cost and time efficient inspection process is initiated only after the production cycle ends.

Manual inspection of items is also considered in Glock and Jaber [26] where the model accounts for learning as well as forgetting. The analysis also deals with how learning can shift the location of the bottleneck in a production system. Several extensions in the recent literature build on the Salameh and Jaber [17] model in an attempt to generalize the model by accounting for more realistic factors. For instance the model in Jaber et al. [27] considers the demand rate as a heat flow and uses the laws of thermodynamics to address extensions of the EOQ model in a novel approach. The work in Konstantaras et al. [25] builds on the EOQ model with imperfect items by accounting for learning in the inspection process. The model in Nasr et al. [28] accounts for correlation among the imperfect items. Deterioration within an EOQ model with imperfect items is accounted for in Moussawi-Haidar et al. [29]. Also recently, the authors in Chang et al. [30] develop an EOQ model with imperfect quality which also accounts for the inspection error as well as considering the option where a supplier can provide a permissible delay in payments. An EOQ with imperfect items which considers investing in the speed of the quality control check is presented in Hauck and Voros [31]. The authors in Zhou et al. [32] consider the case where the manufacturer has the option for a make-or-buy decision under a one-time discount as offered by the original equipment manufacturer. Extensions of the EOQ model with imperfect items also include investigating the relationship between the buyer and supplier in relation to conducting the inspection under different conditions which depends on the relationship between the buyer's selling price, buyer's purchasing price, and customer demand (Rezaei and Salimi [33]).

Many of the studies on defective items consider the situation that defective items can be reworked at a cost. Among the earliest work that incorporates reworking defective items is that of Rosenblatt and Lee [34] who propose a production system producing only perfect quality items up to a point when the system becomes out of control and starts producing defective items until the end of the production cycle. They assume exponential distribution for the time that elapses until the system is out of control, and do not allow for backordering. Wang [35] considers a model with preventive maintenance and incorporates the possibilities of minimal repair and rework. Wee et al. [36] assume non-synchronized screening and rework. Motivated by the wafer probe operation in the semiconductor industry, Lee [9] models a production process with defective items, process corrections and rework, with the objective being to reduce the total processing time. Chiu and Chiu [37] consider the EPQ model with the rework of the repairable portion of the defective items, while scrapping the remaining portion. They assume that reworkable items are immediately identified once production stops, which is unrealistic, as a screening process needs to be conducted to identify those items. Also, they fail to model that the demand during production is met from perfect items only. Jamal et al. [2] find the optimal batch size in a single-stage system, where rework is done according to two different policies to minimize the total system cost. The first policy deals with reworking the defective items in the same cycle. The second policy deals with the rework being done after a fixed number of cycles. Biswas and Sarker [38] consider a single-stage manufacturing system with in-cycle rework policy and scrap with 100% inspection. They assume that defective items are reworked within the same cycle and scrap detection is to comply with the objectives of a lean production system. Three types of scrap detection scenarios are studied: detection before, during, and after rework. To avoid shortages, finished good stocks ordered from various suppliers are kept to satisfy demand. Sarker et al. [3] gives several examples of situations in which defective items are not scrapped, but reworked, and model the problem of finding the optimal batch quantity for a manufacturing system in a multi-stage system with no shortages, when rework occurs within the same cycle and after a given number of cycles. They formulate the problem as a non-linear mathematical program for which they propose a closed-form solution. Chiu et al. [39] determine the optimal run time for an EPQ model with scrap, rework, and stochastic machine breakdowns. They assume that a portion of the defective items is scrap, while the other portion is repairable. They formulate the production-inventory cost expressions with the scenarios of breakdown and no breakdown. Integrating the two functions, they find the optimal cycle length. Jaber and Guiffrida [40] apply the learning curve to the rework time per unit by modifying the Wright's learning curve for processes with repairable defectives. Flapper and Teunter [41] consider the rework of production defects that deteriorate while waiting to be reworked, which is common in the food industry. Inderfurth et al. [42] consider the problem of planning the production of new and recovering defective items, with items being produced in batches. They assume that the processing of each batch includes two stages: In the first stage, production occurs and good items are used to satisfy demand. In the second stage, defective items are reworked, and are subject to deterioration while waiting to be reworked, which increases the time and cost for performing the rework processes. Inderfurth et al. [43] analyze a production system with rework and product deterioration, and the effect of the timing of operations. They develop optimization algorithms covering different planning situations, and obtain closed-form solutions. Ojha et al. [44] consider the rework problem for a manufacturer receiving raw material from the supplier. They analyze three scenarios: (i) a single lot of raw material for multiple lots of finished products, and delivery in multiple installments, (ii) a single lot of raw material for multiple lots of finished products and delivery in a single installment, and (iii) lot-for-lot delivery of finished products in a single installment. Giannakis et al. [45] use queuing theory to study the flows of rework loops in serial manufacturing systems with inspection stations. Glock and Jaber [46] study a serial production line with defective items, learning, forgetting, and rework. Sarkar et al. [47] model an EPQ problem with rework process in a single-stage manufacturing system with planned backordering. They assume uniform, triangular, and beta distributions for the defective rate. Chen and Tsao [48] consider a manufacturer-retailer supply chain with learning and rework of defectives, using the Nash equilibrium theory.

Most of the economic order and production quantity models with imperfect production and rework processes studied in the literature, ignore the impact of screening. Thus, our model presents a more realistic approach for modeling the lot sizing problem with random yield and rework. We demonstrate the significant effect that screening has on the optimal lot size, both analytically and numerically. Our results indicate that the closed-form expression we derive for the optimal lot size depends on the screening rate. Moreover, we numerically analyze the effect of the screening rate on the optimal lot size. To the best of our knowledge, the only work that considers the effect of screening is that of Ullah and Kang [49]; however, they consider the lot sizing problem of the work in process inventory rather than the inventory of finished goods.

Our work is the closest to Hayek and Salameh [1] and Rosenblatt and Lee [34] who analyze a finite production model with the defective items identified during production, and reworked at the end of the production period. Rosenblatt and Lee [34] acknowledge that, in some cases, defective items might not be reworked or replaced, so they may be sold in the secondary market. Rosenblatt and Lee [34] find that ordering smaller lot sizes results in a lower total cost. Similar conclusions have been reported in the literature by Hayek and Salameh [1]. Contrary to the literature, we demonstrate in this paper that, in the presence of defective items that are carried in inventory and salvaged at the end of the production cycle, the optimal inventory policy is to order in larger lot sizes.

All the above-cited literature ignores the screening time necessary to identify the defective items. They assume that the screening happens continuously during production, and the rework starts immediately at the end of the production period (e.g., Jamal et al. [2], Sarker et al. [3], Hayek and Salameh [1]). This is non-practical for most production systems, as the production rate tends to be high, so continuous screening becomes extremely expensive. Our paper offers a practical approach for dealing with this problem, by explicitly modeling the screening time along with the rework of defectives, thus, offering a major contribution to the stream of related work.

3. Mathematical model

Consider a production quantity model, in which production occurs at a rate α, and demand occurs at rate β units per unit time, $\alpha > \beta$. The inventory builds up at the rate $\alpha - \beta$. During production, a random proportion P of defective items is produced, with a known probability density function $f(P)$. Demand during production is met from non-defective items only, which requires the units demanded to be screened before they are sold to customers. During this process, if an item is found to be defective, it is replaced with a non-defective item. The number of defective items accumulated when production stops, and before screening is conducted, is equal to the total number of items screened from the total demand. As soon as production stops, screening the remaining units of the produced lot is conducted at the rate x per unit per unit time, where $x > \beta$. We make the following assumptions:

- No shortages are allowed.
- The screening cost during production is higher than that after production, i.e. $d_1 > d_2$.
- The production rate is higher than the demand rate, i.e. $\alpha > \beta$.
- Demand during production is met from non-defective items only.
- The screening rate is higher than the demand rate, i.e. $x > \beta$.
- The holding cost of defective items being reworked is higher than that of the non-defective items.

We analyze two models that differently address the defective items identified during production and screening. The first model assumes that defective items are sold at a discount at the end of the production cycle. The second model assumes that defective items are reworked at a constant rate.

3.1. Model 1: Salvaging of defective items

The defective items accumulated at the end of the screening period are sold at a discount price v. The following notation will be used to develop the mathematical model.

α: *Production rate per unit time.*
α_1: Rework rate of defective items per unit time.
β: Demand rate of the final product per unit time.
c_p: Unit production cost.
d: Production rate of defective items per unit time.
d_1: Screening cost per item during production.

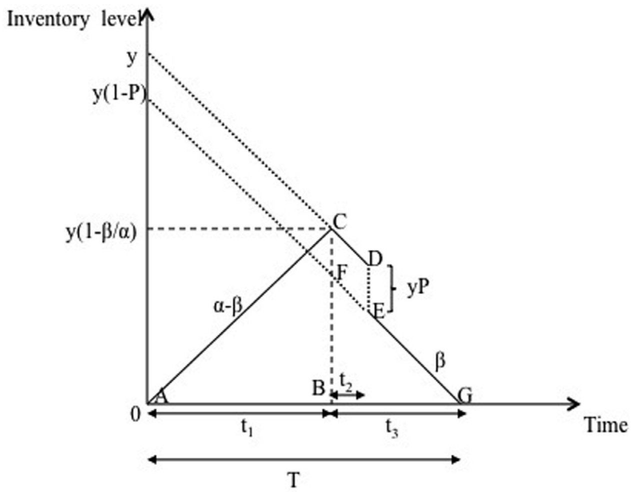

Fig. 1. The behavior of the inventory level over a production cycle.

d_2: Screening cost per item after production stops.
h: Holding cost per unit per unit time.
h_1: Holding cost of defective items being reworked per unit per unit time.
K: Fixed production setup cost.
P: Random proportion of defective items, with probability density function $f(P)$.
$f(P)$: Probability density function of P.
s: Unit selling price of good quality items, $s > v$.
t_1: Production time, $t_1 = y/\alpha$.
t_2: Screening time.
T: Production cycle.
v: Unit discounted selling price of defective items.
x: Screening rate per unit per unit time.
y: Total number of items produced during a production cycle.

Demand during production is met using good items only. Therefore, in $[0, t_1]$, a number of units are screened before they are sold to customers. To be able to satisfy demand from good items only, more than the demand is screened. The total number of units screened can be computed as follows

$$\text{Number of units screened during } t_1 = [\beta + \beta P + \beta P^2 + \cdots]t_1 = \frac{\beta}{1-P}t_1. \tag{1}$$

At the end of t_1, the number of defective items identified is the total number of units screened during the interval $[0, t_1]$, as given in (1), less the demand during this period. This is written as

$$\begin{aligned}\text{Number of defectives at the end of } t_1 &= \left[\frac{\beta}{1-P} - \beta\right]t_1 \\ &= \frac{P\beta}{1-P}\frac{y}{\alpha}.\end{aligned} \tag{2}$$

The on-hand inventory not screened at the end of t_1 is equal to the maximum inventory level, $y(1-\beta/\alpha)$, less the number of defective items identified at the end of t_1, as given in (2). This is depicted in Fig. 1.

$$\text{On-hand inventory not screened at } t_1 = y\left(1 - \frac{\beta}{\alpha}\right) - \frac{P\beta}{1-P}\frac{y}{\alpha}. \tag{3}$$

At t_1, the on-hand inventory not screened in $[0, t_1]$ is screened at the rate x. It can be easily checked that the total number of defective items in a cycle, yP, is the summation of the defective items found during the interval $[0, t_1]$, $\frac{P\beta}{1-P}\frac{y}{\alpha}$, and those found during the screening period t, $P[y(1-\frac{\beta}{\alpha}) - \frac{P\beta}{1-P}\frac{y}{\alpha}]$.

Two conditions are required. First, to avoid shortages during production, the number of good items produced should meet demand during production, i.e. $N(y, P) \geq \beta t_1$, which implies the following condition on P

$$P \leq 1 - \beta/\alpha. \tag{4}$$

The on-hand inventory not screened at the end of production is expressed in (3), and requires t_2 units of time to be screened at rate x per unit per unit time. Thus, t_2 can be written as

$$t_2 = \frac{y(1-\beta/\alpha) - (P\beta/(1-P))(y/\alpha)}{x}. \tag{5}$$

We let t_3 be the time from when production stops until the end of the cycle, i.e. $t_3 = T - t_1$. Then t_3 can be written as

$$t_3 = \frac{y(1-\beta/\alpha) - yP}{\beta}. \tag{6}$$

The second condition is a limit on the screening time t_2. Naturally, we require $t_2 < t_3$, which, after some term arrangement, implies the following condition on the screening rate, x

$$x > \frac{\beta(1-\beta/\alpha) - P\beta^2/(1-P)}{1-\beta/\alpha - P}. \tag{7}$$

Let $TR(y)$ be the total revenue per cycle. $TR(y)$ is the summation of the selling price of good quality items and the discounted selling price of defective items. Thus, it is written as

$$TR(y) = sy(1-P) + vyP. \tag{8}$$

Also, let $TC(y)$ be the total cost per cycle. $TC(y)$ is the summation of the production setup cost, unit production cost, screening cost during and after production, and inventory holding cost. The number of units screened at the end of production, i.e. at time t_1, is equal to the inventory level at t_1 less the total number of defectives identified during production and given in (2). To compute the holding cost expression, we refer to Fig. 1, in which the average inventory is the summation of the three areas, ABC, CDEF, and BGF. From Fig. 1, the cycle time T can be found as $T = y(1-P)/\beta$. Computing the areas of the three triangles, the total cost per cycle, $TC(y)$, can be written as follows

$$\begin{aligned} TC(y) = {} & K + c_p y + d_1 \frac{\beta}{(1-P)}\frac{y}{\alpha} + d_2 y\left[(1-\beta/\alpha) - \frac{P\beta}{\alpha(1-P)}\right] \\ & + h\left[\frac{y^2(1-\beta/\alpha-P)^2}{2\beta} + \frac{y^2(1-\beta/\alpha)}{2\alpha} + \frac{y^2P\left(1-\beta/\alpha - \frac{P\beta}{\alpha(1-P)}\right)}{x}\right]. \end{aligned} \tag{9}$$

The total profit per cycle is the total revenue less the total cost, and is given as

$$\begin{aligned} TP(y) = {} & sy(1-P) + vyP - \left[K + c_p y + d_1 \frac{\beta}{(1-P)}\frac{y}{\alpha} + d_2 y\left\{(1-\beta/\alpha) - \frac{P\beta}{\alpha(1-P)}\right\}\right. \\ & \left. + h\left\{\frac{y^2(1-\beta/\alpha-P)^2}{2\beta} + \frac{y^2(1-\beta/\alpha)}{2\alpha} + \frac{y^2P\left(1-\beta/\alpha - \frac{P\beta}{\alpha(1-P)}\right)}{x}\right\}\right]. \end{aligned} \tag{10}$$

Taking the expected value of the total profit per cycle $ETP(y)$ with respect to P, we get the following expression for the expected total profit per cycle $ETP(y)$

$$\begin{aligned} ETP(y) = {} & sy(1-E(P)) + vyE(P) - \left[K + c_p y + d_1\beta \mathrm{E}\left(\frac{1}{1-P}\right)\frac{y}{\alpha} + d_2 y\left\{1-\beta/\alpha - \frac{\beta}{\alpha}\mathrm{E}\left(\frac{P}{1-P}\right)\right\}\right. \\ & \left. + h\left\{\frac{y^2E[(1-\beta/\alpha-P)^2]}{2\beta} + \frac{y^2(1-\beta/\alpha)}{2\alpha} + \frac{y^2E(P)\left(1-\beta/\alpha - \frac{\beta}{\alpha}\mathrm{E}(P/(1-P))\right)}{x}\right\}\right]. \end{aligned} \tag{11}$$

Using the renewal-reward theorem, (see [51], Theorem 3.6.1), we find the expected profit per unit time as follows:

$$ETPU(y) = \frac{ETP(y)}{\mathrm{E}[T]},$$

where the expected duration of the production cycle is $\mathrm{E}(T) = [y(1-\mathrm{E}(P))]/\beta$. This gives the following expression for the expected profit per unit time, $ETPU(y)$:

$$\begin{aligned} ETPU(y) = {} & s\beta + v\frac{\beta E(P)}{1-E(P)} - c_p\frac{\beta}{1-E(P)} - d_1\frac{\beta^2}{\alpha(1-E(P))}\mathrm{E}\left(\frac{1}{1-P}\right) - d_2\frac{\beta}{1-E(P)}\left(1-\beta/\alpha - \frac{\beta}{\alpha}\mathrm{E}\left(\frac{P}{1-P}\right)\right) \\ & - K\frac{\beta}{y(1-E(P))} - h\frac{y}{1-E(P)}\left[\frac{\mathrm{E}\{(1-\beta/\alpha-P)^2\}}{2} + \frac{\beta(1-\beta/\alpha)}{2\alpha} + \frac{\beta E(P)\left(1-\beta/\alpha - \frac{\beta}{\alpha}\mathrm{E}\left(\frac{P}{1-P}\right)\right)}{x}\right]. \end{aligned} \tag{12}$$

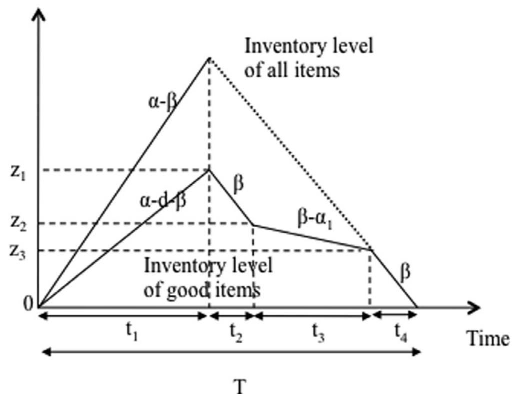

Fig. 2. The behavior of the inventory level over a production cycle when defective items are reworked.

Setting the unit discounted price v to zero results in a model that allows scrapping the defective items, rather than seeling them at discount. Thus, our model in this case reduces to one that allows scrapping defective items. All the mathematical expressions will remain unchanged.

The first order conditions of $ETPU(y)$ with respect to y give the optimal production lot size as

$$y^* = \sqrt{\frac{K\beta}{h\left[\frac{E[(1-\beta/\alpha-P)^2]}{2} + \frac{\beta(1-\beta/\alpha)}{2\alpha} + \frac{\beta E(P)\left(1-\beta/\alpha-\frac{\beta}{\alpha}E(P/(1-P))\right)}{x}\right.}}. \tag{13}$$

Next, we validate our model by comparing it to the EOQ model when all the items are of perfect quality, and by considering the case of infinite production rate.

Comparison to the Economic Production Quantity model. When all the items are of perfect quality, that is, when $P = 0$, our model reduces to the classical economic production quantity model . Setting $P = 0$, Eq. (13) reduces to

$$y^* = \sqrt{(2K\beta)/h(1-\beta/\alpha)}. \tag{14}$$

Case of infinite production rate. For a sufficiently large production rate, $\alpha \to \infty$, the optimal order quantity in Eq. (13) becomes

$$y^* = \sqrt{\frac{2K\beta}{h\{2E(P)\beta/x + E[(1-P)^2]\}}}, \tag{15}$$

which is equivalent to the results obtained in Maddah and Jaber [50] (Eq. (7), p. 810).

3.2. Model 2: Reworking of defective items

In this section, we assume that defective items are reworked at a constant rate α_1, with $\alpha_1 < \beta$. The time period needed to rework all defective items is t_3. At the end of t_3, the reworked items are added to the inventory, and are used to satisfy demand during t_4, the remaining of the production cycle.

Let d be the production rate of defective items (units per unit time). d can be expressed as the production rate multiplied by the proportion of defective items, as follows

$$d = \alpha p. \tag{16}$$

Also, we let h_1 be the holding cost of defective items being reworked per unit per unit time, $h_1 > h$, and c_r the rework cost per unit.

The inventory level of all items, good and defective, is depicted in Fig. 2. The larger triangle depicts the behavior of the inventory level for all items, which increases at the production rate minus the demand rate, $\alpha - \beta$, until the end of production, after which it decreases at the demand rate until the end of the production cycle. Fig. 2 also shows the inventory level for good items, which increases at the rate $\alpha - d - \beta$ in $[0, t_1]$. During the screening period, $[t_1, t_1 + t_2]$, the inventory of good items is depleted by the demand. During the rework period, $[t_1 + t_2, t_1 + t_2 + t_3]$, it increases by the items reworked and decreases due to demand, so the net effect is a change rate of $\beta - \alpha_1$. At the end of the rework period, i.e. in t_4, the inventory level of good items decreases at the demand rate β.

Since demand during production is met from good items, as before, we assume that items are screened before they are sold to customers.The total number of units screened is computed in (1), and the number on hand inventory not screened at the end of production is given in (3). The amount of time required to screen the on-hand inventory not screened at the end of production is found as before, so we have

$$t_2 = \frac{y(1-\beta/\alpha) - y[(P\beta)/(\alpha(1-P))]}{x}. \tag{17}$$

To avoid shortages, the number of non-defective items produced should be greater or equal to the demand during production, i.e. $N(y, t) \geq \beta t_1$, which reduces to $P \leq 1 - \beta/\alpha$, which is Condition (4). Also, we require that screening finishes before the end of the cycle, so $t_2 < t_3 + t_4$. Noting that the remaining of the cycle, $t_3 + t_4$ is also equal to $T - (t_1 + t_2)$, and replacing t_1 and t_2 by their respective expressions, we get after some term arrangement, the following lower bound on x:

$$x > \frac{\alpha\beta[1-\beta/\alpha - (P\beta)/(\alpha(1-P))]}{\alpha - \beta}. \tag{18}$$

At time t_1, the inventory level of good items is z_1, such that

$$t_1 = \frac{y}{\alpha} = \frac{z_1}{\alpha - d - \beta}, \tag{19}$$

where

$$z_1 = y\left(1 - \frac{\beta}{\alpha} - \frac{d}{\alpha}\right). \tag{20}$$

Referring to Fig. 2, the time to rework the defective items, t_3, is found as follows

$$t_3 = \frac{Py}{\alpha_1} = \frac{d}{\alpha\alpha_1}y. \tag{21}$$

The inventory level after screening is z_2, and is found as follows:

$$\begin{aligned} z_2 &= z_1 - \beta t_2 \\ &= y\left[\left(1 - \frac{\beta}{\alpha} - \frac{d}{\alpha}\right) - \frac{\beta}{x}\left(1 - \frac{\beta}{\alpha} - \frac{P\beta}{\alpha(1-P)}\right)\right]. \end{aligned} \tag{22}$$

The second inequality in (22) is obtained after replacing z_1 and t_2 by their respective expressions. Finally, the time between completing rework and the end of the cycle is $t_4 = z_3/\beta$, where z_3 is obtained as

$$\begin{aligned} z_3 &= z_2 - \beta t_3 \\ &= y\left[\left(1 - \frac{\beta}{\alpha} - \frac{d}{\alpha}\right) - \frac{\beta}{x}\left(1 - \frac{\beta}{\alpha} - \frac{P\beta}{\alpha(1-P)}\right) - \beta\frac{d}{\alpha\alpha_1}\right]. \end{aligned} \tag{23}$$

Fig. 3 depicts the behavior of the inventory level separately for non-defective, defective, and all items. The inventory of non-defective items increases at the rate $\alpha - d - \beta$ up to t_1, after which screening is conducted during $[t_1, t_1 + t_2]$, and the inventory of non-defective items decreases at the demand rate in this interval. In $[t_1 + t_2, t_1 + t_2 + t_3]$, the inventory level of non-defective items increases at the rework rate, so the net effect is a change at the rate $\beta - \alpha_1$. After rework is completed, and until the end of the production cycle, i.e. in t_4, the inventory level decreases at the demand rate. On the other hand, the inventory level of defective items increases at the production rate of defective items, d, during $[0, t_1]$. During the screening period, $[t_1, t_1 + t_2]$, the inventory level of defective items is the same as the one at t_1, and stays at the same level until the end of the screening period, since defective items identified will remain in the inventory. At the end of the screening period, i.e. at $t_1 + t_2$, rework starts, and the inventory of defective items decreases at the rework rate α_1. This continues until the end of the rework period, $t_1 + t_2 + t_3$, when all defective items are reworked, and the inventory level of defective items reduces to zero during t_4. The summation of the inventory level of non-defective and defective items gives the inventory of all items which increases at the rate $\alpha - \beta$ during the interval $[0, t_1]$, and decreases at rate β until the end of the production cycle, so the cycle length is $T = y/\beta$.

The holding cost is the summation of the holding cost of non-defective and defective items, and the cost of defective items being reworked. Using Fig. 1, the holding cost of non-defective items is

$$\text{Holding cost of non-defective items } = h\left[\frac{z_1t_1}{2} + \frac{(z_1+z_2)t_2}{2} + \frac{(z_2+z_3)t_3}{2} + \frac{z_3t_4}{2}\right].$$

The holding cost of defective items can be found from 3, as follows

$$\text{Holding cost of defective items } = h\left(\frac{t_1^2 d}{2} + t_1t_2d\right) + h_1\frac{\alpha_1 t_3^2}{2}.$$

As a result, the total cost expression is written as

$$TC(y) = K + c_py + c_rpy + d_1\frac{\beta}{(1-P)}\frac{y}{\alpha} + d_2y\left[(1-\beta/\alpha) - \frac{P\beta}{\alpha(1-P)}\right]$$

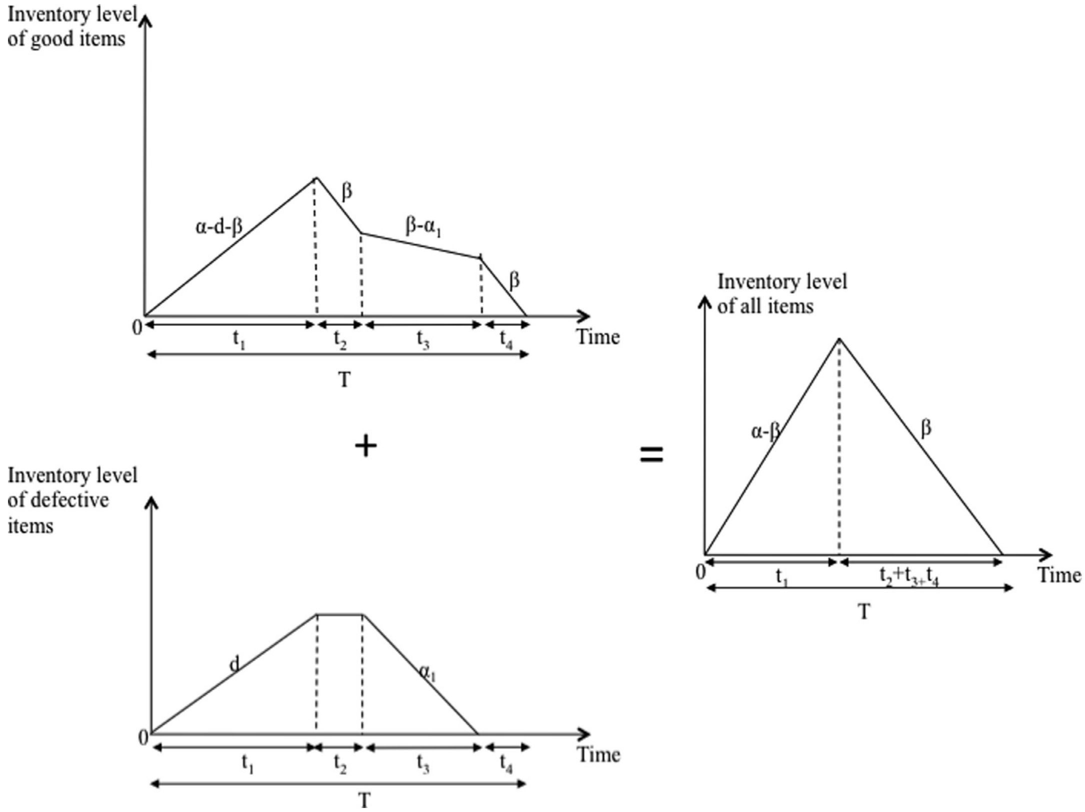

Fig. 3. The inventory of good, defective, and all items.

$$+h\left[\frac{z_1t_1}{2}+\frac{(z_1+z_2)t_2}{2}+\frac{(z_2+z_3)t_3}{2}+\frac{z_3t_4}{2}+\frac{t_1^2d}{2}+t_1t_2d\right]+h_1\frac{\alpha_1t_3^2}{2}. \tag{24}$$

The total profit per cycle is as follows

$$TP(y)=sy-\left[K+c_py+c_rpy+d_1\frac{\beta}{(1-P)}\frac{y}{\alpha}+d_2y\left\{(1-\beta/\alpha)-\frac{P\beta}{\alpha(1-P)}\right\}\right]$$
$$-h\left[\frac{z_1t_1}{2}+\frac{(z_1+z_2)t_2}{2}+\frac{(z_2+z_3)t_3}{2}+\frac{z_3t_4}{2}+\frac{t_1^2d}{2}+t_1t_2d\right]-h_1\frac{\alpha_1t_3^2}{2}. \tag{25}$$

Using the renewal-reward theorem, ([51], Theorem 3.6.1), the expected profit per unit time is the following

$$ETPU(y)=\frac{ETP(y)}{\mathrm{E}[T]},$$

where $ETP(y)$ is the expected profit per cycle, and E(T) the expected duration of the production cycle is $\mathrm{E}(T)=y/\beta$. We get

$$ETPU(y)=s\beta-c_p\beta-c_rp\beta-K\frac{\beta}{y}-d_1\frac{\beta^2}{\alpha}\mathrm{E}\left(\frac{1}{1-P}\right)-d_2\beta\left[1-\frac{\beta}{\alpha}-\frac{\beta}{\alpha}\mathrm{E}\left(\frac{P}{1-P}\right)\right]$$
$$-hy\left[\frac{\beta}{2\alpha}\left(1-\frac{\beta}{\alpha}-\frac{d}{\alpha}\right)+\frac{\beta}{x}\left(1-\frac{\beta}{\alpha}-\frac{\beta}{\alpha}\mathrm{E}\left(\frac{P}{1-P}\right)\right)\left\{\left(1-\frac{\beta}{\alpha}-\frac{d}{\alpha}\right)-\beta\frac{1-\frac{\beta}{\alpha}-\frac{\beta}{\alpha}\mathrm{E}(\frac{P}{1-P})}{2x}\right\}\right.$$
$$+\left\{\left(1-\frac{\beta}{\alpha}-\frac{d}{\alpha}\right)-\frac{\beta}{x}\left(1-\frac{\beta}{\alpha}-\frac{\beta}{\alpha}\mathrm{E}\left(\frac{P}{1-P}\right)\right)\right\}\frac{\beta d}{\alpha\alpha_1}$$
$$+\left\{\left(1-\frac{\beta}{\alpha}-\frac{d}{\alpha}\right)-\frac{\beta}{x}\left(1-\frac{\beta}{\alpha}-\frac{\beta}{\alpha}\mathrm{E}\left(\frac{P}{1-P}\right)\right)-\frac{\beta d}{\alpha\alpha_1}\right\}^2/2$$
$$\left.+\frac{\beta d}{2\alpha^2}+\left(1-\frac{\beta}{\alpha}-\frac{\beta}{\alpha}\mathrm{E}\left(\frac{P}{1-P}\right)\right)\frac{\beta d}{\alpha x}\right]-h_1y\frac{\beta d^2}{2\alpha^2\alpha_1}. \tag{26}$$

To simplify the above expression, we define the following two expressions J and $\tilde{J}$ as

$$J := 1 - \frac{\beta}{\alpha} - \frac{\beta}{\alpha}\mathrm{E}\left(\frac{P}{1-P}\right),$$
$$\tilde{J} := 1 - \frac{\beta}{\alpha} - \frac{d}{\alpha} = 1 - \frac{\beta}{\alpha} - E(P).$$

Then, the expected profit per unit time, $ETPU(y)$, in (26), becomes

$$ETPU(y) = s\beta - c_p\beta - c_r p\beta - K\frac{\beta}{y} - d_1\frac{\beta^2}{\alpha}\mathrm{E}\left(\frac{1}{1-P}\right) - d_2\beta J$$
$$- hy\left[\frac{\beta}{2\alpha}\tilde{J} + \frac{\beta}{x}J\left(\tilde{J} - \beta\frac{J}{2x}\right) + \left(\tilde{J} - \frac{\beta}{x}J\right)\frac{\beta d}{\alpha\alpha_1} + \left(\tilde{J} - \frac{\beta}{x}J - \frac{\beta d}{\alpha\alpha_1}\right)^2/2 + \frac{\beta d}{2\alpha^2} + J\frac{\beta d}{\alpha x}\right] - h_1 y\frac{\beta d^2}{2\alpha^2\alpha_1}. \quad (27)$$

The optimal production quantity y^* is obtained by taking the derivative of the expected profit per unit time in (27). Then, we get

$$\frac{k\beta}{y^2} = h\left[\frac{\beta}{2\alpha}\left(1 - \frac{\beta}{\alpha} - \frac{d}{\alpha}\right) + \frac{\beta}{x}\left(1 - \frac{\beta}{\alpha} - \frac{\beta}{\alpha}\mathrm{E}\left(\frac{P}{1-P}\right)\right)\left\{\left(1 - \frac{\beta}{\alpha} - \frac{d}{\alpha}\right) - \beta\frac{1 - \frac{\beta}{\alpha} - \frac{\beta}{\alpha}\mathrm{E}\left(\frac{P}{1-P}\right)}{2x}\right\}\right.$$
$$+ \left\{\left(1 - \frac{\beta}{\alpha} - \frac{d}{\alpha}\right) - \frac{\beta}{x}\left(1 - \frac{\beta}{\alpha} - \frac{\beta}{\alpha}\mathrm{E}\left(\frac{P}{1-P}\right)\right)\right\}\frac{\beta d}{\alpha\alpha_1}$$
$$+ \left\{\left(1 - \frac{\beta}{\alpha} - \frac{d}{\alpha}\right) - \frac{\beta}{x}\left(1 - \frac{\beta}{\alpha} - \frac{\beta}{\alpha}\mathrm{E}\left(\frac{P}{1-P}\right)\right) - \frac{\beta d}{\alpha\alpha_1}\right\}^2/2$$
$$\left. + \frac{\beta d}{2\alpha^2} + \left(1 - \frac{\beta}{\alpha} - \frac{\beta}{\alpha}\mathrm{E}\left(\frac{P}{1-P}\right)\right)\frac{\beta d}{\alpha x}\right] + h_1\frac{\beta d^2}{2\alpha^2\alpha_1}. \quad (28)$$

Then, Eq. (28) becomes

$$\frac{k\beta}{y^2} = h\left[\frac{\beta}{2\alpha}\tilde{J} + \frac{\beta}{x}J\left(\tilde{J} - \beta\frac{J}{2x}\right) + \left(\tilde{J} - \frac{\beta}{x}J\right)\frac{\beta d}{\alpha\alpha_1} + \left(\tilde{J} - \frac{\beta}{x}J - \frac{\beta d}{\alpha\alpha_1}\right)^2/2 + \frac{\beta d}{2\alpha^2} + J\frac{\beta d}{\alpha x}\right] + h_1\frac{\beta d^2}{2\alpha^2\alpha_1}. \quad (29)$$

As a result, the optimal production quantity y^* has the following expression

$$y^* = \sqrt{\frac{K\beta}{h\left[\frac{\beta}{2\alpha}\tilde{J} + \frac{\beta}{x}J\left(\tilde{J} - \beta\frac{J}{2x}\right) + \left(\tilde{J} - \frac{\beta}{x}J\right)\frac{\beta E(P)}{\alpha_1} + \left(\tilde{J} - \frac{\beta}{x}J - \frac{\beta E(P)}{\alpha_1}\right)^2/2 + \frac{\beta E(P)}{2\alpha} + J\frac{\beta E(P)}{x}\right] + h_1\frac{\beta E(P)^2}{2\alpha_1}}}. \quad (30)$$

Special cases. A special case of our model occurs when all the items are of perfect quality, or when the production rate is infinite. In the case when all items are of perfect quality, we will check next that the optimal quantity obtained in (30) reduces to the economic production quantity. In the case when the production rate is infinite, we recover the economic order quantity.

All items are of perfect quality. In this case, we have $P = 0$, $d = 0$, and $\alpha_1 \longrightarrow \infty$. Then we get $J \longrightarrow 1 - \frac{\beta}{\alpha}$ and $\tilde{J} \longrightarrow 1 - \frac{\beta}{\alpha}$. replacing J and $\tilde{J}$ by their respective expressions into (30), and rearranging the terms, we get

$$y^* = \sqrt{\frac{K\beta}{h((\beta/2\alpha)(1-\beta/\alpha) + (1-\beta/\alpha)^2/2)}}$$
$$= \sqrt{\frac{2\tilde{}K\beta}{h(1-\beta/\alpha)}}, \quad (31)$$

which is the optimal quantity for the classical production problem.

Case of infinite production rate and perfect quality items. For a sufficiently large production rate, $\alpha \longrightarrow \infty$, and no defective items ($P = 0$), we get $J \longrightarrow 1$ and $\tilde{J} \longrightarrow 1$. Replacing J and $\tilde{J}$ by their values into (30), and simplifying similar terms, the optimal quantity becomes

$$y^* = \sqrt{\frac{K\beta}{h((\beta/x)(1-\beta/2x) + (1-\beta/x)^2/2)}}$$
$$= \sqrt{\frac{2K\beta}{h}}.$$

Case of infinite production rate. For a sufficiently large production rate, $\alpha \longrightarrow \infty$, we get $J \longrightarrow 1$ and $\tilde{J} \longrightarrow 1 - E(P)$. Replacing J and $\tilde{J}$ by their values into (30), and simplifying similar terms, the optimal quantity becomes

$$y^* = \sqrt{\frac{K\beta}{h\left(\frac{\beta}{\alpha} + \frac{(1-E(P))^2}{2} + \frac{\beta E(P)}{x} + \frac{\beta^2 E(P)^2}{2\alpha_1^2}\right) + h_1 \frac{\beta E(P)^2}{2\alpha_1}}}. \quad (32)$$

4. Numerical analysis

In this section, we analyze how the optimal production quantity and optimal profit vary with the model parameters, for each of the models in Sections 3.1 and 3.2. Note that in each model, we need to make sure the conditions on the expected proportion of defective items and the screening rate hold. We develop numerical results similar to those in Hayek and Salameh (2001). This illustrates the application of our model and allows comparing our results with those of Hayek and Salameh (2001) for the model with rework. We consider constant demand rate $\beta = 1200$ units/year, and a production rate $\alpha = 1600$ units/year. The production cost per unit is $c_p = \$104$, selling price of non-defective items is $s = \$200$ /unit, and discount price of defective items is $v = \$80$/unit. The screening rate is 1 unit/min. If the production operates 8 hours/day for 365 days a year, then the annual screening rate is 175200 units. The screening cost per item during production is $d_1 = \$0.5$ and the screening cost per item when production stops is $d_2 = \$0.6$. The machine setup cost $K = \$1500$. The holding cost of non-defective items is $h = \$20/$ unit/year, and the holding cost of defective items is $h_1 = \$22/$ unit/year. The defective items are reworked at the rate $\alpha_1 = 100$ units/year. The repair cost per defective item is $c_r = \$8$. The proportion of defectives items is uniformly distributed over the range [0, 0.1], with the probability distribution function $f(P)$ as follows:

$$f(P) = \begin{Bmatrix} 10, & \text{for } 0 \le P \le 0.1, \\ 0, & \text{otherwise.} \end{Bmatrix}.$$

Using (33), we can compute the following expected value expressions: $E(P) = 0.05$, $E(\frac{1}{1-P}) = 1.0536$, and $E(\frac{P}{1-P}) = 0.0536$.

When the defective items are salvaged, the optimal production quantity is derived in (13). We check that the special case when P is set to zero, the optimal solution reduces to that of the classical economic production quantity model, $y^* = 848.52$ and the optimal expected profit is \$110332. Clearly, y^* increases as the order cost K increases or the holding cost h decreases. The sensitivity with respect to the other parameters is not necessarily intuitive, so we perform numerical analysis. Fig. 4 illustrates the sensitivity of y^* and the optimal profit for the model when the defective items are salvaged, with respect to the demand, production rate, average proportion of defectives, and screening rate. Note that the range of the parameters considered should satisfy the two condition in (4) and (7). Condition (4) implies

$$\beta \le \alpha(1 - E(P)) = 1520,$$

which is the maximum value that the demand can assume. Also, we have

$$\alpha \ge \beta = 1200 \text{ and } \alpha \ge \beta/(1-P) = 1263$$

which gives a lower bound on the values that the production rate can assume. Condition (4) on the expected proportion of defectives also implies

$$E(P) \le 1 - \beta/\alpha = 0.25.$$

The second condition (7) sets a lower limit on the screening rate, which is negative, so Condition (7) is also satisfied. In Fig. 4, y^* is shown on the primary vertical axis, while the optimal expected profit is on the secondary axis, in thousand dollars.

The numerical examples presented in this section consider the sensitivity of the two models to the system parameters where Fig. 5 reports the numerical results for the model with no rework and Fig. 6 reports the results for the model where rework is allowed. The system parameters investigated are the demand rate β, the production rate α, the proportion of defective items E[P] and the screening rate x. The system parameters in many cases are exogenous factors that might be out of a manager's control such as the demand rate or the proportion of defective items. In some cases it might be possible to improve the production rate or speed up the screening rate if the marginal cost associated with this increase is justified by the increase in profit. Even if these parameters are out of a manager's control, there is still value in investigating the sensitivity of the system to changes in the environment. For example, investigating the behavior of the optimal profit to a decrease or increase in the market demand can provide a level of risk assessment in a highly fluctuating market. In Fig. 5, we vary the demand rate β over the range [0, 1520]. As expected, the demand rate increases the optimal amount to be produced as well as the optimal profit where for this set of numerical examples the optimal production quantity increases from 0 to 140 and the optimal profit increases linearly. As the production rate α increases from a starting value of 1200, the optimal profit and production quantity decrease as shown in Fig. 5 where both measures graphically approach a limit value. For this particular set of numerical examples, this justifies lower production rates to offset the higher holding cost of producing most of the items at the beginning of the cycle. As the proportion of defective items is increased to 0.25, it is optimal to produce larger lot sizes, and compensate for the defective items produced. This is consistent with results in the literature (Moussawi-Haidar et al. [29], and Salameh and Jaber [17]). In the bottom right plot of Fig. 5, the optimal quantity and profit increase significantly as the screening rate reaches 0.5 item/unit time. For values of x greater than 0.5 the increase in profit becomes negligible and the optimal profit converges to a value of \$108770.

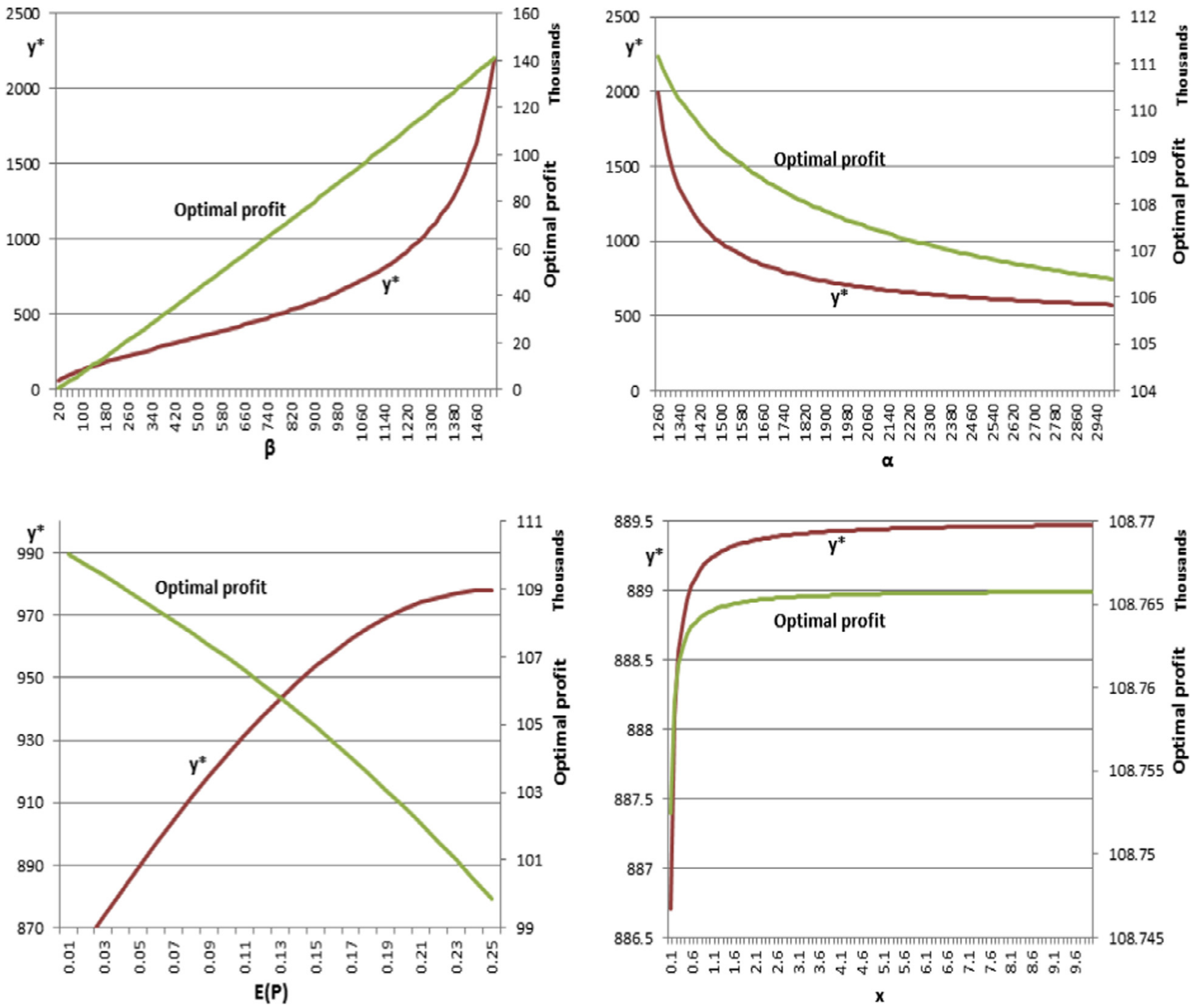

Fig. 4. The sensitivity of the optimal quantity and optimal profit of the model in Section 3.1 versus demand β, production rate α, expected proportion of defective items $E(P)$, and screening rate x.

Next we consider the model with rework where the numerical results are plotted in Fig. 5. Here we note that the conditions on the expected proportion of defective items, demand, production rate, and screening rate should be satisfied. That is, we have $\beta \le \alpha(1 - E(P)) = 1520,\ \alpha \ge \beta = 1200,\ \alpha \ge \beta/(1-P) = 1263,\ P \le 1 - (\beta/\alpha) = 0.25$, and $x > -377447$, which is checked to be satisfied for each instance. We can observe from a comparison of Figs. 5 and 6 that the behavior of the plots are similar in most cases. Specifically, the optimal profit and production quantity are increasing with the demand rate and decreasing with the production rate to offset high holding costs. The behavior of the optimal profit when compared to the screening rate is also negligible for values of x greater than 0.5 where the optimal profit converges to $107000. The main difference observed from comparing Figs. 5 and 6 is that the optimal production quantity decreases as the proportion of defective items increases. In the no re-work case, a higher proportion of defectives resulted in a larger production quantities to compensate for the scrapped items which are not utilized to satisfy the demand. When items are allowed to be reworked, all items produced eventually are used to satisfy demand and an increase in the production lot size to compensate for scrapped items is no longer required.

5. Conclusions

In this work we consider the realistic case where defective items undergo quality control by the consumer or seller during the purchasing process. As soon as production is completed, a cheaper and faster screening process identifies all the defective items. We investigate two realistic cases where defective items are scrapped and when defective items are reworked. The equations to calculate the optimal total profit per unit time and order quantities are presented. A set of numerical example is presented to

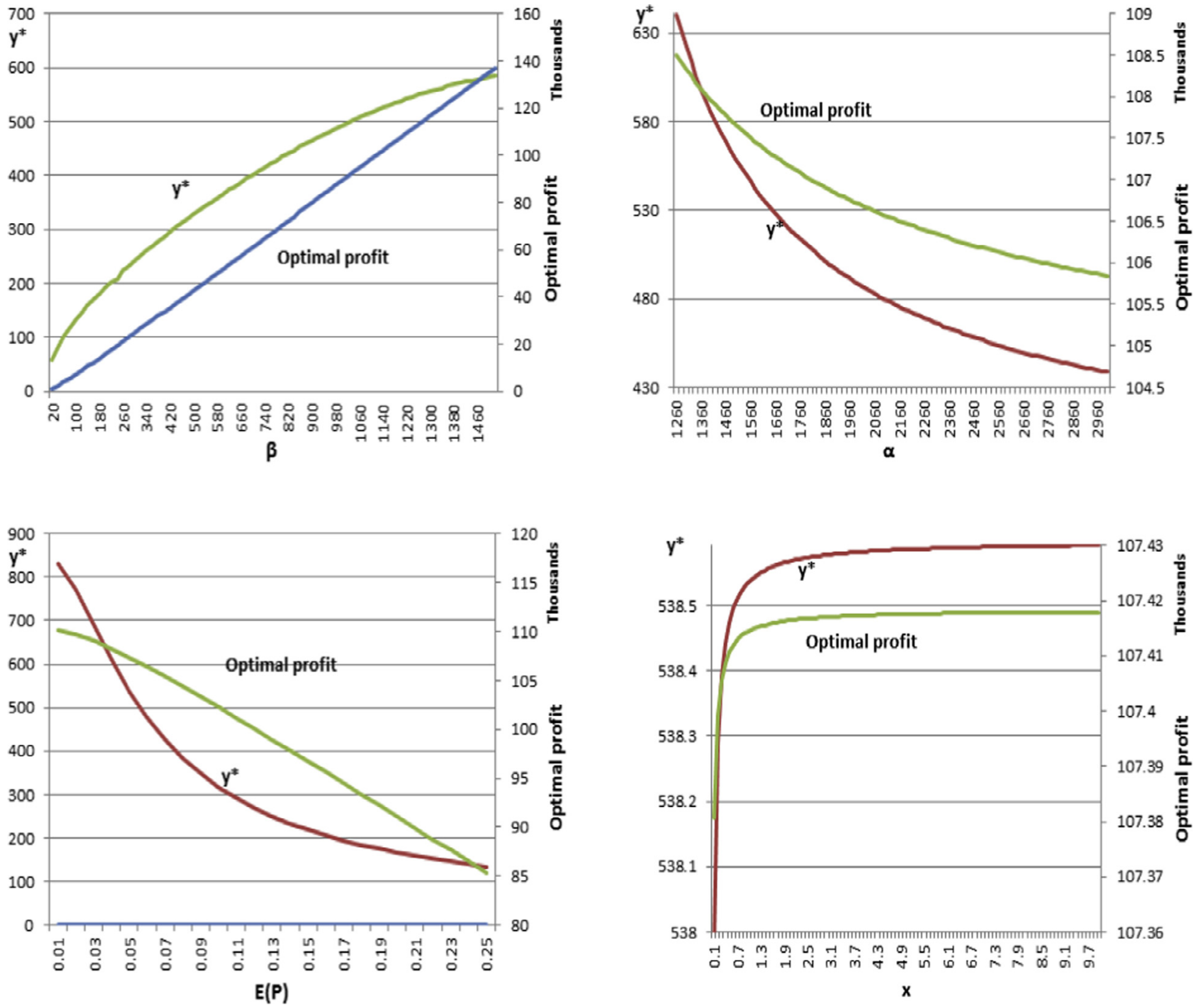

Fig. 5. The sensitivity of the optimal quantity and optimal profit of the model in Section 3.2 versus demand β, production rate α, expected proportion of defective items $E(P)$, and screening rate x.

test the sensitivity to the system parameters which include the demand rate, production rate, proportion of defective items and screening rate.

The literature on the production model with rework is extensive but has a main limitation, which is the assumption that the screening for imperfect quality items occurs continuously, and the rework follows immediately after an imperfect item is identified. This assumption is not practical, especially for automated high rate production processes. This paper takes into consideration the screening process necessary to identify the items to be reworked, which is conducted right after production stops. Thus, our work significantly contributes to the literature as it integrates the screening time into the production model with rework, which is ignored from the previous works. Furthermore, in our model, the system screens the items as they are being sold, i.e. the screening rate is the same as the demand rate before production ends. After production ends, a screening process is initiated for all remaining products. Screening the items during production to meet demand is also new to the related literature. We assume that no items are scrapped, and all items can be repaired, which is true for expensive products such as metal book shelves, defective book shelves and the wafer fabrication in the semiconductor manufacturing. Our work can be extended to consider that a portion of defectives items can be scrapped.

An alternative and practical approach is to postpone the screening process until after production, and maintain a safety stock to satisfy demand during production. Utilizing a safety stock to meet demand has been considered in Gurnani et al. [4], as they analyze the problem of locating inspecting stations in a serial production line and how much safety stock is required to meet the demand. Our paper can be extended to consider such an alternative approach to meet demand during production. Another extension of our work would be to consider a multi-stage manufacturing system rather than a single-stage system. In a multi-stage manufacturing system, two scenarios can be analyzed: When screening is carried out after each stage and at the end of the final stage. Under each scenario, the optimal lot size can be derived.

The sensitivity analysis of the optimal profit and optimal quantity with respect to the demand, production rate, expected proportion of defective items, and screening rate is performed. For both the no-rework and rework models, we observe that the optimal profit and production quantity are increasing with the demand rate and decreasing with the production rate to offset high holding costs. The optimal profit and quantity are highly insensitive to the screening rate. The main difference observed is that the optimal production quantity decreases as the proportion of defective items increases. In the no re-work case, a higher proportion of defectives resulted in a larger production quantities to compensate for the scrapped items which are not utilized to satisfy the demand.

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