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Coordination of a three-level supply chain (supplier–manufacturer–retailer) with permissible delay in payments and price discounts

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ABSTRACT

Different coordination mechanisms are used to manage supply chains. This paper investigates a coordination of a three-level supply chain (supplier–manufacturer–retailer) by coupling two well-known trade credit mechanisms that are widely used in practice, permissible delay in payments and price discounts, where the length of the delay period and discounts offered along the supply chain are decision variables. The paper investigates nine different cases of delay in payments along with eight cases of price discounts among the three players in the supply chain. A numerical example was presented to compare between the cases considered. Also, extensive sensitivity analyses were performed to study the effect of changing the model parameters on the optimal decisions. In addition, we point out the limitations of each model developed in this paper. The numerical examples and the sensitivity analyses conclude that the coupling of delay in payments and price discounts increases the profit of the supply chain more than using only a single mechanism at a time.

1. Introduction

Global market changes, such as the drop in oil prices, rapid product development, high product variety, and short product life cycle, have dramatically increased competition among companies, making it more difficult to sustain the desired levels of profitability. One of the most useful tools that help industries to survive are improving and optimizing their supply chains.

Supply Chain Management (SCM) is defined as the planning and analysis of all processes involved in acquiring and delivering raw material and components, transforming them into finished products and selling them to customers. SCM has been the focus of researchers and practitioners, as effective supply chain management tools have resulted in improvements in performance. A supply chain can be coordinated through adopting different mechanisms. The Joint Economic Lot Sizing Problem (JELSP), which was first studied by Banerjee [1] and Goyal [2], is perhaps the most common coordination mechanism used in the literature. It minimizes the supply chain cost by treating the lot size and the number of shipments as decision variables (Jaber and Zolfaghari [3], Ben-Daya et al. [4] and Glock [5]). Although JELSP minimizes the supply chain cost, not all members would benefit as the savings would cluster with just a few players. The buyer is the one who is always at a disadvantage, as the JELSP suggests a lot size larger than the buyer's economic order quantity. Thus, the coordination of

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a supply chain has to include incentive schemes to entice the buyer and to make it work. Many incentive schemes were introduced and studied in the literature, e.g., permissible delay in payments, price/quantity discounts, common replenishment period and consignment stocks [3].

Permissible delay in payments is a coordination mechanism that allows a buyer (customer) to settle its balance with a supplier (vendor) by a specified period without being charged interest on the amount owed (an interest-free period). However, if the buyer settles its payment after the permitted period, interest is charged on the unpaid balance until it is settled. The advantage for the buyer is that the cash owed to the supplier/vendor can be invested during the permissible delay period for some return. Many researchers have used delay in payments to coordinate orders and shipments in supply chains. It has been commonly assumed in the literature that a delay in payment is an input parameter of a fixed length; see for examples Goyal [8], Boyac and Gallego [6], Jaggi et al. [7] and Kouvelis and Zhao [8]. Recent publications focus on studying cash flow management in supply chains in the presence of delay in payments as in Moussawi-Haidar and Jaber [9] and Moussawi-Haidar et al. [10]. The first work that uses JELSP in a two-level supply chain where the length of a permissible delay in payments is treated as a decision variable is that of Jaber and Osman [11]. Their objective was to minimize the total cost of the supply chain system by jointly optimizing the total cost for the lot size quantity, the number of shipments and the length of the permissible delay, which consists of an interest-free period and a period over which an interest is charged, both of which are decision variables. Aljazzar et al. [12] modified the work of Jaber and Osman [14] by treating the supplier as a manufacturer and investigated the problem for three well-known production policies. Readers may refer to Chang, et al. [13], Seifert et al. [14] and Molamohamadi et al. [15] for a review of inventory and supply chain models with delay in payments .

Reduction in price, or price discounts, which is widely applied in practice, is usually offered by a supplier/vendor to its customers (buyers) to entice them to order in larger quantities. There are two types of price discounts, all-unit discount, which applies to all items purchased in every order, and incremental discount, which applies only if the customer's order quantity falls within a certain quantity range. Accordingly, incorporating price discounts in supply chain research has been the attention of researchers. Crowther [16] was the first to investigate quantity discounts from the perspectives of a buyer and a seller. Many researchers investigated price discounts in a two-level supply chain, e.g., Monahan [17], Lee and Rosenblatt [18], Kim and Hwang [19] and Viswanathan and Wang [20]. Munson and Rosenblatt [21] are the first to investigate quantity discounts in a three-level supply chain (supplier–manufacturer–retailer). They assumed all parameters to be deterministic and the retailer's lot size to be based on its EOQ. Jaber et al. [22] investigated coordinating a three-level supply chain with a price discount, with profit being the performance measure. They assumed that demand is sensitive to a price discount offered by the buyer to its customers, where discounting price increases demand and, subsequently, alters the order quantities and the frequency of shipments. They also assumed price discounts are offered along the supply chain (supplier to manufacturer to retailer to customers) and are decision variables. In addition, they presented a profit sharing scenario in which the most powerful player gets a higher portion of the profit and another scenario in which the system profit is allocated proportionally to the cost of each player.

To the best of our knowledge, very few studies exist in the literature on coupling permissible delay in payments with price discounts. Huang et al. [23] investigated a two-level supply chain (supplier– retailer) with trade credit and cash discounts, but did not consider the joint coupling of permissible delay in payments and discounts. Feng et al. [24] investigated the effect of utilizing cash discount with permissible delay in payments between a retailer and its customers, with a cost minimization approach. They assumed that the retailer offers a trade credit to its loyal customers and requests its bad customers to pay once they receive the purchased items. Thangam [25] explored the changes in a retailer's inventory when the supplier offers the retailer a trade credit, and the latter offers both a price discount and trade credit to its customers.

Offering a price discount along with permissible delay in payments was suggested by many consultants and practitioners. Deloitte indicated that some businesses offer a discount for early paying customers [26]. Similarly, Bertsch, the founder of Dryun, a software developer of accounting and cash flow management, recommends business owners to speed up the payments of their customer by offering price discounts [27]. To the best of our knowledge, integrating price discounts along with a permissible delay in payment has not been investigated in the literature. Herein is the contribution of our work. This paper presents a coordination mechanism of a three-level (supplier–manufacturer–retailer) supply chain system by integrating two coordination mechanisms, delay in payments and price discounts. To reflect the real-life scenarios that are encountered in practice, the paper analyzes detailed cases of permissible delay and price discounts. The remainder of this paper is organized as follows. Section 2 lists the notation and assumptions. Section 3 presents the mathematical models. Section 4 provides numerical examples and sensitivity analysis. We conclude our work in Section 5.

2. Notations and assumptions

The following notations have been used to develop the mathematical models:

i	Corresponds to the supply chain member as indicated by the subscript (i.e. '*s*' for supplier, '*w*' for manufacturer's raw material, '*m*' for manufacturer's finished goods, '*r*' for retailer and 'c' for customer)
A_i	Setup/order cost for player '*i*'
C_i	Production/Purchasing cost per item for player '*i*'
h_i	Financial holding cost per item for player '*i*'
si	Physical (storage) holding cost per item for player '*i*'

Q	The buyer's order quantity
n_1	Number of shipments delivered by the supplier to the manufacturer, per each manufacturer's raw material cycle
n_2	Number of shipments delivered by the manufacturer to the retailer, per each retailer's cycle
α	Amount of raw material required to produce one finished product
t_i	Permissible delay in payments offered by player 'i'
τ_i	Balance settlement time for player 'i'
k_i	Return on investment for player 'i'
P	Manufacturer's annual production rate
d_i	Discount in '$' offered by player i to its customer (m,r or c)
D_o	Retailer's base annual demand rate, $D_o < P$
D_1	Demand elasticity to the discount
$f(d_r)$	Retailer's annual demand, assumed to be a linear function of the discount $f(d_r)=D_o+\mathrm{D}_1 d_r$
T	Common cycle length $=\frac{n_2 Q}{D}$
ψ_i	Annual profit for player 'i'

Furthermore, the paper considers the following assumptions:

- The supply chain consists of a supplier, a manufacturer and a retailer (buyer).
- A single product consists of multiple (α) raw materials.
- The demand is discount dependent.
- The supplier's production rate is greater than the manufacturer's demand for raw materials, and the production rate of finished goods at the manufacturer is faster than the retailer's demand.
- Shortages are not allowed.
- The manufacturer's production policy follows that of Hill [28], where equal batches are produced, and equal-sized shipments are made.
- The holding cost consists of two components: physical and financial.
- A price discount and permissible delay in payments are offered respectively by the supplier to the manufacturer and by the manufacturer to the retailer.
- Price discounts and permissible delay in payments are decision variables.
- During the permissible delay period, the manufacturer and the retailer invest the accumulated balances.
- The manufacturer and the retailer pay their balances in single payments.
- The maximum price discount that is offered by the supplier, manufacturer, or retailer cannot exceed the profit margin.

The supply chain system presented in this paper consists of a three-level supply chain (supplier–manufacturer–retailer), as illustrated in Fig. 1. In this system, the retailer orders a lot size Q of finished items for an annual demand rate of D units. The manufacturer produces finished items at an annual rate of P units, where $P > D$. In addition, the manufacturer orders αQ from the supplier, where α is the number of raw material units required to manufacture one finished product. The manufacturer settles its payments to the supplier by time t_s, an interest-free period. If the manufacturer settles its payment after t_s, but by time τ_m, where $\tau_m > t_s$, an interest rate, k_s, is charged on the outstanding balance for $\tau_m - t_s$, units of time. During τ_m or t_s, the manufacturer invests the balance owed to the supplier at an interest rate, k_m. Similarly, the manufacturer allows the retailer to settle its payment by time, t_m an interest-free period. The retailer may postpone its payment to the manufacturer till after t_m, when the manufacturer charges an interest rate, k_m, on any outstanding balance. The retailer invests the balance it owes to the manufacturer at an interest rate, k_r. Besides offering delay in payments, a price discount is offered from each upstream supply chain party to the downstream one.

3. Mathematical model

This section presents the mathematical models of coordinating a three-level supply chain (supplier–manufacturer–retailer) with delay in payment and price discount. There are nine possible scenarios of delay in payments that would occur in real practice between the players in the considered supply chain. The profit function for each player in the following models is the annual revenue minus the annual cost. Therefore, the supplier's annual revenue (ψ_s) includes revenue from selling raw materials to the manufacturer ($(C_{m,w}-d_s)\alpha f(d_r)$) and from interest charges that will be paid by the manufacturer ($(C_{m,w}-d_s)\alpha f(d_r)(e^{k_s(\tau_{m,w}-t_s)}-1)$). Accordingly, the annual revenue for the supplier can be written as follows:

$$\psi_s(t_s,\ \tau_{m,w}, d_s, d_r) = (C_{m,w}-d_s)\alpha f(d_r) + (C_{m,w}-d_s)\alpha f(d_r)\left(e^{k_s(\tau_{m,w}-t_s)}-1\right). \tag{1}$$

Note that for very small values of the interest rate k_s, the term $e^{k_s(\tau_{m,w}-t_s)}-1$ in Eq. (1) can be approximated as $k_s(\tau_{m,w}-t_s)$. Readers who wish to use the approximate model can simply replace $e^{k_s t_s}-1$ with $k_s(\tau_{m,w}-t_s)$. The annual cost for the supplier (γ_s) includes setup cost ($\frac{A_s f(d_r)}{n_2 Q}$), production cost of raw materials ($C_s\alpha f(d_r)$), financial and storage holding costs ($\frac{(n_1-1)}{2}(h_s+s_s)\frac{\alpha n_2 Q f(d_r)}{P n_1}$), financial holding cost due to the delay offered to the manufacturer ($h_s\tau_{m,w}\alpha f(d_r)$) and losing opportunity of investing the profit from sales in a risk-free investment due to the delay in

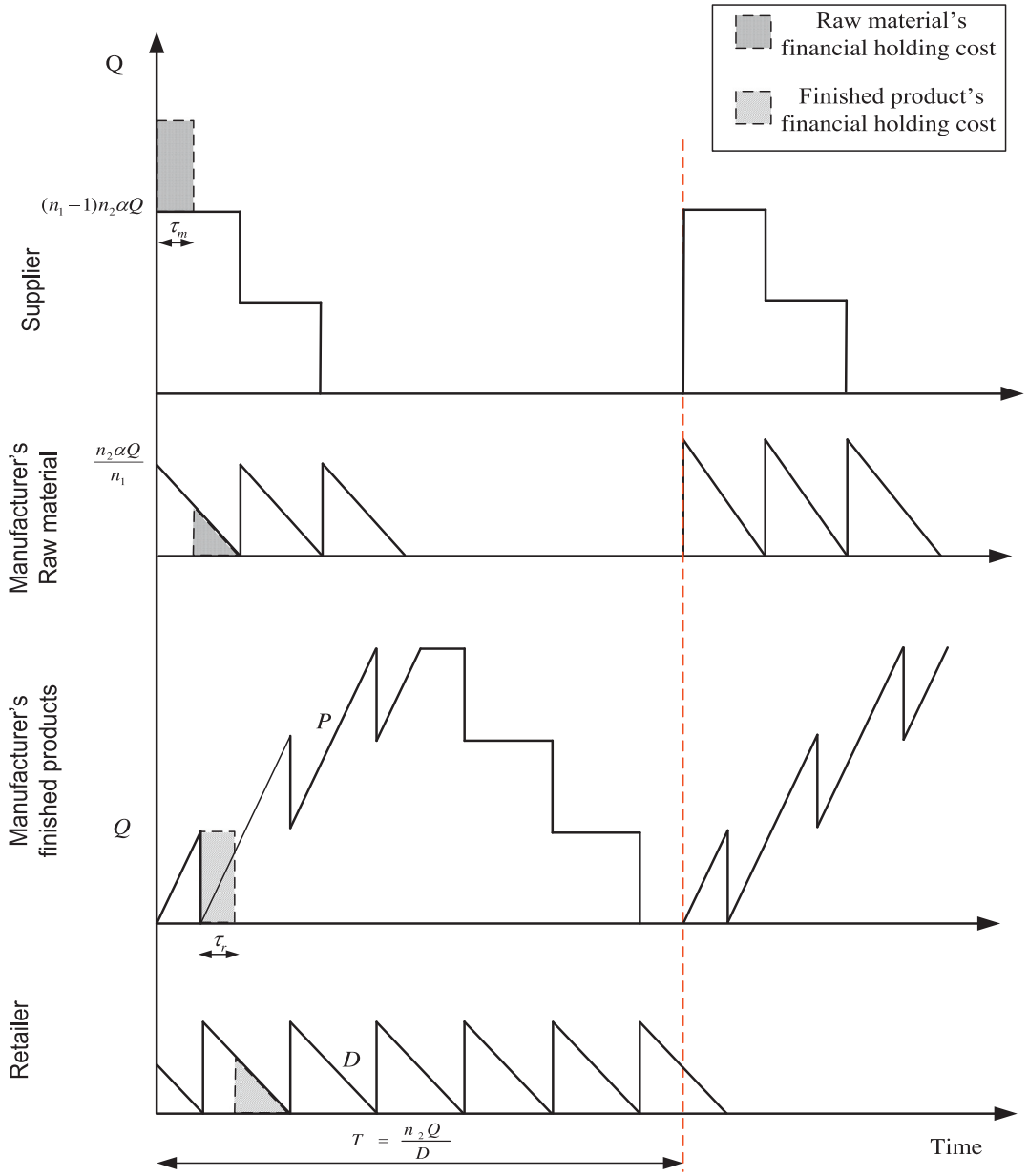

Fig. 1. Inventory behavior for a three-level (supplier–manufacturer–retailer) supply chain.

payments $((C_{m,w} - d_s - C_s)\alpha f(d_r)(e^{k_s t_s} - 1))$. Thus, the annual cost for the supplier is as follows:

$$\gamma_s(t_s,\ \tau_{m,w}, n_1, n_2, d_r) = \frac{A_s\ f(d_r)}{n_2 Q} + C_s\alpha f(d_r) + \frac{(n_1-1)}{2}(h_s + s_s)\frac{\alpha n_2 Q f(d_r)}{P n_1}, \\ + h_s\tau_{m,w}\alpha f(d_r) + (C_{m,w} - d_s - C_s)\alpha f(d_r)\left(e^{k_s t_s} - 1\right) \quad (2)$$

, where $e^{k_s t_s} - 1 \approx k_s t_s$ when $k_s \ll 0$.

As a result, the annual profit for the supplier (φ_s) is equal to Eq. (1) minus Eq. (2)

$$\varphi_s(t_s,\ \tau_{m,w}, n_1, n_2, d_s, d_r) = \psi_s(t_s,\ \tau_{m,w}, d_s, d_r) - \gamma_s(t_s,\ \tau_{m,w}, n_1, n_2, d_r). \quad (3)$$

Please refer to Appendix B for the full expression of Eq. (3).

In addition, the annual revenue for the manufacturer (ψ_m) includes the revenue from selling the finished products to the retailer $((C_r - d_m)\, f(d_r))$, interest charges paid by the retailer to the manufacturer if the retailer settles its balance with the manufacturer beyond the permitted period to delay settling the balance $((C_r - d_m)f(d_r)(e^{k_m(\tau_r - t_m)} - 1))$ and the interest

earned during the permissible delay period offered by the supplier ($(C_{m,w}-d_s)\alpha f(d_r)(e^{k_m\tau_{m,w}}-1)$). The annual revenue for the manufacturer can be written as follows:

$$\psi_m(t_m,\ \tau_{m,w},\ \tau_r, d_s,\ d_m, d_r) = (C_r-d_m)f(d_r) + (C_r-d_m)f(d_r)\left(e^{k_m(\tau_r-t_m)}-1\right), \\ +\ (C_{m,w}-d_s)\alpha f(d_r)\left(e^{k_m\tau_{m,w}}-1\right) \quad (4)$$

,where $e^{k_m(\tau_r-t_m)}-1\approx k_m(\tau_r-t_m)$ and $e^{k_m\tau_{m,w}}-1\approx k_m\tau_{m,w}$ when $k_s \ll 0$. Moreover, the annual cost for the manufacturer (γ_m) includes the order cost of raw materials ($\frac{n_1A_{m,w}f(d_r)}{n_2Q}$), purchasing cost of raw materials ($(C_{m,w}-d_s)\alpha f(d_r)$), storage holding cost for the raw materials in inventory ($\frac{S_{m,w}\alpha n_2Qf(d_r)}{2Pn_1}$), financial holding costs of raw materials in inventory ($h_{m,w}(Q,\ t_s, \tau_{m,w})$), interest charges if the manufacturer settles its balance with the supplier past the permissible delay period offered by the supplier ($(C_{m,w}-d_s)\alpha f(d_r)(e^{ks(\tau_{m,w}-t_s)}-1)$), setup cost of finished products ($\frac{A_mf(d_r)}{n_2Q}$), production cost of finished products ($C_m\ f(d_r)$), financial and storage holding costs for finished goods in the inventory ($(h_m+S_m)[\frac{Q(2f(d_r)+(P-f(d_r))n_2-P)}{2P}]$), financial holding costs due to offering a permissible delay for the retailer ($h_m\tau_rf(d_r)$) and the opportunity cost of investing the profit of the manufacturer during the permissible delay period offered to the retailer ($(C_r-d_m-C_m)f(d_r)(e^{k_mt_m}-1)$). Thus, the annual cost for the manufacturer is as follows:

$$\gamma_m(Q,t_s,\ \tau_{m,w},n_1,n_2,d_s,d_m,d_r) = \frac{(n_1A_{m,w}+A_m)f(d_r)}{n_2Q} + (C_{m,w}-d_s)\alpha f(d_r) \\ +\ \frac{S_{m,w}\alpha n_2Qf(d_r)}{2Pn_1} + h_{m,w}\ (Q,t_s,\ \tau_{m,w}) + (C_{m,w}-d_s)\alpha f(d_r)\left(e^{ks(\tau_{m,w}-t_s)}-1\right) \\ +\ C_mf(d_r) + (h_m+S_m)\left[\frac{Q(2f(d_r)+(P-f(d_r))n_2-P)}{2P}\right] \\ +\ h_m\tau_rf(d_r) + (C_r-d_m-C_m),\quad f(d_r)\left(e^{k_mt_m}-1\right) \quad (5)$$

where $e^{ks(\tau_{m,w}-t_s)}-1\approx ks(\tau_{m,w}-t_s)$ and $e^{k_mt_m}-1\approx k_mt_m$ when ks and $k_m<<0$. These conditions also apply to Eq. (6) below.

Accordingly, the annual profit for the manufacturer (φ_m) is equal to Eq. (4) minus Eq. (5)

$$\varphi_m(Q,t_s,t_m,\tau_{m,w},\tau_r,n_1,n_2,d_m,d_r,d_s) = \psi_m(t_m,\ \tau_{m,w},\ \tau_r,d_s,\ d_m,d_r) - \gamma_m(Q,t_s,\ \tau_{m,w},n_1,n_2,d_s,d_m,d_r) \quad (6)$$

Please refer to Appendix C for the full expression of Eq. (6)

The annual revenue for the retailer (ψ_r) includes selling finished goods ($(C_c-d_r)f(d_r)$), interest earned during the permissible delay period offered by the manufacturer ($(C_r-d_m)f(d_r)(e^{k_rt_m}-1)$) and interest earned from investing the revenue of selling finished goods. The annual revenue for the retailer is as follows:

$$\psi_r(Q,t_m,d_r,d_m) = (C_c-d_r)f(d_r) + (C_r-d_m)f(d_r)\left(e^{k_rt_m}-1\right) + (C_c-d_r)\frac{Q}{2}\left(e^{k_r\left(\frac{Q}{f(d_r)}\right)}-1\right), \quad (7)$$

where $e^{k_mt_m}-1\approx k_mt_m$ when $k_m<<0$.

The annual costs for the retailer (γ_r) includes the order cost ($\frac{A_rf(d_r)}{Q}$), the purchasing cost of finished goods ($(C_r-d_m)\ f(d_r)$), financial holding cost of finished goods ($h_r(Q,t_m,\tau_r)$), storage holding cost of finished goods ($\frac{S_rQ}{2}$) and interest charges t paid to the manufacturer if the retailer settles its balance past the permissible delay period. ($(C_r-d_m)f(d_r)(e^{k_m(\tau_r-t_m)}-1)$). Thus, the annual cost for the retailer is written as follows:

$$\gamma_r(Q,t_m,\tau_r) = \frac{A_rf(d_r)}{Q} + (C_r-d_m)f(d_r) + h_r(Q,t_m,\ \tau_r) + \frac{S_rQ}{2} + (C_r-d_m)f(d_r)\left(e^{k_m(\tau_r-t_m)}-1\right), \quad (8)$$

where $e^{k_mt_m}-1\approx k_mt_m$ when $k_m<<0$.

The annual profit for the retailer (φ_r) is equal to Eq. (7) minus Eq. (8), which is

$$\varphi_r(Q,t_m,\tau_r,d_r,d_m) = \psi_r(Q,t_m,d_r,d_m) - \gamma_r(Q,t_m,\tau_r). \quad (9)$$

Please refer to Appendix D for the full expression of Eq. (9)

As a result, the annual profit for the whole supply chain system (φ_{system}) is equal to Eq. (3) plus Eq. (6) plus Eq. (9)

$$\varphi_{system}(Q,t_s,t_m,\tau_{m,w},\tau_r,n_1,n_2,d_s,d_m,d_r) = \varphi_s(t_s,\ \tau_{m,w},n_1,n_2,d_s,d_r) + \varphi_m(Q,t_s,t_m,\tau_{m,w},\tau_r,n_1,n_2,d_m,d_r,d_s) \\ +\ \varphi_r(Q,t_m,\tau_r,d_r,d_m).$$

The objective is to maximize the profit of the supply chain system $\varphi_{system}(Q,t_s,t_m,\tau_{m,w},\ \tau_r,n_1,n_2,d_s,d_m,d_r)$ subject to:Q, $n_1,n_2 \geq 1$ and integers

$$t_s, \tau_{m,w}, t_m, \tau_r \geq 0$$
$$d_s, d_m, d_r \geq 0$$
$$d_s \leq c_m - c_s,\quad d_m \leq c_r - c_m,\ d_r \leq c_c - c_r$$

Table 1
Holding costs for each delay in payments' scenario.

Delay in payments' scenario	$h_{m,w}(Q, t_s, \tau_{m,w})$	$h_r(Q, t_m, \tau_r)$
I-I $0 \le t_s = \tau_m \le \frac{n_2 Q}{P n_1}$ & $0 \le t_m = \tau_r \le \frac{Q}{D}$	$h_{m,w}\frac{f(d_r)}{n_2 Q}\frac{(n_2\alpha Q - \alpha P t_s n_1)^2}{2\alpha P n_1}$	$h_r\frac{(Q - f(d_r)t_m)^2}{2Q}$
I-II $0 \le t_s = \tau_m \le \frac{n_2 Q}{P n_1}$ & $0 \le t_m < \tau_r \le \frac{Q}{D}$	$h_{m,w}\frac{f(d_r)}{n_2 Q}\frac{(n_2\alpha Q - \alpha P t_s n_1)^2}{2\alpha P n_1}$	$h_r\frac{(Q - f(d_r)\tau_r)^2}{2Q}$
I-III $0 \le t_s = \tau_m \le \frac{n_2 Q}{P n_1}$ & $0 \le t_m < \frac{Q}{D} \le \tau_r$	$h_{m,w}\frac{f(d_r)}{n_2 Q}\frac{(n_2\alpha Q - \alpha P t_s n_1)^2}{2\alpha P n_1}$	0
II-I $0 \le t_s < \tau_m \le \frac{n_2 Q}{P n_1}$ & $0 \le t_m = \tau_r \le \frac{Q}{D}$	$h_{m,w}\frac{f(d_r)}{n_2 Q}\frac{(n_2\alpha Q - \alpha P \tau_{m,w} n_1)^2}{2\alpha P n_1}$	$h_r\frac{(Q - f(d_r)t_m)^2}{2Q}$
II-II $0 \le t_s < \tau_{m,w} \le \frac{n_2 Q}{P n_1}$ & $0 \le t_m < \tau_r \le \frac{Q}{D}$	$h_{m,w}\frac{f(d_r)}{n_2 Q}\frac{(n_2\alpha Q - \alpha P \tau_{m,w} n_1)^2}{2\alpha P n_1}$	$h_r\frac{(Q - f(d_r)\tau_r)^2}{2Q}$
II-III $0 \le t_s < \tau_m \le \frac{n_2 Q}{P n_1}$ & $0 \le t_m \le \frac{Q}{D} \le \tau_r$	$h_{m,w}\frac{f(d_r)}{n_2 Q}\frac{(n_2\alpha Q - \alpha P \tau_{m,w} n_1)^2}{2\alpha P n_1}$	0
III-I $0 \le t_s \le \frac{n_2 Q}{P n_1} \le \tau_m$ & $0 \le t_m = \tau_r \le \frac{Q}{D}$	0	$h_r\frac{(Q - f(d_r)t_m)^2}{2Q}$
III-II $0 \le t_s \le \frac{n_2 Q}{P n_1} \le \tau_m$ & $0 \le t_m < \tau_r \le T_r$	0	$h_r\frac{(Q - f(d_r)\tau_r)^2}{2Q}$
III-III $0 \le t_s \le \frac{n_2 Q}{P n_1} \le \tau_m$ & $0 \le t_m \le \frac{Q}{D} \le \tau_r$	0	0

Table 1 illustrates the different calculations for the holding costs of the manufacturer's raw materials and the retailer's inventory based on each delay in payments' scenario (refer to Appendix A for a detailed explanation about the scenarios of delay in payments).

In the case of settling the balance after receiving the next shipment in a single payment, the inventory of the player will be zero. Therefore, some of financial holding costs illustrated in Table 1 are zeros.

Lemma 1. Under the condition $(h_{m,w}+s_{m,w}) > (h_s+s_s)$, the profit function is concave in n_1. The optimal value for n_1 that maximizes the expected annual profit of the supply chain is

$$n_1 = n_2 Q \sqrt{\frac{\frac{S_{m,w}\alpha}{P} + \frac{h_{m,w}\alpha}{P}}{2A_{m,w} + h_{m,w}(\tau_{m,w})^2 \alpha P}}.$$

Refer to Appendix E for the proof.

Lemma 2. The profit function is concave in n_2. The optimal value for n_2 that maximizes the expected annual profit of the supply chain is

$$n_2 = \frac{1}{Q}\sqrt{\frac{2(n_1 A_{m,w} + A_m) + h_{m,w}\alpha P n_1 (\tau_{m,w})^2}{\frac{S_{m,w}\alpha}{P n_1} + h_{m,w}\frac{\alpha}{P n_1} + (h_m + S_m)\left(\frac{1}{f(d_r)} - \frac{1}{P}\right)}}.$$

Refer to Appendix F for the proof.

It was difficult to obtain a closed form expression for Q. Therefore, a numerical search is necessary to obtain Q. An excel solver enhanced with VBA codes was used for this purpose.

4. Numerical results

In this section, we first solve a numerical example for a given set of parameter values for the nine scenarios of delay in payments presented in Section 3 and investigate them for eight cases of price discounts. Then, we performed a sensitivity analysis to analyze all models developed in this paper when varying certain parameters.

Table 2
Results of the numerical example.

	(d_s,d_m,d_r)	t_s	τ_m	n_1	d_s	t_m	τ_r	n_2	d_m	Q	d_r	Total profit ($)
No delay	**(0,0,0)**	0	0	1	0	0	0	2	0	203	0.00	21,039.88
	(0,0,1)	0	0	1	0	0	0	3	0	181	5.20	22,769.67
	(0,1,0)	0	0	1	0	0	0	2	20	203	0.00	21,039.88
	(0,1,1)	0	0	1	0	0	0	3	13	181	5.20	22,769.67
	(1,0,0)	0	0	1	8	0	0	2	0	215	0.00	21,409.54
	(1,0,1)	0	0	1	8	0	0	3	0	181	5.20	22,769.67
	(1,1,0)	0	0	1	8	0	0	2	20	203	0.00	21,039.88
	(1,1,1)	0	0	1	8	0	0	3	20	181	5.20	22,769.67
I-I	**(0,0,0)**	33	33	1	0	36	36	2	0	214	0.00	21,394.52
	(0,0,1)	37	37	1	0	31	31	2	0	244	5.18	23,213.38
	(0,1,0)	33	33	1	0	42	42	2	20	217	0.00	21,459.10
	(0,1,1)	38	38	1	0	35	35	2	20	247	5.19	23,286.91
	(1,0,0)	35	35	1	8	37	37	2	0	215	0.00	21,409.54
	(1,0,1)	40	40	1	8	31	31	2	0	245	5.19	23,236.77
	(1,1,0)	35	35	1	8	42	42	2	20	217	0.00	21,474.28
	(1,1,1)	40	40	1	8	35	35	2	20	248	5.21	23,310.53
I-II	**(0,0,0)**	34	34	1	0	0	50	2	0	221	0.00	21,584.88
	(0,0,1)	39	39	1	0	0	42	2	0	252	5.21	23,430.06
	(0,1,0)	34	34	1	0	0	50	2	0	221	0.00	21,584.88
	(0,1,1)	39	39	1	0	0	42	2	0	252	5.21	23,430.06
	(1,0,0)	36	36	1	8	0	50	2	0	221	0.00	21,600.34
	(1,0,1)	41	41	1	8	0	42	2	0	252	5.22	23,454.17*
	(1,1,0)	36	36	1	8	0	50	2	0	221	0.00	21,600.34
	(1,1,1)	41	41	1	8	0	42	2	0	252	5.22	23,454.17*
I-III	**(0,0,0)**	33	33	1	0	0	80	2	0	215	0.00	21,409.37
	(0,0,1)	38	38	1	0	0	67	2	0	246	5.18	23,226.93
	(0,1,0)	33	33	1	0	0	80	2	0	215	0.00	21,409.37
	(0,1,1)	36	36	1	0	0	83	2	0	231	0.29	21,603.66
	(1,0,0)	35	35	1	8	0	80	2	0	216	0.00	21,424.47
	(1,0,1)	40	40	1	8	0	67	2	0	246	5.19	23,250.41
	(1,1,0)	35	35	1	8	0	80	2	0	216	0.00	21,424.47
	(1,1,1)	40	40	1	8	0	67	2	0	246	5.19	23,250.41
II-I	**(0,0,0)**	0	44	1	0	37	37	2	0	217	0.00	21,478.68
	(0,0,1)	0	59	1	0	24	24	3	0	195	5.43	23,349.92
	(0,1,0)	0	44	1	0	42	42	2	20	219	0.00	21,544.13
	(0,1,1)	0	51	1	0	35	35	2	20	252	5.27	23,419.85
	(1,0,0)	0	44	1	0	37	37	2	0	217	0.00	21,478.68
	(1,0,1)	0	59	1	0	24	24	3	0	195	5.43	23,349.92
	(1,1,0)	0	44	1	0	42	42	2	20	219	0.00	21,544.13
	(1,1,1)	0	51	1	0	35	35	2	20	252	5.27	23,419.85

Our objective in the numerical analysis is to maximize the supply chain annual profit. The decision variables are the lengths of the permissible delay in payment (t_s and t_m), the extended (τ_m and τ_r) delay periods, and the price discounts (d_s,d_m,d_r). Now, we explain the notation used in differentiating the eight cases of price discounts. For example, the case denoted by (**0,0,1**) indicates that the supplier does not offer (0) the manufacturer a price discount, the manufacturer does not offer (0) the retailer a price discount, and the retailer offers price discounts (1) to customers, while (**1,1,1**) means that each player (the supplier, the manufacturer, or the retailer) offers price discounts to its adjacent downstream player (a customer is a downstream player for a retailer). Accordingly, 0 means price discounts is offered, and 1 otherwise. The nine scenarios and the eight cases result in a combination of seventy-two combinations of delay in payments and price discounts. These combinations help in bringing some managerial insights that could guide the actions of decision makers in a supply chain of a similar structure.

First, we show one numerical example that has been setup based on surveying couple of papers that investigated the coordination of a three-level supply chain e.g. Jaber et al. [22], Moussawi-Haidar et al. [10], and Aljazzar et al. [29]. The parameters of the numerical example are as follows: D_o= 1000 units/year, D_1= 70, P= 3000 unit/year, α= 1, As= 264 $/order, Cs= 20 $/unit, h_s= 3 $/unit/year, s_s= 2 $/unit/year, k_s= 10%, A_w= 355 $/order, C_w= 28 $/unit, h_w= 4.2 $/unit/year, s_w= 2.8 $/unit/year, A_m= 125 $/order, C_m= 50 $/unit, h_m= 7.5 $/unit/year, s_m= 5 $/unit/year, k_m= 8%, A_r= 242 $/order, C_r= 70 $/unit, h_r= 10.5 $/unit/year, s_r= 7 $/unit/year, k_r= 5%, C_c= 98$/unit. Table 2 illustrates the results of the numerical example under all scenarios and price discount combinations. Solver (add-on Microsoft Excel) was utilized to maximize the supply chain profit of the proposed supply chain by setting n_1, n_2, t_s, τ_w, d_s, t_m, τ_r, d_m and d_r as decision variables and finding their optimal values. The optimal values for $n_1$and n_2 that were found by Excel Solver were very close to the values calculated using the closed-forms of n_1 and n_2 on page 11, i.e. in one of the numerical examples the optimal value for n_1 that was obtained by Solver was 1 and the calculated value (closed form) was 1.02. In addition, the optimal value obtained from

Table 3
(cont'd): Results of the numerical example.

	(d_s,d_m,d_r)	t_s	τ_m	n_1	d_s	t_m	τ_r	n_2	d_m	Q	d_r	Total profit ($)
II-II	**(0,0,0)**	0	44	1	0	36	37	2	0	217	0.00	21,472.97
	(0,0,1)	0	59	1	0	23	24	3	0	195	5.43	23,342.04
	(0,1,0)	0	44	1	0	41	42	2	20	219	0.00	21,536.58
	(0,1,1)	0	51	1	0	34	35	2	20	252	5.26	23,409.54
	(1,0,0)	0	44	1	0	36	37	2	0	217	0.00	21,472.97
	(1,0,1)	0	59	1	0	23	24	3	0	195	5.43	23,342.04
	(1,1,0)	34	35	1	8	41	42	2	20	217	0.00	21,466.74
	(1,1,1)	0	59	1	0	23	24	3	0.03	195	5.43	23,342.11
II-III	**(0,0,0)**	0	42	1	0	74	77	2	0	207	0.00	21,134.66
	(0,0,1)	0	57	1	0	49	51	3	0	188	5.37	23,025.42
	(0,1,0)	0	42	1	0	76	78	2	20	211	0.00	21,259.96
	(0,1,1)	0	58	1	0	50	52	3	20	191	5.39	23,137.57
	(1,0,0)	0	42	1	0	74	77	2	0	207	0.00	21,134.66
	(1,0,1)	0	57	1	0	49	51	3	0	188	5.37	23,025.42
	(1,1,0)	0	42	1	0	76	78	2	20	211	0.00	21,259.96
	(1,1,1)	0	58	1	0	50	52	3	20	191	5.39	23,137.57
III-I	**(0,0,0)**	0	54	1	0	37	37	2	0	217	0.00	21,467.14
	(0,0,1)	0	72	1	0	24	24	3	0	195	5.43	23,329.95
	(0,1,0)	0	54	1	0	42	42	2	20	219	0.00	21,532.51
	(0,1,1)	0	62	1	0	35	35	2	20	251	5.26	23,402.22
	(1,0,0)	0	54	1	0	37	37	2	0	217	0.00	21,467.14
	(1,0,1)	0	72	1	0	24	24	3	0	195	5.43	23,329.95
	(1,1,0)	0	54	1	0	42	42	2	20	219	0.00	21,532.51
	(1,1,1)	0	62	1	0	35	35	2	20	251	5.26	23,402.22
III-II	**(0,0,0)**	0	54	1	0	36	37	2	0	217	0.00	21,461.43
	(0,0,1)	0	72	1	0	23	24	3	0	195	5.43	23,322.07
	(0,1,0)	0	54	1	0	41	42	2	20	219	0.00	21,524.97
	(0,1,1)	0	72	1	0	23	24	3	0.06	195	5.43	23,322.22
	(1,0,0)	0	54	1	0	36	37	2	0	217	0.00	21,461.43
	(1,0,1)	0	72	1	0	23	24	3	0	195	5.43	23,322.07
	(1,1,0)	0	54	1	0	41	42	2	20	219	0.00	21,524.97
	(1,1,1)	0	72	1	0	23	24	3	0.06	195	5.43	23,322.22
III-III	**(0,0,0)**	0	51	1	0	74	77	2	0	207	0	21,123.52
	(0,0,1)	0	70	1	0	49	51	3	0	188	5.36	23,006.05
	(0,1,0)	0	52	1	0	76	78	2	20	211	0	21,248.67
	(0,1,1)	0	70	1	0	49	51	3	0.10	188	5.36	23,006.61
	(1,0,0)	0	51	1	0	74	77	2	0	207	0	21,123.52
	(1,0,1)	0	70	1	0	49	51	3	0	188	5.36	23,006.05
	(1,1,0)	0	52	1	0	76	78	2	20	211	0	21,248.67
	(1,1,1)	0	70	1	0	49	51	3	0.10	188	5.36	23,006.61

Note: Asterisk (*) indicates the optimal values, the values of delay in payments are in days

Table 4
Results of the intendent decision's numerical example.

	(d_s,d_m,d_r)	t_s	τ_m	n_1	d_s	t_m	τ_r	n_2	d_m	Q	d_r	Total profit ($)
I-II	**(1,1,0)**	36	36	1	8	0	50	2	0	574	0	19,472.62

Solver for n_2 was 3 and the calculated value was 2.5. Tables 2 and 3 illustrate the optimal results of the numerical example for all scenarios.

Adopting a centralized decision process would increase the profit for the supply chain as compared to an independent decision making process. For illustration purposes, we solved the same numerical example for an independent (decentralized) decision process by assuming the manufacturer to be the supply chain leader. The results are reported Table 4 and indicate that a centralized decision policy is more profitable for the supply chain than a decentralized one.

The results show that most of the delay in payments scenarios generated maximum profit when the manufacturer and the retailer, but not the supplier, offer price discounts (**0,1,1**). However, the maximum profit for the supply chain is attained when the manufacturer settles its balance with the supplier by the end of the permissible delay period, and the retailer settles its balance with the manufacturer past the permissible delay period and before receiving the next shipment (**I-II**) where, only the supplier and the retaielr offer a price discount (**1,0,1**). The results also show that offering a mixed trade credit of delay in payment and price discounts increases the profit of the supply chain. It is noticeable that the optimal solution for many cases is not to offer a price discount e.g. the supply chain profit of scenario I-II under case (**0,0,0**) is the same as scenario I-II case (**0,1,0**). The reason is that the optimal solution in scenario I-II for the case (**0,1,0**) suggests not to offer price discounts to the retailer, which produces an identical result to case (**0,0,0**). Tables 5 and 6 rank the solutions for

Table 5
Best discounts' cases for each delay scenario.

Delay scenario	No delay	I-I	I-II	I-III	II-I
discount's case	(1/0,1/0,1)	(1,1,1)	(1,1/0,1)	(1,1/0,1)	(1/0,1,1)
Total profit	22,769.67	23,310.53	23,454.17	23,250.41	23,419.85
Rank	10	6	1	7	2

Table 6
(cont'd): Best discounts' cases for each delay scenario.

Delay scenario	II-II	II-III	III-I	III-II	III-III
discount's case	(1/0,1,1)	(1/0,1,1)	(1/0,1,1)	(1/0,1,1)	(1/0,1,1)
Total profit	23,409.54	23,137.57	23,402.22	23,322.22	23,006.61
Rank	3	8	4	5	9

Table 7
Illustration of the interest rates' combinations.

No.	Combinations of interest rates	k_s	k_m	k_r
1	$k_r = k_m > k_s$	5	10	10
2	$k_r = k_s > k_m$	10	5	10
3	$k_m > k_r > k_s$	5	10	8
4	$k_m > k_s > k_r$	8	10	5
5	$k_r > k_s > k_m$	8	5	10
6	$k_s > k_m = k_r$	10	5	5
7	$k_r > k_m = k_s$	5	5	10
8	$k_r > k_m > k_s$	5	8	10
9	$k_s > k_m > k_r$	10	8	5
10	$k_s > k_r > k_m$	10	5	8
11	$k_s > k_r = k_m$	10	5	5
12	$k_m = k_s > k_r$	10	10	5
13	$k_s = k_m = k_r$	8	8	8

different combinations of delay in payments and price discounts from the highest to lowest profit. The results show that case (**1,1,1**), offering delay in payments and price discounts, is dominant.

Next, we perform sensitivity analysis to investigate the behavior of the developed models for changes in the values of the critical independent variables, i.e., elasticity of the demand function to a discount in price, D_1, and the interest rates for the players, k_s, k_m, and k_r. First, D_1 is varied from 0 to 100 with an increment of one. All the scenarios of delay in payments are insensitive to D_1 when in the range 0–40. Scenario I-I attained its maximum the supply chain profit when the supplier and the manufacturer offer price discounts; i.e. case (**1,1,0**). Scenarios I-II and I-III attained their maximum profit when the supplier offers price discounts to the manufacturer for the cases (**1,0,0**), (**1,0,1**), (**1,1,0**) and (**1,1,1**). The remaining scenarios of delay in payments attained their maximum profit when the manufacturer offers the retailer price discounts, i.e. (**0,1,0**), (**0,1,1**), (**1,1,0**) and (**1,1,1**). Incurring holding costs and interest charges for longer periods, as in scenario III-III, prevents the system from offering a discount to maximize the supply chain profit. The maximum supply chain profit is attained when the elasticity is between 0 and 40, in which case the profit reported was $ 21,600.34 under scenario I-II and a discount was offered by the supplier to the manufacturer. However, all cases are sensitive to D_1 when in the range 50–100, where the supply chain profit increases as D_1 increases. The highest system profit, $ 27,100.13, is attained for scenario I-II when $D_1 = 100$ for the cases (**1,0,1**) and (**1,1,1**).

Now, we investigate the behavior of the model by varying the interest rates among the three players. Thirteen possible combinations, shown in Table 7, have been considered for the 72 combined cases of delay in payments and price discounts.

We observe that when the retailer's interest rate is equal to the manufacturer's interest rate but greater than the supplier's interest rate ($\boldsymbol{k_r = k_m > k_s}$), all the combined cases of delay in payments and price discounts, i.e. (**1,1,1**), have the highest supply chain profit when compared to the other interest combinations. However, the cases of delay in payments and no price discounts (**0,0,0**) have their highest profits when the interest rate of the retailer is higher than the interest rate of the manufacturer, and that of the manufacturer is higher than that of the supplier ($\boldsymbol{k_r > k_m > k_s}$). The highest profit of all cases occurs when the manufacturer settles its balance with the supplier by the end of the permissible delay period, the retailer settles its balance with the manufacturer past the permissible delay period but before receiving the next shipment, and the retailer offers price discounts to customers; i.e., scenario **I-II** and case (**0,0,1**) with a profit of 24,399.42 $/year. Although scenario I-II for case (**0,0,1**) attained the highest profit, the difference in profits with scenario I-II for case (**1,0,1**) is 0.01%. The worst supply chain profit was attained when the manufacturer settles its balance past the permissible delay period, after receiving the next shipment and when price discounts is not offered along the supply chain, i.e., (**0,0,0**) with a profit of 21,081.78 $/year. Table 8 shows selected cases of best and worst profits for the nine scenarios.

The following recommendations are based on the numerical and sensitivity analysis performed in the previous section.

Table 8
Best and worst total profit.

No.	Combination of interest rates	Scenario of delay in payments	Price discount's case	Total profit ($)
Best total profit				
1	$k_r = k_m > k_s$	I-II	(0,0,1)	24,399.42
2	$k_r = k_m > k_s$	I-III	(0,0,1)	24,396.25
3	$k_r = k_s > k_m$	I-II	(1,0,1)	24,332.14
4	$k_r = k_s > k_m$	I-III	(1,0,1)	24,328.97
Worst total profit				
5	$k_m > k_s > k_r$	II-III	(0,0,0)	21,121.29
6	$k_m > k_s > k_r$	III-III	(0,0,0)	21,120.38
7	$k_m = k_s > k_r$	II-III	(0,0,0)	21,097.14
8	$k_m = k_s > k_r$	III-III	(0,0,0)	21,081.78

- We numerically show that offering delay in payments in conjunctions with price discounts entices coordination and increases the profit of the supply chain more than offering only one of the two coordination mechanisms. This creates higher savings for the supply chain members that could be used to compensate the players who are worse off from coordination (usually the player that will operate away from its optimal policy), to invest in programs that improve supply chain performance, and to have some additional profits.
- Scenario I-II attains the highest system profit for different cases of price discounts. In this scenario, the manufacturer settles its balance with the supplier by the end of the permissible delay period and the retailer settles its balance with the manufacturer past the permissible delay period but before receiving the next shipment
- Since customer demand is the driver of the supply chain, offering a price discount by the retailer will always increase the chances of achieving the maximum profit possible for the system. It also increases the retailer's, and subsequently the manufacturer's and supplier's, market share.
- Although the supply chain is making a profit, some players in a few cases were losing money. In this case, it would be fair to have profit sharing agreements among the players, whether it is based on the investment that each player puts in the chain (ROI) or by compensating the losing player from the supply chain profit and then sharing the remaining profit.

5. Summary and conclusions

This paper presents a model for integrating two coordination mechanisms to coordinate a three-level-supply chain (supplier–manufacturer–retailer). The coordination mechanisms, price discounts and delay in payments, were treated as decision variables and were implemented between all players in the supply chain. The paper investigates nine possible scenarios of delay in payments along with eight cases of a price discount, representing all cases that might occur in practice.

A numerical example was solved to compare combinations of all scenarios and cases studied in this paper. The numerical example was selected based on surveying several papers that investigated the coordination of a three-level supply chain. The best scenario was I-II, in which the manufacturer settles its balance with the supplier by the end of the permissible delay period and the retailer settles its balance with the manufacturer past the permissible delay period but before receiving the next shipment, for the discount case (**0,0,1**), i.e. the case in which the retailer is the only player in the supply chain offering price discounts to its customers. Adopting this scenario increases the system profit by about 10.3%.

A sensitivity analysis was performed to identify the limitations of the models developed in this paper for varying the elasticity of the demand function and for the interest rates of the players. Scenario I-II for case (**1,0,1**), the supplier and the retailer offer price discounts, attained the highest profit of all combinations. The results also showed that for a range of values of the interest rates varying the interest rates, scenario I-II for case (**0,0,1**) attained the highest supply chain profit.

In summary, we conclude that offering period of permissible delay in payments in conjunction with price discounts increase the supply chain profit more than implementing each mechanism separately. We found that the best scenario to follow when coordinating a three-level of a supply chain with permissible delay in payments and price discount is I-II for (**1,0,1**). It showed to be the most profitable and flexible for the ranges of values for demand elasticity and the interest rates.

One could extend the models developed in this paper in several directions. One study could focus on comparing the results when the delay in payments is fixed and imposed upon the downstream player by an upstream one, as it is probably a common business practice, with the results of this paper. The results from such a study may guide managers when jointly deciding on inventory, pricing, and incentives in a supply chain. Assuming stochastic demand in the presence of multiple players at the three supply chain levels is another interesting and challenging extension. This model can also be extended by considering a dual-channel strategy with or without accounting for reverse logistics [30,31]

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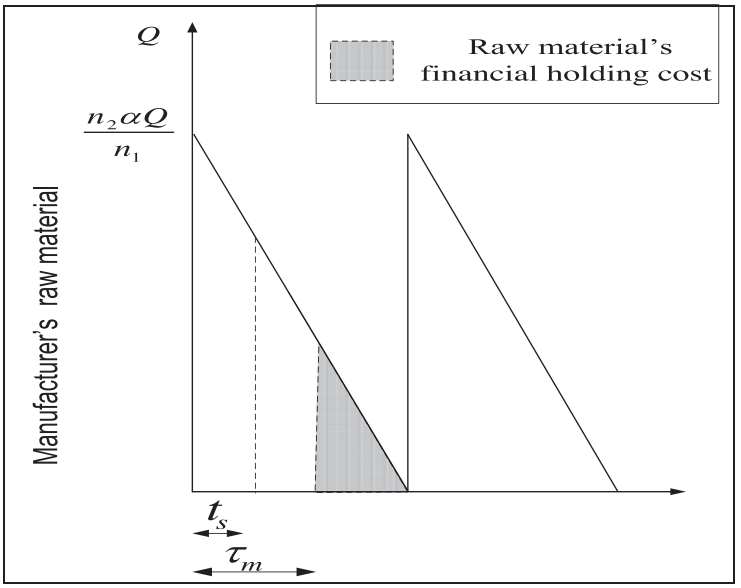

Fig. A1. Illustration of the effect of different delay in payments' scenario.

Appendix A

Fig. A1 illustrates the behavior of the raw material inventory at the manufacturer's side. The figure presents the following scenarios II-I, II-II, II-III, where the manufacturer's settles its balance with the supplier beyond the permitted period (t_s) and before receiving the next shipment, which can be written mathematically as follows: $0 \le \boldsymbol{t_s} < \boldsymbol{\tau_m} \le \frac{\boldsymbol{n_2 Q}}{\boldsymbol{P n_1}}$. However, this scenario does not show the real practice, i.e. the manufacturer settles its balance by the end of the permissible delay period ($t_s = \tau_m$) or the manufacturer settles its balance beyond the permissible delay period and after receiving the next shipment ($0 \le \boldsymbol{t_s} \le \frac{\boldsymbol{n_2 Q}}{\boldsymbol{P n_1}} \le \boldsymbol{\tau_m}$). In any of the previous scenarios, the manufacturer will incur the financial holding cost for the raw materials after the full payment is made. Similarly, this paper considered different delay in payments' scenarios between the manufacturer and the retailer.

Appendix B

$$\begin{aligned}\varphi_s(t_s, \tau_{m,w}, n_1, n_2, d_s, d_r) = {} & \left((C_{m,w} - d_s)\alpha f(d_r) + (C_{m,w} - d_s)\alpha f(d_r)\left(e^{k_s(\tau_{m,w} - t_s)} - 1\right)\right) \\ & - \left(\frac{A_s\ f(d_r)}{n_2 Q} + C_s \alpha f(d_r) + \frac{(n_1 - 1)}{2}(h_s + s_s)\frac{\alpha n_2 Q f(d_r)}{P n_1} \right. \\ & \left. + h_s \tau_{m,w} \alpha f(d_r) + (C_{m,w} - d_s - C_s)\alpha f(d_r)\left(e^{k_s t_s} - 1\right) \right) \end{aligned} \tag{3}$$

where $e^{k_s(\tau_{m,w} - t_s)} - 1 \approx k_s(\tau_{m,w} - t_s)$ and $e^{k_s t_s} - 1 \approx k_s t_s$ when $k_s \ll 0$.

Appendix C

$$\begin{aligned}&\varphi_m(Q, t_s, t_m, \tau_{m,w}, \tau_r, n_1, n_2, d_m, d_r, d_s) \\ &= \left((C_r - d_m) f(d_r) + (C_r - d_m) f(d_r)\left(e^{k_m(\tau_r - t_m)} - 1\right) + (C_{m,w} - d_s)\alpha f(d_r)\left(e^{k_m \tau_{m,w}} - 1\right)\right) \\ &\quad - \left(\frac{(n_1 A_{m,w} + A_m) f(d_r)}{n_2 Q} + (C_{m,w} - d_s)\alpha f(d_r) + \frac{S_{m,w}\alpha n_2 Q f(d_r)}{2 P n_1} + h_{m,w}(Q, t_s, \tau_{m,w}) \right. \\ &\quad + (C_{m,w} - d_S)\alpha f(d_r)(e^{k_S(\tau_{m,w} - t_s)} - 1) + C_m f(d_r) + (h_m + S_m)\left[\frac{Q(2f(d_r) + (P - f(d_r))n_2 - P)}{2P}\right] \\ &\quad \left. + h_m \tau_r f(d_r) + (C_r - d_m - C_m) f(d_r)(e^{k_m t_m} - 1) \right) \end{aligned} \tag{6}$$

Appendix D

$$\varphi_r(Q,t_m,\tau_r,d_r,d_m)=\left((C_c-d_r)f(d_r)+(C_r-d_m)f(d_r)\left(e^{k_r t_m}-1\right)+(C_c-d_r)\frac{Q}{2}\left(e^{k_r\left(\frac{Q}{f(d_r)}\right)}-1\right)\right)$$
$$-\left(\frac{A_r f(d_r)}{Q}+(C_r-d_m)f(d_r)+h_r(Q,t_m,\ \tau_r)+\frac{S_r Q}{2}+(C_r-d_m)f(d_r)\left(e^{k_m(\tau_r-t_m)}-1\right)\right) \quad (9)$$

where $e^{k_r t_m}-1\approx k_r t_m$ and $e^{k_m(\tau_r-t_m)}-1\approx k_m(\tau_r-t_m)$ when k_r and $k_m<<0$.

Appendix E

Derivations with respect to n_1 for the supplier

$$\varphi_s(t_s,\tau_{m,w},n_1,n_2,d_s,d_r)=\left((C_{m,w}-d_s)\alpha f(d_r)+(C_{m,w}-d_s)\alpha f(d_r)\left(e^{k_s(\tau_{m,w}-t_s)}-1\right)\right)$$
$$-\left(\frac{A_s\ f(d_r)}{n_2 Q}+C_s\alpha f(d_r)+\frac{(n_1-1)}{2}(h_s+s_s)\frac{\alpha n_2 Q f(d_r)}{P n_1}\right.$$
$$\left.+h_s\tau_{m,w}\alpha f(d_r)+(C_{m,w}-d_s-C_s)\alpha f(d_r)\left(e^{k_s t_s}-1\right)\right) \quad (3a)$$

First derivative

$$=-(h_s+s_s)\frac{\alpha n_2 Q f(d_r)}{2P n_1^2}$$

Second derivative

$$=(h_s+s_s)\frac{\alpha n_2 Q f(d_r)}{P n_1^3}$$

Derivations with respect to n_1 for the manufacturer

$$\varphi_m(Q,t_s,t_m,\tau_{m,w},\tau_r,n_1,n_2,d_m,d_r,d_s)$$
$$=\left((C_r-d_m)f(d_r)+(C_r-d_m)f(d_r)\left(e^{k_m(\tau_r-t_m)}-1\right)+(C_{m,w}-d_s)\alpha f(d_r)\left(e^{k_m\tau_{m,w}}-1\right)\right)$$
$$-\left(\frac{(n_1A_{m,w}+A_m)f(d_r)}{n_2 Q}+(C_{m,w}-d_s)\alpha f(d_r)+\frac{S_{m,w}\alpha n_2 Q f(d_r)}{2P n_1}+h_{m,w}\frac{f(d_r)}{n_2 Q}\frac{n_2^2\alpha^2 Q^2}{2\alpha P n_1}\right.$$
$$-h_{m,w}\frac{f(d_r)}{n_2 Q}\frac{n_2\alpha Q\alpha P\tau_{m,w}n_1}{\alpha P n_1}+h_{m,w}\frac{f(d_r)}{n_2 Q}\frac{(\alpha P\tau_{m,w}n_1)^2}{2\alpha P n_1}$$
$$+(C_{m,w}-d_s)\alpha f(d_r)\left(e^{ks(\tau_{m,w}-t_s)}-1\right)+C_m f(d_r)+(h_m+S_m)\left[\frac{Q(2f(d_r)+(P-f(d_r))n_2-P)}{2P}\right]$$
$$\left.+h_m\tau_r f(d_r)+(C_r-d_m-C_m)f(d_r)\left(e^{k_m t_m}-1\right)\right)$$

First derivative

$$=-\frac{A_{m,w}f(d_r)}{n_2 Q}+\frac{s_{m,w}\ \alpha n_2 Q f(d_r)}{2pn_1^2}+\frac{h_{m,w}f(d_r)n_2 Q\alpha}{2pn_1^2}-\frac{h_{m,w}f(d_r)\alpha p\tau_{m,w}^2}{2n_2 Q}$$

Second derivative

$$=-\frac{(s_{m,w}+h_{m,w})\alpha n_2 Q f(d_r)}{pn_1^3}$$

Derivations with respect to n_1 for the retailer

$$\varphi_r(Q,t_m,\tau_r,d_r,d_m)=\left((C_c-d_r)f(d_r)+(C_r-d_m)f(d_r)\left(e^{k_r t_m}-1\right)+(C_c-d_r)\frac{Q}{2}\left(e^{k_r\left(\frac{Q}{f(d_r)}\right)}-1\right)\right)$$
$$-\left(\frac{A_r f(d_r)}{Q}+(C_r-d_m)f(d_r)+h_r(Q,t_m,\ \tau_r)+\frac{S_r Q}{2}+(C_r-d_m)f(d_r)\left(e^{k_m(\tau_r-t_m)}-1\right)\right)$$
$$=0$$

Then to get the optimal solution the following condition should be satisfied

$$(h_{m,w}+s_{m,w})>(h_s+s_s)$$

Appendix F

Derivations with respect to n_2 for the supplier

$$\varphi_s(t_s,\ \tau_{m,w}, n_1, n_2, d_s, d_r) = \left((C_{m,w}-d_s)\alpha f(d_r) + (C_{m,w}-d_s)\alpha f(d_r)\left(e^{k_s(\tau_{m,w}-t_s)}-1\right)\right) - \left(\frac{A_s\ f(d_r)}{n_2 Q} + C_s\alpha f(d_r) + \frac{(n_1-1)}{2}(h_s+s_s)\frac{\alpha n_2 Q f(d_r)}{Pn_1} + h_s\tau_{m,w}\alpha f(d_r) + (C_{m,w}-d_s-C_s)\alpha f(d_r)\left(e^{k_s t_s}-1\right)\right) \tag{3b}$$

First derivative

$$\frac{A_s f(d_r)}{n_2^2 Q} - \frac{(n_1-1)(h_s+s_s)\alpha Q f(d_r)}{2Pn_1}$$

Second derivative

$$= -\frac{2A_s f(d_r)}{n_2^3 Q}$$

Derivations with respect to n_2 for the manufacturer

$$\begin{aligned}
&\varphi_m(Q, t_s, t_m, \tau_{m,w}, \tau_r, n_1, n_2, d_m, d_r, d_s)\\
&= \left((C_r-d_m)f(d_r) + (C_r-d_m)f(d_r)\left(e^{k_m(\tau_r-t_m)}-1\right) + (C_{m,w}-d_s)\alpha f(d_r)\left(e^{k_m\tau_{m,w}}-1\right)\right)\\
&\quad -\left(\frac{(n_1A_{m,w}+A_m)f(d_r)}{n_2Q} + (C_{m,w}-d_s)\alpha f(d_r) + \frac{S_{m,w}\alpha n_2 Q f(d_r)}{2Pn_1} + h_{m,w}\frac{f(d_r)}{n_2Q}\frac{n_2^2\alpha^2Q^2}{2\alpha Pn_1}\right.\\
&\quad -h_{m,w}\frac{f(d_r)}{n_2Q}\frac{n_2\alpha Q\alpha P\tau_{m,w}n_1}{\alpha Pn_1} + h_{m,w}\frac{f(d_r)}{n_2Q}\frac{(\alpha P\tau_{m,w}n_1)^2}{2\alpha Pn_1}\\
&\quad +(C_{m,w}-d_s)\alpha f(d_r)\left(e^{ks(\tau_{m,w}-t_s)}-1\right) + C_m f(d_r) + (h_m+S_m)\left[\frac{Q(2f(d_r)+(P-f(d_r))n_2-P)}{2P}\right]\\
&\quad \left.+h_m\tau_r f(d_r) + (C_r-d_m-C_m)f(d_r)\left(e^{k_m t_m}-1\right)\right)
\end{aligned}$$

First derivative

$$(n_1A_{m,\ w}+A_m)\frac{f(d_r)}{n_2^2Q} - \frac{h_{m,\ w}f(d_r)Q\alpha}{2Pn_1} + \frac{h_{m,\ w}f(d_r)\alpha P\tau_{m,w}^2 n_1}{2n_2^2Q}$$

Second derivative

$$-2\frac{(n_1A_{m,\ w}\ +\ A_m)f(d_r)}{n_2^3Q} - \frac{h_{m,w}f(d_r)\alpha p\tau_{m,w}^2 n_1}{n_2^3Q}$$

Derivations with respect to n_2 for the retailer

$$\begin{aligned}
\varphi_r(Q, t_m, \tau_r, d_r, d_m) &= \left((C_c-d_r)f(d_r) + (C_r-d_m)f(d_r)\left(e^{k_r t_m}-1\right) + (C_c-d_r)\frac{Q}{2}\left(e^{k_r\left(\frac{Q}{f(d_r)}\right)}-1\right)\right)\\
&\quad -\left(\frac{A_r f(d_r)}{Q} + (C_r-d_m)f(d_r) + h_r(Q, t_m,\ \tau_r) + \frac{S_r Q}{2} + (C_r-d_m)f(d_r)\left(e^{k_m(\tau_r-t_m)}-1\right)\right)\\
&= 0
\end{aligned}$$

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