Compensation in Complex Variables for Microgrid Power Flow

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***Abstract*—The solution of the distribution network power flow in practical applications is based either on the forward–backward sweep method for radial networks, or the current injection method for meshed networks. While the power flow in microgrids that operate in grid-connected mode could be resolved using the above-mentioned approaches, their operation in island mode would require simulating local generation droop controllers for sharing the complex power load and network loss among the generators. This letter proposes a complex power compensation approach, which is based on Wirtinger calculus, for extending the applicability of practical distribution power flow methods to microgrids operating in island mode. Supporting numerical results are reported on microgrids with up to 3139 nodes.**

***Index Terms*—Load flow control, microgrids, power system analysis computing.**

I. Introduction

The power flow solution of microgrids operating in island mode requires simulating the operation of local generator controllers, such as those implementing power sharing droop control [1]–[5]. The state-of-the-art in distribution network power flow is based on the forward-backward sweep [6] and current injection [7] techniques; these solvers are adopted by the industry and can readily simulate microgrid operation in grid-connected mode. Refs. [1], [2] report the application of the forward-backward sweep power flow to island microgrids with radial structure, however [2] notes that the simulation of the Q-V droop control is very much dependent on the initial choice of the reactive power. The use of the classical transmission network power flow in island networks has been recently discussed [3]–[5]. Ref. [3] used a Gauss-Seidel approach, but it is expected to scale poorly on large networks; [5] proposed a Trust-Region Newton-Raphson solver to alleviate ill conditioning that is attributed to the narrow region of convergence in island mode. The islanded operation poses special challenges to conventional distribution power flow solvers, mainly the operation without a slack node and the simulation of droop control. This letter proposes a complex compensation approach for modeling local power sharing in the industry adopted power flow techniques [6], [7], and presents an effective solution for power flow simulation with a distributed slack node. The compensation approach is based on sensitivity equations derived via Wirtinger calculus [8].

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II. Microgrid Power Flow

Consider a distribution network having n nodes, with node 1 being the slack in grid-connected operation and nodes $2,\ldots,n$ representing PQ power injection connections; there is no requirement for the network to be radial. The power flow solution via the current injection method is based on iterating the current injection computation (1) with the voltage update (2) until the power mismatches at all PQ nodes are satisfied within tolerance. The bar sign in (1) represents complex conjugation, and V_{sl} in (2) is the slack node voltage; the voltage update solution in (2) is effectively computed by LU decomposition of the nodal admittance matrix followed by forward/backward substitution.

$$I_i = \frac{P_i - \mathtt{i}Q_i}{\overline{V}_i} \tag{1}$$

$$\begin{bmatrix} V_1 \\ \vdots \\ V_i \\ \vdots \\ V_n \end{bmatrix} = \begin{bmatrix} \gamma_{11} & \cdots & 0 & \cdots & 0 \\ \vdots & & \vdots & & \vdots \\ \gamma_{i1} & \cdots & Z_{ii} & \cdots & Z_{in} \\ \vdots & & \vdots & & \vdots \\ \gamma_{in} & \cdots & Z_{ni} & \cdots & Z_{nn} \end{bmatrix} \begin{bmatrix} V_{sl} \\ \vdots \\ I_i \\ \vdots \\ I_n \end{bmatrix} \tag{2}$$

The above iterative setup is also applicable to radial networks; however, the voltage update (2) in the radial case is commonly computed through forward-backward sweeps. In islanded operation, the generators at nodes $1,\ldots,m$ have their real (P_{Gi}) and reactive (Q_{Gi}) powers dependent on the droop control laws (3)-(4); k_{pi}/k_{qi} are the droop control settings for real/reactive power, ω is the microgrid frequency, and the star superscript denotes nominal values:

$$P_{Gi} = P_{Gi}^{\star} + \frac{1}{k_{pi}}\left(\omega^{\star} - \omega\right),\ i = 1,\ldots,m \tag{3}$$

$$|V_i| = |V_i|^{\star} - k_{qi}\left(Q_{Gi} - Q_{Gi}^{\star}\right),\ i = 1,\ldots,m \tag{4}$$

By substituting (3) in the real power balance equation (5), the expression for the steady-state frequency deviation $\Delta\omega = \omega^{\star} - \omega$ can be computed and replaced back in (3) to give the real power generation sharing law (6):

$$\sum_{j=1}^{m} P_{Gj} = P_{Load} + P_{Loss} \tag{5}$$

$$\Longrightarrow P_{Gi} = P_{Gi}^{\circ} + \alpha_i P_{Loss} \tag{6}$$

$$\alpha_i = \frac{1}{\sum_{j=1}^{m} \frac{k_{pi}}{k_{pj}}} \tag{7}$$

$$P_{Gi}^{\circ} = P_{Gi}^{\star} + \alpha_i \left(P_{Load} - \sum_{j=1}^{m} P_{Gj}^{\star}\right) \tag{8}$$

By noting that $P_{Loss} = \sum_{k=1}^{n} P_k$, (6) can be now linearized around the current operating solution via a complex Taylor series

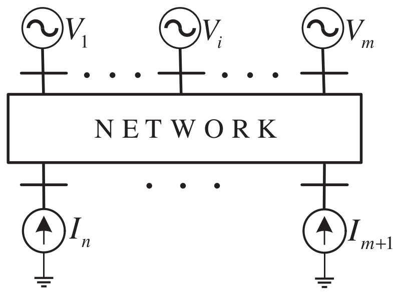

Fig. 1. Node partitioning for sensitivity computation; the indicated variables denote independent quantities.

expansion in terms of the generator node voltage corrections ΔV_j and their complex conjugates $\Delta \overline{V}_j$ [8]:

$$(1-\alpha_i)\sum_{j=1}^{m}\left[\frac{\partial P_i}{\partial V_j}\Delta V_j + \frac{\partial P_i}{\partial \overline{V}_j}\Delta \overline{V}_j\right] - \alpha_i \sum_{k=1,k\neq i}^{m}\sum_{j=1}^{m}\left[\frac{\partial P_k}{\partial V_j}\Delta V_j + \frac{\partial P_k}{\partial \overline{V}_j}\Delta \overline{V}_j\right] = P_{Gi}^{\circ} + \alpha_i \sum_{k=1}^{n} P_k - P_{Gi} \tag{9}$$

Similarly, the reactive power droop control law (4) can be linearized around the current operating point:

$$\left(\frac{1}{2}\sqrt{\frac{\overline{V}_i}{V_i}} + k_{qi}\frac{\partial Q_i}{\partial V_i}\right)\Delta V_i + k_{qi}\sum_{j=1,j\neq i}^{m}\frac{\partial Q_i}{\partial V_j}\Delta V_j + \left(\frac{1}{2}\sqrt{\frac{V_i}{\overline{V}_i}} + k_{qi}\frac{\partial Q_i}{\partial \overline{V}_i}\right)\Delta \overline{V}_i + k_{qi}\sum_{j=1,j\neq i}^{m}\frac{\partial Q_i}{\partial \overline{V}_j}\Delta \overline{V}_j = |V_i|^{\star} - k_{qi}\left(Q_{Gi} - Q_{Gi}^{\star}\right) - \sqrt{V_i \overline{V}_i} \tag{10}$$

The sensitivity (9) ($i = 2, \ldots, m$), (10) ($i = 1, \ldots, m$), and the zero angle condition at node 1 ($\Delta V_i - \Delta \overline{V}_i = 0$) are solved simultaneously at the m generators to give the corrections to the complex generator nodal voltages (ΔV_j) and their conjugates ($\Delta \overline{V}_j$). The voltage corrections and sensitivity factors are then used to compute ΔP_{G_i} and ΔQ_{Gi}, and to update the real and reactive power generation values:

$$P_{Gi} \leftarrow P_{Gi}^{\circ} + \alpha_i\left(\sum_{k=1}^{n} P_k + \sum_{j=1}^{m}\Delta P_{Gj}\right) \tag{11}$$

$$Q_{Gi} \leftarrow Q_{Gi} + \Delta Q_{Gi} \tag{12}$$

The new generation values in (11)–(12) together with the updated generator voltages are subsequently employed to compute the injection currents (1), and to proceed with the voltage updates (2). The convergence condition would require (4) and (6) to be satisfied within tolerance, in addition to the classical current injection convergence criterion on complex power injection mismatch.

III. Sensitivity Computation

For computing the sensitivity coefficients in (9)–(10), the nodes $m+1, \ldots, n$ are assumed to have constant current injections as shown in Fig. 1, i.e., the dependence of the load current injections on the generator voltage corrections is neglected. Subsection III-A and III-B derive the expressions for $\frac{\partial \overline{S}_i}{\partial V_j}$ and $\frac{\partial \overline{S}_i}{\partial \overline{V}_j}$, respectively. These are sufficient to compute the desired complex sensitivity coefficients, because the Wirtinger derivatives satisfy [8]:

$$\frac{\partial S_i}{\partial \overline{V}_j} = \overline{\left(\frac{\partial \overline{S}_i}{\partial V_j}\right)},\ \frac{\partial S_i}{\partial V_j} = \overline{\left(\frac{\partial \overline{S}_i}{\partial \overline{V}_j}\right)} \tag{13}$$

$$\frac{\partial P_i}{\partial x} = \frac{1}{2}\left(\frac{\partial S_i}{\partial x} + \frac{\partial \overline{S}_i}{\partial x}\right),\ \frac{\partial Q_i}{\partial x} = \frac{\mathtt{i}}{2}\left(\frac{\partial \overline{S}_i}{\partial x} - \frac{\partial S_i}{\partial x}\right) \tag{14}$$

A. Sensitivity of the Conjugate Complex Power Injection at Generator Nodes to Phasor Voltages

1) Sensitivity of $\left(\overline{S}_1, \ldots, \overline{S}_m\right)$ *to* $(V_2, \ldots, V_m)$*:* Consider the equations for voltages at nodes i and k, as given by (2):

$$V_i = \gamma_{i1}V_1 + \sum_{j=2}^{m} Z_{ij}\frac{\overline{S}_j}{\overline{V}_j} + \sum_{j=m+1}^{n} Z_{ij}I_j \tag{15}$$

$$V_k = \gamma_{k1}V_1 + \sum_{j=2}^{m} Z_{kj}\frac{\overline{S}_j}{\overline{V}_j} + \sum_{j=m+1}^{n} Z_{kj}I_j \tag{16}$$

Taking the derivative with respect to V_i gives:

$$\sum_{j=2}^{m}\frac{Z_{ij}}{\overline{V}_j}\frac{\partial \overline{S}_j}{\partial V_i} = 1,\ i = 2, \ldots, m \tag{17}$$

$$\sum_{j=2}^{m}\frac{Z_{kj}}{\overline{V}_j}\frac{\partial \overline{S}_j}{\partial V_i} = 0,\ k = 2, \ldots, m,\ k \neq i \tag{18}$$

For each generator $i = 2, \ldots, m$, the above equations are solved for the sensitivity coefficients $\frac{\partial \overline{S}_j}{\partial V_i}$:

$$\begin{bmatrix} Z_{22} & \cdots & Z_{2i} & \cdots & Z_{2m} \\ \vdots & & \vdots & & \vdots \\ Z_{i2} & \cdots & Z_{ii} & \cdots & Z_{im} \\ \vdots & & \vdots & & \vdots \\ Z_{m2} & \cdots & Z_{mi} & \cdots & Z_{mm} \end{bmatrix} \begin{bmatrix} \frac{1}{\overline{V}_2}\frac{\partial \overline{S}_2}{\partial V_i} \\ \vdots \\ \frac{1}{\overline{V}_i}\frac{\partial \overline{S}_i}{\partial V_i} \\ \vdots \\ \frac{1}{\overline{V}_m}\frac{\partial \overline{S}_m}{\partial V_i} \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} \tag{19}$$

The sensitivity of the complex slack node power to any generator voltage (excluding the slack node) is obtained by taking the derivative of the KCL equation at the network level:

$$\frac{\overline{S}_1}{\overline{V}_1} + \sum_{j=2}^{m}\frac{\overline{S}_j}{\overline{V}_j} + \sum_{j=m+1}^{n} I_j = 0 \tag{20}$$

$$\Rightarrow \frac{\partial \overline{S}_1}{\partial V_i} = -\overline{V}_1\sum_{j=2}^{m}\frac{1}{\overline{V}_j}\frac{\partial \overline{S}_j}{\partial V_i},\ i = 2, \ldots, m \tag{21}$$

2) Sensitivity of $\left(\overline{S}_1, \ldots, \overline{S}_m\right)$ *to* V_1*:* The sensitivities of the complex conjugate power injections at nodes $2, \ldots, m$ to V_1 are computed by taking the derivative of (15) with respect to V_1, and solving for the system in matrix form (23):

$$\sum_{j=2}^{m} \frac{Z_{ij}}{\overline{V}_j} \frac{\partial \overline{S}_j}{\partial V_1} = -\gamma_{i1},\ i = 2, \ldots, m \implies \tag{22}$$

$$\begin{bmatrix} Z_{22} & \cdots & Z_{2i} & \cdots & Z_{2m} \\ \vdots & & \vdots & & \vdots \\ Z_{i2} & \cdots & Z_{ii} & \cdots & Z_{im} \\ \vdots & & \vdots & & \vdots \\ Z_{m2} & \cdots & Z_{mi} & \cdots & Z_{mm} \end{bmatrix} \begin{bmatrix} \frac{1}{\overline{V}_2} \frac{\partial \overline{S}_2}{\partial V_1} \\ \vdots \\ \frac{1}{\overline{V}_i} \frac{\partial \overline{S}_i}{\partial V_1} \\ \vdots \\ \frac{1}{\overline{V}_m} \frac{\partial \overline{S}_m}{\partial V_1} \end{bmatrix} = \begin{bmatrix} -\gamma_{21} \\ \vdots \\ -\gamma_{i1} \\ \vdots \\ -\gamma_{m1} \end{bmatrix} \tag{23}$$

The sensitivity of $\overline{S}_1$ to V_1 is then computed from (20):

$$\frac{\partial \overline{S}_1}{\partial V_1} = -\overline{V}_1 \sum_{j=2}^{m} \frac{1}{\overline{V}_j} \frac{\partial \overline{S}_j}{\partial V_1} \tag{24}$$

B. Sensitivity of the Conjugate Complex Power Injection at Generator Nodes to Conjugate Phasor Voltages

1) Sensitivity of $(\overline{S}_1, \ldots, \overline{S}_m)$ *to* $(\overline{V}_2, \ldots, \overline{V}_m)$*:* Following the same procedure in subsection III-A1, but taking the derivative with respect to the conjugate voltage gives ($i = 2, \ldots, m$):

$$\begin{bmatrix} Z_{22} & \cdots & Z_{2i} & \cdots & Z_{2m} \\ \vdots & & \vdots & & \vdots \\ Z_{i2} & \cdots & Z_{ii} & \cdots & Z_{im} \\ \vdots & & \vdots & & \vdots \\ Z_{m2} & \cdots & Z_{mi} & \cdots & Z_{mm} \end{bmatrix} \begin{bmatrix} \frac{1}{\overline{V}_2} \frac{\partial \overline{S}_2}{\partial \overline{V}_i} \\ \vdots \\ \frac{1}{\overline{V}_i} \frac{\partial \overline{S}_i}{\partial \overline{V}_i} \\ \vdots \\ \frac{1}{\overline{V}_m} \frac{\partial \overline{S}_m}{\partial \overline{V}_i} \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ \frac{Z_{ii} \overline{S}_i}{\overline{V}_i^2} \\ \vdots \\ 0 \end{bmatrix} \tag{25}$$

$$\frac{\partial \overline{S}_1}{\partial \overline{V}_i} = \frac{\overline{V}_1}{\overline{V}_i^2} \overline{S}_i - \overline{V}_1 \sum_{j=2}^{m} \frac{1}{\overline{V}_j} \frac{\partial \overline{S}_j}{\partial \overline{V}_i},\ i = 2, \ldots, m \tag{26}$$

2) Sensitivity of $(\overline{S}_1, \ldots, \overline{S}_m)$ *to* $\overline{V}_1$*:* Computing the sensitivity to $\overline{V}_1$ proceeds as in subsection III-A2:

$$\sum_{j=2}^{m} \frac{Z_{ij}}{\overline{V}_j} \frac{\partial \overline{S}_j}{\partial \overline{V}_1} = 0,\ i = 2, \ldots, m \tag{27}$$

The system of equations (27) in this case is however homogenous, giving a zero solution for the derivatives:

$$\frac{\partial \overline{S}_i}{\partial \overline{V}_1} = 0,\ i = 2, \ldots, m \tag{28}$$

The sensitivity $\frac{\partial \overline{S}_1}{\partial \overline{V}_1}$ (derived from (20)) therefore reduces to:

$$\frac{\partial \overline{S}_1}{\partial \overline{V}_1} = \frac{\overline{S}_1}{\overline{V}_1} - \overline{V}_1 \sum_{j=2}^{m} \frac{1}{\overline{V}_j} \frac{\partial \overline{S}_j}{\partial \overline{V}_1} = \frac{\overline{S}_1}{\overline{V}_1} \tag{29}$$

IV. Numerical Results & Conclusion

A computer program was implemented in Matlab and tested on a modified version of a realistic Brazilian distribution system (BR:160 nodes) in addition to two test networks with 1458 (1k5) and 3139 (3k) nodes; the three networks are weakly meshed and their complete data sets can be downloaded from [9]. The implementation employs a single LU factorization for the network nodal admittance matrix, and a single LU factorization for the common coefficient matrix (19), (23), (25) used in the sensitivity computations. The program was run on an iMAC having a 2.7 GHz quad-core Intel Core i5 processor with 4 MB L3 cache and 8 GB of RAM. The stopping tolerance was set to 10^{-8} pu. Testing was carried out for both the island ($\alpha_i \neq 0$, $k_{qi} \neq 0$) and grid-connected ($\alpha_i = 0$ except at the slack node, $k_{qi} = 0$) operation modes. The numerical results in Table I show that the computational effort increases moderately with problem size. For grid-connected operation, a comparison was carried out with the classical compensation method for handling generator nodes [10]; both methods gave exactly the same power flow results with the same order of computational time. Therefore, the proposed compensation method can be viewed as a generalization of classical compensation technique for simulating power flow in island microgrids; it is particularly useful as it extends the applicability of the state-of-the-art distribution network power flow techniques [6], [7] to islanded operation. In comparison with a Newton-Raphson based approach for island microgrid operation [4] (last two columns of Table I), the proposed method exhibits significant speed-up as it does not require computing and factorizing the Jacobian matrix at every iteration.

TABLE I
Computational Effort of the Sensitivity-Based Current Injection in Island and Grid-Connected Operation, and Comparison With [4]

Net.	n	Br.	Grid-Connected		Island		Ref. [4]	
			iter.	time [ms]	iter.	time [ms]	iter.	time [ms]
BR	160	160	11	78.1	12	83.8	4	103.8
1k5	1458	1475	18	91.0	29	105.0	5	657.7
3k	3139	3158	27	146.9	36	163.5	5	1427.2

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