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Coordination of a three-level supply chain (supplier–manufacturer–retailer) with permissible delay in payments

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ABSTRACT

Different trade credits can be used to coordinate a supply chain. One of the well-known mechanisms that is widely used in practice is the permissible delay in payments. This paper proposes models for coordinating a three-level (supplier–manufacturer–retailer) supply chain with permissible delay in payments, with its length (time to settle a payment) being a decision variable. It investigates the permissible delay among the three players, where the supplier offers a delay in payment to the manufacturer and the manufacturer offers another one to the retailer. The paper investigates nine different scenarios of permissible delay among the three players. A simulation study was performed and a thorough analysis of the results was used to identify the limitations of all scenarios and to draw some managerial insights and findings.

1. Introduction

Supply chain management is considered as one of the most useful management practices for industries to increase their sustainability and competitiveness. In today's globalized economy, supply chain sustainability is key to ensuring business continuity and managing operational costs [1]. With the growing focus on sustainable supply chain management, firms realize that inventories across the entire supply chain can be more efficiently managed through greater cooperation and better coordination. The collaborative paradigm in supply chain management is a crucial source of competitive advantage, as it can increase the impact and efficiency of the supply chain. As collaboration increases among supply chain players, the total costs reduce by up to 30% [2,3].

Coordination in supply chain management is based on centralized and decentralized decision making. The objective of supply chain coordination is to minimize the total supply chain cost. In a decentralized supply chain, each player tries to maximize his/her own performance, resulting in an inefficient supply chain. In centralized decision making, coordination allows supply chain players to work closely to streamline their decision making, with the objective being to maximize the entire supply chain performance. Inefficiencies reduce throughout the entire supply chain, with better matching of supply and demand. As a result, costs reduce and sales increase, leading to increased supply chain profitability.

Permissible delay in payments or trade-credit option is one of the coordination mechanisms that can be used to entice the buyer (retailer) to buy in larger lots than his/her Economic Order Quantity (EOQ), whereby the buyer is allowed by

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the vendor (manufacturer) to settle its balance by a certain time at no additional charges (interest-free period). Wal-Mart supplier reduced its capital expenditure by 1.2% by offering Wal-Mart a one month delay in payment [4]. In addition, the buyer will benefit from the permissible delay by investing his/her owed balance in a risk-free investment during the interest-free period. However, interest charges will be applied to the buyer if the settlement occurs after the permissible period. The benefits for the vendor when offering the permissible period for the customer are increasing its sales and moving the inventory to the buyer.

The EOQ inventory problem faced by a retailer under the assumption of permissible delay in payment offered by the supplier has been widely studied in the literature. In this problem, the supplier grants the retailer a certain amount of time during which the retailer needs to settle his/her account. No interest is charged during the permissible delay period, beyond which interest is charged under the terms agreed upon, and interest is earned on the revenue from sales received during the credit period. Among the earliest work in this field is that of Haley and Higgins [5], who consider a retailer's lot-sizing problem with trade credit financing, and find the optimal order quantity and optimal credit settlement time. Many research studies on inventory control and delay in payment followed, the most notable is the work of Goyal [6] who develop an EOQ model with delay in payment, and show that the optimal quantity and replenishment interval increase when the retailer can delay the payment without paying any interest. Other research work that analyze the lot-sizing problem faced by a retailer under permissible delay in payments for different inventory situations and/or assumptions include, but not limited to, Aggarwal and Jaggi [7]; Chung and Liao [8]; Luo [9]; Jaber [10]; Chung [11]; Gupta and Wang [12]; Teng [13]; Balkhi [14].

Joint economic lot sizing (JELS) between the players in the supply chain is one of the most efficient mechanisms to reduce the total costs in a supply chain [15,16]. JELS models determine the order, production and shipment quantity from a supply chain perspective, with the objective being to minimize the total supply chain costs [15]. JELSP initiated in the literature by Goyal [17]. However, not all players involved in JELSP will benefit from the savings generated, as some players will not operate to their optimal policies. In this case, a profit-sharing policy could be used to compensate the losing players. Utilizing permissible delay in payments with JELS enriches the benefits gained from coordination. The first paper that considered coupling permissible delay in payments with JELSP was Goyal [6], where the length of the permissible period was fixed. Similar approaches were explored in the literature with different assumptions, such as considering defective items, stochastic demand, deteriorating items, to name few. Among the important work in this area is Jaber and Osman [18] who develop a two-level supply chain model with permissible delay in payments and profit sharing. They show that coordination results in larger orders for the retailer, with savings to the entire supply chain. Abad and Jaggi [19] consider a supplier–retailer channel and assume demand is price sensitive. They develop policies for the buyer and seller under coordination and no-coordination. Viswanathan and Piplani [20] analyze the benefit of coordinating supply chain inventories through the use of common replenishment epochs. The recent work of Moussawi-Haidar and Jaber [21] and Moussawi-Haidar et al. [22] incorporate cash management into the inventory lot sizing problem under delay in payment.

Recently, Aljazzar et al. [23] propose the coordination of a two-level (manufacturer–retailer) supply chain with permissible delay in payments as a decision variable. They study the effect of implementing three well-known production policies, Goyal's [24], Hill's [25] and Jaber et al.'s [26], and conclude that the minimum total cost was obtained when Hill's [25] production policy was adopted. Other research work on supply chain coordination include those of Chen and Kang [27], Lu [28] and Goyal [29]. Readers may refer to Jaber and Zolfaghari [16] and Glock [15] for a detailed review. For more details on the literature of permissible delay, see, for example, Chang et al. [30], Seifert et al. [31], and Molamohamadi et al. [32].

The main contribution of this paper is that it investigates the coordination of a three-level (supplier–manufacturer–retailer) supply chain, using the permissible delay in payments as a decision variable. Although in real life practices, the length of the permissible delay in payments is not considered as a decision variable, treating it as such will shed the light on the optimal permissible period that would minimize the total cost of the supply chain. Coordinating a three level supply chain with permissible delay in payment between every two levels has not been considered in the literature. Such a situation is very common in real life and a thorough analysis of the real scenarios involved would allow us to provide useful recommendations that more accurately reflect the structure and practices of a three-level supply chain. In this model, the manufacturer purchases raw material from the supplier and converts it into finished goods, following Hill's [25] policy, according to which the manufacturer produces and ships to the retailer during the production period. Specifically, we consider a supplier, a manufacturer, and a retailer coordinating their decisions through delay in payment. During each period, the supplier offers the manufacturer a delay in payment period, after which the supplier starts charging interest to the manufacturer on any outstanding balance. Also, the manufacturer offers the retailer a permissible delay in payment period during which the retailer pays no interest on any outstanding balance, and after which the manufacturer starts charging the retailer an interest on any outstanding amount. We consider a centralized decision making model with delay in payment, in which the three players coordinate their decisions of how much to order from the upstream player, and how to set the length of the permissible delay in payment. In our paper, the duration of the delay in payment period is a decision variable to be determined by the upstream player. Nine real-life scenarios are considered in this study to cover all possible real situations that might occur in practice, when permissible delay in payments is integrated into the supply chain.

The problem in this paper is modeled and solved, first by assuming a common cycle length for all the three different players, and then, by assuming uncommon cycle length. Using simulation, we compare the performance of the two models corresponding to each of the two assumptions, and conclude that adopting the common cycle length assumption considerably reduces the total supply chain costs when compared to uncommon cycle length. Adopting the common cycle length assumption, we ran simulation for 10,000 numerical examples that were randomly generated and solved for each of the

nine scenarios, with the independent variables varying randomly in specific ranges. First, we compare the total supply chain cost of the case when no delay in payment is offered and each of the nine scenarios of the model with delay in payment. We observe that the total supply chain cost is the highest when no permissible delay in payment is offered. Among the nine scenarios of the delay in payment model, the total cost is the lowest in the case when the manufacturer settles its balance to the supplier beyond the permissible delay period and before receiving the next shipment from the supplier, and the retailer settles his/her balance by the end of the delay in payment period. We then analyze the effect of each independent variable on the total supply chain's cost under each scenario, and identify the variables with insignificant effect on the total cost. Finally, we perform sensitivity analysis for each of the nine scenarios, by varying the interest rates and holding costs of each player, and the demand to production ratio. For each sensitivity case, the total cost is obtained for all the different scenarios and the lowest cost is identified. This allows the different players in practice to identify the best scenario that will optimize the entire supply chain cost, based on the specific values of their interest rates, holding costs, and demand to production ratio.

The remainder of this paper is organized as follows: Section 2 introduces the notations and assumptions. Section 3 provides the mathematical models for all scenarios. Section 4 provides a numerical example, the simulation results, and sensitivity analysis. Section 5 is for managerial insights. Finally, the conclusion and discussion are provided in Section 6.

2. Notations and assumptions

The following notations have been used to develop the mathematical models:

i Corresponds to the supply chain member as indicated by the subscript (i.e. 's' for supplier, 'm' for manufacturer and 'r' for retailer)

j Corresponds to the inventory level at the supply chain member as indicated by the subscript (i.e. 'w' for raw material and 'f ' for finished goods), where $j = 0$ when the member has a single inventory e.g. A_s stands for the setup cost for the supplier and $A_{m,w}$ stands for the order cost of the manufacturer's raw material.

$A_{i,j}$ Setup/order cost for player 'i'

$C_{i,j}$ Production/purchasing cost per item for player 'i'

$h_{i,j}$ Financial holding cost per item for the player 'i'

$S_{i,j}$ Physical (storage) holding cost per item for the player 'i'

Q Order quantity for the buyer

n_1 Number of shipments delivered by the supplier to the manufacturer per the manufacturer's raw material cycle

n_2 Number of shipment delivered by the manufacturer to the retailer per retailer's cycle

A Number of raw materials required to produce one finished product

t_i Permissible delay in payments offered by player 'i'

τ_i Time of balance settlement by the player 'i'

k_i Return on investment for player 'i'

P Manufacturer's annual production rate

D Retailer's annual demand rate, $D < P$

T Common cycle length $= \frac{n_2 Q}{D}$

T_s Supplier's cycle length $= \frac{n_2 Q}{P}$

T_w Manufacturer's raw material cycle length $= \frac{n_2 Q}{n_1 P}$

T_m Manufacturer's finished goods cycle length $= \frac{n_2 Q}{D}$

T_r Retailer's cycle length $= \frac{Q}{D}$

This paper considers the following assumptions:

- A single supplier, a single manufacturer and a single retailer.
- Demand at the manufacturer and retailer is deterministic and constant over time.
- Production rate of the supplier is greater than the demand of the manufacturer for raw material, and the manufacturer's production rate is greater than and retailer's demand.
- Shortages are not allowed.
- All shipments in this supply chain are equal-sized shipments.
- The system has a single product.
- The holding costs consist of two components: financial and physical holding costs.
- A permissible delay in payment is offered by the supplier to the manufacturer and by the manufacturer to the retailer.
- The permissible delay is assumed to be a decision variable.
- The manufacturer and the retailer invest what they owe to their vendors (supplier or manufacturer) in a risk-free investment during the permissible period.
- The retailer and the manufacturer pay the balance on their account in a single payment by τ.
- The manufacturer has two individual warehouses: one for the raw materials and the other for the finished goods.

The system that is presented in this paper is a three-level supply chain (supplier–manufacturer–retailer), as illustrated in Fig. 1. The retailer orders a lot size Q from the manufacturer at the beginning of each retailer's cycle, and the manufacturer

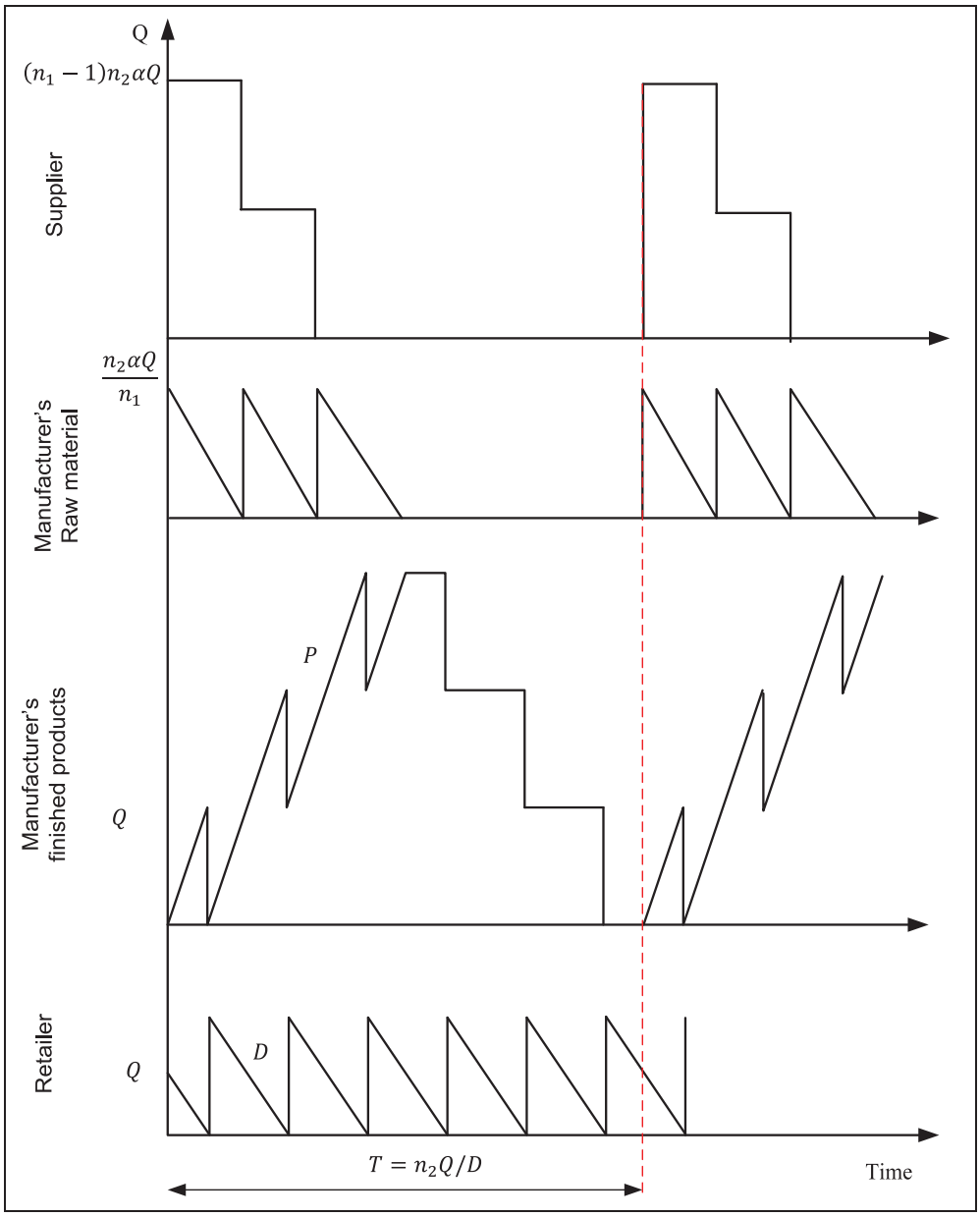

Fig. 1. Inventory behavior for the supplier, manufacturer and the retailer under a common cycle assumption.

produces and then delivers an equal lot size shipment by the end of each retailer's cycle $\frac{Q}{D}$. The retailer is permitted by the manufacturer to delay the payment of each shipment for an interest-free period (t_m). Accordingly, the retailer will not pay anything at the time of receiving the shipment. Similarly, the manufacturer orders from the supplier $\frac{n_2\alpha Q}{n_1}$ at the beginning of each raw material's cycle, and the supplier ships the equal lot size shipment accordingly. The manufacturer will not pay the supplier at the time of the receiving the raw materials due to the permissible delay that is offered by the supplier. To investigate all the possible scenarios that could be encountered in a real-life setting, this paper considers nine scenarios as illustrated in next section.

3. Mathematical model

This section provides the mathematical models of a coordinated three-level supply chain, with the permissible delay in payment considered as a decision variable. In order to adequately compare between the scenarios, two major assumptions

considered in the calculations, a common cycle length between the players, and an uncommon cycle length. Then, the assumption with the minimal resulting total cost will be adopted for the rest of the paper.

3.1. Common cycle length

In this case a common cycle length is considered when formulating the mathematical models, where the cycle length is $T = \frac{n_2 Q}{D}$. Fig. 1 shows the inventory and the common cycle length.

The manufacturer adopts the production policy of Hill [25], where the manufacturer delivers shipments of equal sizes at equal intervals of produced items to the retailer over its cycle including during the production segment. The manufacturer builds inventory, to a maximum, after the first shipment, where the manufacturer's inventory hits zero after it is shipped out to the retailer. The next manufacturer's cycle starts once all shipments are delivered to the retailer. The different scenarios under this assumption are as follows:

3.1.1. Scenario I-I: $0 \le t_s = \tau_m \le \frac{n_2 Q}{P n_1}$ and $0 \le t_m = \tau_r \le \frac{Q}{D}$

In this scenario, it is assumed that the supplier grants the manufacturer t_s units of time to settle its payment without interest charges. The manufacturer pays its balance by the end of this period (τ_m). In addition, it is assumed that the manufacturer grants the retailer t_m units of time to settle its payment without interest charges; however, the retailer may find it beneficial to settle it payment by time τ_r. For this scenario, we consider the case when $t_s = \tau_m$ and $t_m = \tau_r$, (refer to Appendix A for detailed derivations). Therefore, the supplier's total annual cost, that includes setup, production, physical and financial holding costs, can be written as follows:

$$\psi_s^{1,\mathrm{I}} = \frac{A_s D}{n_2 Q} + C_s \alpha D + \frac{(n_1 - 1)(h_s + S_s)\ \alpha n_2 Q D}{2 P n_1} + h_s \tau_m \alpha D + (C_w - C_s)\ \alpha D e^{k_s t_s} - C_{m,w} \alpha D e^{k_s (\tau_m - t_s)}. \tag{1}$$

It is assumed that the manufacturer invests $C_{m,w}\alpha Q$, which is owed to the supplier, in a risk-free investment. Correspondingly, the manufacturer's raw material annual cost, which includes order, purchasing, storage and financial holding costs and interest earnings, is as follows:

$$\psi_{m,w}^{\mathrm{I-I}} = \frac{n_1 A_{m,w} D}{n_2 Q} + C_{m,w}\alpha D + \frac{h_{m,w}\alpha n_2 Q D}{2 P n_1} - h_{m,w}\alpha t_s D + \frac{h_{m,w}\ \alpha P n_1 D\left(t_s^2\right)}{2 n_2 Q} + \frac{S_{m,w}\alpha n_2 Q D}{2 P n_1} + C_{m,w}\alpha D\left(e^{k_s(\tau_m - t_s)} - e^{k_m \tau_m}\right). \tag{2}$$

In addition, it is assumed that the manufacturer allows the retailer a period (interest-free) to settle its payment $C_r Q$. However, this will cost the manufacturer the opportunity to invest the money he should have received from the retailer. Accordingly, the manufacturer's annual cost, which includes setup, production, holding and losing opportunity costs, is:

$$\psi_{m,f}^{\mathrm{I-I}} = \frac{A_{m,f} D}{n_2 Q} + C_{m,f} D + \left(h_{m,f} + S_{m,f}\right)\left(\frac{Q(2D + (P - D)n_2 - P)}{2P}\right) + h_{m,f}\tau_r D + \left(C_r - C_{m,f}\right) D e^{k_m t_m} - C_r D e^{k_m(\tau_r - t_m)}. \tag{3}$$

Similarly to the manufacturer's raw material outstanding balance, it is assumed that the retailer invests the balance owed $C_r Q$ to the manufacturer in a risk-free investment. Therefore, the retailer's annual cost, which includes order, purchasing, holding costs (storage and financial) and interest earnings, is as follows:

$$\psi_r^{\mathrm{I-I}} = \frac{A_r D}{Q} + C_r D + \frac{\left(h_r (Q - D t_m)^2\right)}{2Q} + \frac{S_r Q}{2} + C_r D\left(e^{k_m(\tau_r - t_m)} - e^{k_r t_m}\right). \tag{4}$$

Accordingly, the total annual cost of the system, $\psi_{sc}^{\mathrm{I-I}} = \psi_s^{\mathrm{I-I}} + \psi_{m,w}^{\mathrm{I-I}} + \psi_{m,f}^{\mathrm{I-I}} + \psi_r^{\mathrm{I-I}}$, is:

$$\begin{aligned}
\psi_{sc}^{\mathrm{I-I}} = {} & \left(A_s + n_1 A_{m,w} + A_{m,f} + A_r n_2\right)\frac{D}{n_2 Q} + \left(C_s\alpha + C_{m,w}\alpha + C_{m,f} + C_r\right)D \\
& + (h_s\tau_m - h_{m,w} t_s)\alpha D \\
& + \frac{(n_1 - 1)(h_s + S_s)\ \alpha n_2 Q D + h_{m,w}\alpha n_2 Q D + S_{m,w}\alpha n_2 Q D}{2 P n_1} \\
& + (C_{m,w} - C_s)\ \alpha D e^{k_s t_s} + \frac{h_{m,w}\ \alpha P n_1 D t_s^2 + h_r n_2 (Q - D t_m)^2}{2 n_2 Q} \\
& - C_{m,w}\alpha D e^{k_m \tau_m} + \frac{Q(2D + (P - D)n_2 - P)\left(h_{m,f} + S_{m,f}\right) + S_r P Q}{2P} \\
& + h_{m,f}\tau_r D + \left(C_r - C_{m,f}\right) D e^{k_m t_m} - C_r D e^{k_r t_m}.
\end{aligned} \tag{5}$$

Eq. (5), $\psi_{sc}^{\mathrm{I-I}}$, has a joint minimum for Q, n_1 and n_2 (refer to Appendix B). The optimal order quantity ($Q^{\mathrm{I-I}}$) that minimizes $\psi_{sc}^{\mathrm{I-I}}$ is found by setting its first partial derivative equal to zero and solving for Q to get

$$Q^{\mathrm{I-I}} = \sqrt{\frac{A^{\mathrm{I-I}}\ (n_1,\ n_2,\ t_s,\ t_m)}{h^{\mathrm{I-I}}(n_1, n_2) + S^{\mathrm{I-I}}(n_2)}}, \tag{6}$$

where

$$A^{\text{I–II}}\ (n_1,\ n_2,\ t_s,\ t_m) = \left[A_s + A_{m,f} + n_1 A_{m,w} + n_2 A_r + \frac{\alpha n_1 h_{m,w} P t_s^2}{2} + \frac{n_2 h_r D t_m^2}{2}\right]\frac{D}{n_2},$$

$$h^{\text{I–I}}(n_1, n_2) = [h_{m,w} + S_{m,w} + (n_1 - 1)(h_s + S_s)]\frac{\alpha n_2 D}{2n_1 P},$$

$$S^{\text{I–I}}(n_2) = \left[\frac{S_r P + h_r P + (h_{m,f} + S_{m,f})(2D + n_2(P - D) - P)}{2P}\right].$$

3.1.2. Scenario I-II: $0 \le t_s = \tau_m \le \frac{n_2 Q}{P n_1}$ and $0 \le t_m < \tau_r \le \frac{Q}{D}$

This scenario assumes that the manufacturer settles its balance with the supplier by the end of the permissible period given by the supplier, and the retailer settles its balance payments with the manufacturer beyond the permissible period given by the manufacturer, and before receiving the next shipment. In addition, the retailer invests the balance it owes to the manufacturer ($C_r Q$) in a risk-free investment. Therefore, the annual retailer's cost is as follows:

$$\psi_r^{\text{I–II}} = \frac{A_r D}{Q} + C_r D + \frac{h_r (Q - D\tau_r)^2}{2Q} + \frac{S_r Q}{2} + C_r D\left(e^{k_m(\tau_r - t_m)} - e^{k_r t_m}\right). \tag{7}$$

The expressions for $\psi_s^{\text{I–II}}$, $\psi_{m,w}^{\text{I–II}}$ and $\psi_{m,f}^{\text{I–II}}$ are the same as that for scenario I-I. Thus, the total annual cost for the system is:

$$\begin{aligned}\psi_{sc}^{\text{I–II}} = {} & \left(A_s + n_1 A_{m,w} + A_{m,f} + n_2 A_r\right)\frac{D}{n_2 Q} + (C_s\alpha + C_{m,w}\alpha + C_{m,f} + C_r)D \\ & - h_{m,w}\alpha t_s D + \frac{((h_s + S_s)(n_1 - 1) + h_{m,w} + S_{m,w})\alpha n_2 Q D}{2P n_1} + h_s \tau_m \alpha D \\ & + (C_{m,w} - C_s)\alpha D e^{k_s t_s} + \frac{h_{m,w}\alpha P n_1 D t_s^2}{2n_2 Q} - C_{m,w}\alpha D e^{k_m \tau_m} \\ & + \left(h_{m,f} + S_{m,f}\right)\left(\frac{Q(2D + (P - D)n_2 - P)}{2P}\right) + \left(h_{m,f} - h_r\right)\tau_r D \\ & + \left(C_r - C_{m,f}\right)D e^{k_m t_m} + \frac{(h_r + S_r)Q^2 + h_r(D_r\tau_r)^2}{2Q} - C_r D e^{(k_r t_m)}.\end{aligned} \tag{8}$$

The optimal order quantity, $Q^{\text{I–II}}$, that minimizes Eq. (8), $\psi_{sc}^{\text{I–II}}$, can be found in a similar manner to Eq. (5) and it is given as:

$$Q^{\text{I–II}} = \sqrt{\frac{A^{\text{I–II}}\ (n_1,\ n_2,\ t_s,\ \tau_r)}{h^{\text{I–I}}(n_1, n_2) + S^{\text{I–I}}(n_2)}}, \tag{9}$$

where

$h^{\text{I–I}}(n_1, n_2)$ and $S^{\text{I–I}}(n_2)$ have been defined earlier after Eq. (6)

$$A^{\text{I–II}}\ (n_1,\ n_2,\ t_s,\ \tau_r)\ = \left[A_s + A_{m,f} + n_1 A_{m,w} + n_2 A_r + \frac{\alpha n_1 h_w P t_s^2}{2} + \frac{n_2 h_r D \tau_r^2}{2}\right]\frac{D}{n_2}.$$

3.1.3. Scenario I-III: $0 \le t_s = \tau_m \le \frac{n_2 Q}{P n_1}$ and $0 \le t_m < \frac{Q}{D} \le \tau_r$

The assumptions under this scenario are similar to the previous scenario's assumptions, except that the retailer settles its payments to the manufacturer after receiving the next shipment. Accordingly, the total annual cost of the retailer cost is:

$$\psi_r^{\text{I–III}} = \frac{A_r D}{Q} + C_r D + \frac{S_r Q}{2} + C_r D\left(e^{k_m(\tau_r - t_m)} - e^{k_r t_m}\right). \tag{10}$$

$\psi_s^{\text{I–III}}$, $\psi_{m,w}^{\text{I–III}}$ and $\psi_{m,f}^{\text{I–III}}$ for this scenario are the same as that for scenario I-I. Subsequently, the total annual cost of the system $\psi_{sc}^{\text{I–III}}$ can be written as follows:

$$\begin{aligned}\psi_{sc}^{\text{I–III}} = {} & \left(A_s + n_1 A_{m,w} + A_{m,f} + n_2 A_r\right)\frac{D}{n_2 Q} + (C_s\alpha + C_{m,w}\alpha + C_{m,f} + C_r)D \\ & - h_{m,w}\alpha t_s D + \frac{((h_s + S_s)(n_1 - 1) + h_{m,w} + S_{m,w})\alpha n_2 Q D}{2P n_1} + h_s \tau_m \alpha D \\ & + (C_{m,w} - C_s)\alpha D e^{k_s t_s} + \frac{h_{m,w}\alpha P n_1 D t_s^2}{2n_2 Q} - C_{m,w}\alpha D e^{k_m \tau_m}\end{aligned}$$

$$+\left(h_{m,f}+S_{m,f}\right)\left(\frac{Q(2D+(P-D)n_2-P)}{2P}\right)+h_{m,f}\tau_r D$$
$$+\left(C_r-C_{m,f}\right)De^{k_m t_m}+\frac{S_r Q}{2}-C_r De^{k_r t_m}. \tag{11}$$

The optimal order quantity, $Q^{\text{I-III}}$, that minimizes Eq. (11), $\psi_{sc}^{\text{I-III}}$, can be found in a similar manner to before and it is given as:

$$Q^{\text{I-III}}=\sqrt{\frac{A^{\text{I-III}}\ (n_1,\ n_2,\ t_s)}{h^{\text{I-I}}(n_1,n_2)+S^{\text{I-III}}(n_2)}}, \tag{12}$$

where

$h^{\text{I-I}}(n_1,n_2)$ has been defined earlier after Eq. (6)

$$A^{\text{I-III}}\ (n_1,n_2,t_s)=\left[A_s+A_{m,f}+n_1A_{m,w}+n_2A_r+\frac{\alpha n_1 h_{m,w}Pt_s^2}{2}\right]\frac{D}{n_2},$$
$$S^{\text{I-III}}(n_2)=\left[\frac{S_rP+(h_{m,f}+S_{m,f})(2D+n_2(P-D)-P)}{2P}\right].$$

3.1.4. Scenario II-I: $0\le t_s<\tau_m\le\ \frac{n_2Q}{Pn_1}$ and $0\le t_m=\tau_r\le\ \frac{Q}{D}$

In this scenario, it is assumed that the manufacturer settles the balance ($C_{m,w}\alpha Q$) it owes to the supplier after the permissible period has passed and before receiving the next shipment from the supplier. The retailer settles its balance (C_rQ) by the end of the free-interest period that is permitted by the manufacturer. The total annual cost for the manufacturer's raw material can then be written as follows:

$$\psi_{m,w}^{\text{II-I}}=\frac{n_1A_{m,w}D}{n_2Q}+C_{m,w}\alpha D+\frac{h_{m,w}\alpha n_2QD}{2Pn_1}-h_{m,w}\alpha\tau_mD+\frac{h_{m,w}\alpha Pn_1D\tau_m^2}{2n_2Q}+\frac{S_{m,w}\alpha n_2QD}{2Pn_1}+C_{m,w}\alpha D\left(e^{(k_s(\tau_m-t_s))}-e^{(k_m\tau_m)}\right). \tag{13}$$

$\psi_s^{\text{II-I}}$, $\psi_{m,f}^{\text{II-I}}$ and $\psi_r^{\text{II-I}}$ for this scenario are the same as that for scenario I-I. Accordingly, the total annual cost of the system $\psi_{sc}^{\text{II-I}}$ is:

$$\psi_{sc}^{\text{II-I}}=\left(A_s+n_1A_{m,w}+A_{m,f}+n_2A_r\right)\frac{D}{n_2Q}+(C_s\alpha+C_{m,w}\alpha+C_{m,f}+C_r)D$$
$$-h_w\alpha\tau_{m,w}D+\frac{((h_s+S_s)(n_1-1)+h_{m,w}+S_{m,w})\alpha n_2QD}{2Pn_1}+h_s\tau_m\alpha D$$
$$+(C_{m,w}-C_s)\alpha De^{k_st_s}+\frac{h_{m,w}\alpha Pn_1D\tau_m^2}{2n_2Q}-C_{m,w}\alpha De^{k_m\tau_m}$$
$$+\left(h_{m,f}+S_{m,f}\right)\left(\frac{Q(2D+(P-D)n_2-P)}{2P}\right)+h_{m,f}\tau_rD$$
$$+\left(C_r-C_{m,f}\right)De^{k_mt_m}+\frac{S_rQ}{2}-C_rDe^{(k_rt_m)}. \tag{14}$$

The optimal order quantity that minimizes Eq. (14) is determined in a similar manner to the previous ones and is given as:

$$Q^{\text{II-I}}=\sqrt{\frac{A^{\text{II-I}}(n_1,\ n_2,\ \tau_{m,w},\ t_m)}{h^{\text{I-I}}(n_1,n_2)+S(n_2)}}, \tag{15}$$

where

$h^{\text{I-I}}(n_1,n_2)$ and $S^{\text{I-I}}(n_2)$ have been defined earlier after Eq. (6)

$$A^{\text{II-I}}\ (n_1,n_2,\tau_{m,w},t_m)=\left[A_s+A_m+n_1A_{m,w}+n_2A_r+\frac{h_{m,w}\alpha P\tau_m^2n_1}{2}+\frac{n_2h_rDt_m^2}{2}\right]\frac{D}{n_2}.$$

3.1.5. Scenario II-II: $0\le t_s<\tau_{m,w}\le\ \frac{n_2Q}{Pn_1}$ and $0\le t_m<\tau_r\le\ \frac{Q}{D}$

This scenario assumes that the manufacturer settles its payments for the raw materials to the supplier ($C_{m,w}\alpha Q$) beyond the permitted period and before receiving the next shipment. Similarly, the retailer settles its payments that owed to the manufacturer (C_rQ) past the permissible delay offered by the manufacturer and before receiving the next shipment. Both the manufacturer and the retailer invest their owed balances in free-risk investments. Subsequently, the retailer's total annual cost becomes as follows:

$$\psi_r^{\text{II-II}}=\frac{A_rD}{Q}+C_rD+\frac{h_r\left((Q-(D\tau_r))^2\right)}{2Q}+\frac{S_rQ}{2}+C_rD\left(e^{(k_m(\tau_r-t_m))}-e^{k_rt_m}\right). \tag{16}$$

$\psi_s^{\text{II-II}}$ and $\psi_{m,f}^{\text{II-II}}$ are the same as that for scenario I-I. Moreover, $\psi_{m,w}^{\text{II-II}}$ for this scenario is the same as that for scenario II-I. Accordingly, the total annual cost of the system ($\psi_{sc}^{\text{II-II}}$), can be written as follows:

$$\begin{aligned}\psi_{sc}^{\text{II-II}} = {} & \left(A_s + n_1 A_{m,w} + A_{m,f} + n_2 A_r\right)\frac{D}{n_2 Q} + (C_s\alpha + C_{m,w}\alpha + C_{m,f} + C_r)D \\ & - h_{m,w}\alpha\tau_m D + \frac{((h_s + S_s)(n_1 - 1) + h_{m,w} + S_{m,w})\alpha n_2 QD}{2Pn_1} + h_s\tau_m\alpha D \\ & + (C_{m,w} - C_s)\alpha D e^{k_s t_s} + \frac{h_{m,w}\alpha P n_1 D\tau_m^2}{2n_2 Q} - C_{m,w}\alpha D e^{k_m\tau_m} \\ & + \left(h_{m,f} + S_{m,f}\right)\left(\frac{Q(2D + (P - D)n_2 - P)}{2P}\right) + \left(h_{m,f} - h_r\right)\tau_r D \\ & + \left(C_r - C_{m,f}\right)D e^{k_m t_m} + \frac{(h_r + S_r)Q^2 + h_r(D_r\tau_r)^2}{2Q} - C_r D e^{(k_r t_m)}. \end{aligned} \tag{17}$$

The optimal order quantity that minimizes the total cost of the system is:

$$Q^{\text{II-II}} = \sqrt{\frac{A^{\text{II-II}}(n_1,\ n_2,\ \tau_{m,w},\ \tau_r)}{h(n_1, n_2) + S(n_2)}}, \tag{18}$$

where

$h(n_1, n_2)$ and $S(n_2)$ have been defined earlier after Eq. (6)

$$A^{\text{II-II}}\ (n_1, n_2, \tau_{m,w}, \tau_r) = \left[A_s + A_{m,f} + n_1 A_{m,w} + n_2 A_r + \frac{\alpha n_1 h_{m,w} P\tau_m^2}{2} + \frac{n_2 h_r D\tau_r^2}{2}\right]\frac{D}{n_2}.$$

3.1.6. Scenario II-III: $0 \le t_s < \tau_m \le \frac{n_2 Q}{Pn_1}$ and $0 \le t_m \le \frac{Q}{D} \le \tau_r$

The assumptions of this scenario are similar to the assumptions of the scenario II-II, where the retailer settles its balance payment ($C_r Q$) that is owed to the manufacturer beyond the permissible period and after receiving the next shipment. Thus, the total annual cost of the retailer ($\psi_r^{\text{II-III}}$) can be written as follows:

$$\psi_r^{\text{II-III}} = \frac{A_r D}{Q} + C_r D + \frac{S_r Q}{2} + C_r D\left(e^{(k_m(\tau_r - t_m))} - e^{k_r t_m}\right). \tag{19}$$

Note that, $\psi_s^{\text{II-III}}$, $\psi_{m,w}^{\text{II-III}}$ and $\psi_{m,f}^{\text{II-III}}$ in this scenario are equivalent for those in scenario II-II. As a result, the total annual cost of the system ($\psi_{sc}^{\text{II-III}}$) can be written as follows:

$$\begin{aligned}\psi_{sc}^{\text{II-III}} = {} & \left(A_s + n_1 A_{m,w} + A_{m,f} + n_2 A_r\right)\frac{D}{n_2 Q} + (C_s\alpha + C_{m,w}\alpha + C_{m,f} + C_r)D \\ & - h_{m,w}\alpha\tau_m D + \frac{((h_s + S_s)(n_1 - 1) + h_{m,w} + S_{m,w})\alpha n_2 QD}{2Pn_1} + h_s\tau_m\alpha D \\ & + (C_{m,w} - C_s)\alpha D e^{k_s t_s} + \frac{h_{m,w}\alpha P n_1 D\tau_m^2}{2n_2 Q} - C_{m,w}\alpha D e^{k_m\tau_m} \\ & + \left(h_{m,f} + S_{m,f}\right)\left(\frac{Q(2D + (P - D)n_2 - P)}{2P}\right) + \left(h_{m,f} - h_r\right)\tau_r D \\ & + \left(C_r - C_{m,f}\right)D e^{k_m t_m} + \frac{S_r Q}{2} - C_r D e^{(k_r t_m)}. \end{aligned} \tag{20}$$

The solution of Eq. (20), after taking the first derivative with respect to Q, and then setting it equal to zero is as follows:

$$Q^{\text{II-III}} = \sqrt{\frac{A^{\text{II-III}}(n_1,\ n_2)}{h(n_1, n_2) + S^{\text{I-III}}(n_2)}}, \tag{21}$$

where

$$A^{\text{II-III}}(n_1, n_2) = \left[A_s + A_{m,f} + n_1 A_{m,w} + n_2 A_r\right]\frac{D}{n_2}.$$

3.1.7. Scenario III-I: $0 \le t_s \le \frac{n_2 Q}{Pn_1} \le \tau_m$ and $0 \le t_m = \tau_r \le \frac{Q}{D}$

This scenario considers that the manufacturer settles its raw materials balance ($C_w\alpha Q$) with the supplier beyond the permissible period and after receiving the next shipment. In addition, the retailer settles its payments with the manufacturer by the end of the permissible period (interest-free period). Accordingly, the total annual cost of the manufacturer at the raw material side ($\psi_w^{\text{III-I}}$) is as follows:

$$\psi_w^{\text{III-I}} = \frac{n_1 A_{m,w} D}{n_2 Q} + C_{m,w}\alpha D + \frac{S_{m,w}\alpha n_2 QD}{2Pn_1} + C_{m,w}\alpha D\left(e^{k_s(\tau_m - t_s)} - e^{k_m\tau_m}\right). \tag{22}$$

In this scenario, $\psi_s^{\text{III}-\text{I}}$, $\psi_{m,f}^{\text{III}-\text{I}}$ and $\psi_r^{\text{III}-\text{I}}$ are similar to that for scenario I-I. Accordingly, the total annual cost of the system is as follows:

$$\begin{aligned}\psi_{sc}^{\text{III}-\text{I}} = {} & \left(A_s + n_1 A_{m,w} + A_{m,f} + n_2 A_r\right)\frac{D}{n_2 Q} + (C_s\alpha + C_{m,w}\alpha + C_{m,f} + C_r)D \\ & + h_s\tau_m\alpha D + \frac{((h_s + S_s)(n_1 - 1) + S_{m,w})\alpha n_2 QD}{2Pn_1} \\ & + (C_{m,w} - C_s)\alpha D e^{k_s t_s} - C_{m,w}\alpha D e^{k_m\tau_m} \\ & + \left(h_{m,f} + S_{m,f}\right)\left(\frac{Q(2D + (P - D)n_2 - P)}{2P}\right) + h_{m,f}\tau_r D \\ & + \left(C_r - C_{m,f}\right)D e^{k_m t_m} + \frac{S_r Q}{2} - C_r D e^{(k_r t_m)}. \end{aligned} \tag{23}$$

The optimal order quantity found by solving Eq. (23), after setting the first derivative equal to zero and solving for Q, is as follows:

$$Q^{\text{III}-\text{I}} = \sqrt{\frac{A^{\text{III}-\text{I}}(n_1,\ n_2,\ t_m)}{h^{\text{I}-\text{I}}(n_1, n_2) + S^{\text{I}-\text{I}}(n_2)}}, \tag{24}$$

where

$$A^{\text{III}-\text{I}}\ (n_1,\ n_2,\ t_m) = \left[A_s + A_{m,f} + n_1 A_{m,w} + n_2 A_r + \frac{n_2 h_r D t_m^2}{2}\right]\frac{D}{n_2}.$$

3.1.8. Scenario III-II: $0 \le t_s \le \frac{n_2 Q}{Pn_1} \le \tau_m$ and $0 \le t_m < \tau_r \le T_r$

This scenario considers that the manufacturer settles its balance of purchased raw materials with the supplier after receiving the next shipment and after the permissible period has passed. Also, it assumed that the retailer settles its payment with the manufacturer after the permissible period has passed and before receiving the next shipment. Therefore, the total annual cost of the retailer that includes the order cost, purchasing cost, holding cost and interest charges is given as:

$$\psi_r^{\text{III}-\text{II}} = \frac{A_r D}{Q} + C_r D + \frac{h_r(Q - D\tau_r)^2}{2Q} + \frac{S_r Q}{2} + C_r D\left(e^{k_m(\tau_r - t_m)} - e^{k_r t_m}\right). \tag{25}$$

$\psi_s^{\text{III}-\text{II}}$ is the same as that for scenario I-I. Also, $\psi_{m,w}^{\text{III}-\text{II}}$ and $\psi_{m,f}^{\text{III}-\text{II}}$ are the same as that in scenario I-II. The total annual cost of the system can be written as:

$$\begin{aligned}\psi_{sc}^{\text{III}-\text{II}} = {} & \left(A_s + n_1 A_{m,w} + A_{m,f} + n_2 A_r\right)\frac{D}{n_2 Q} + (C_s\alpha + C_{m,w}\alpha + C_{m,f} + C_r)D \\ & + h_s\tau_m\alpha D + \frac{((h_s + S_s)(n_1 - 1) + S_{m,w})\alpha n_2 QD}{2Pn_1} + h_s\tau_m\alpha D \\ & + (C_{m,w} - C_s)\alpha D e^{k_s t_s} - C_{m,w}\alpha D e^{k_m\tau_m} \\ & + \left(h_{m,f} + S_{m,f}\right)\left(\frac{Q(2D + (P - D)n_2 - P)}{2P}\right) + \left(h_{m,f} - h_r\right)\tau_r D \\ & + \frac{(h_r + S_r)Q^2 + h_r(D_r\tau_r)^2}{2Q} + \left(C_r - C_{m,f}\right)D e^{k_m t_m} + \frac{S_r Q}{2} \\ & - C_r D e^{(k_r t_m)}. \end{aligned} \tag{26}$$

The solution to Eq. (26) is obtained as before and it is:

$$Q^{\text{III}-\text{II}} = \sqrt{\frac{A^{\text{III}-\text{II}}\ (n_1,\ n_2,\ \tau_r)}{h^{\text{III}-\text{II}}\ (n_1, n_2) + S^{\text{I}-\text{I}}(n_2)}}, \tag{27}$$

where

$$A^{\text{III}-\text{II}}\ (n_1,\ n_2,\ \tau_r) = \left[A_s + A_{m,f} + n_1 A_{m,w} + n_2 A_r + h_r\frac{n_2 D\tau_r^2}{2}\right]\frac{D}{n_2},$$

$$h^{\text{III}-\text{II}}(n_1, n_2) = [\ S_{m,w} + (n_1 - 1)(h_s + S_s)]\frac{\alpha n_2 D}{2n_1 P}.$$

3.1.9. Scenario III-III: $0 \le t_s \le \frac{n_2 Q}{P n_1} \le \tau_m$ and $0 \le t_m \le \frac{Q}{D} \le \tau_r$

Here, the manufacturer is assumed to settle its payment for purchased raw material with the supplier after receiving the next shipment and after the permissible period has passed. Also, the retailer settles its payment after receiving the next shipment from the manufacturer. Thus, the total annual cost of the retailer is given as:

$$\psi_r^{\text{III-III}} = \frac{A_r D}{Q} + C_r D + \frac{S_r Q}{2} + C_r D\left(e^{k_m(\tau_r - t_m)} - e^{k_r t_m}\right). \tag{28}$$

$\psi_s^{\text{III-III}}$, $\psi_{m,w}^{\text{III-III}}$ and $\psi_{m,f}^{\text{III-III}}$ for this scenario are the same as that for Case III-I. Subsequently, the total annual cost ($\psi_{sc}^{\text{III-III}}$) of this supply chain can be written as follows:

$$\begin{aligned}\psi_{sc}^{\text{III-III}} = {} & \left(A_s + n_1 A_{m,w} + A_{m,f} + n_2 A_r\right)\frac{D}{n_2 Q} + (C_s\alpha + C_{m,w}\alpha + C_{m,f} + C_r)D \\ & + h_s\tau_m\alpha D + \frac{((h_s + S_s)(n_1 - 1) + S_w)\alpha n_2 Q D}{2Pn_1} + h_s\tau_m\alpha D \\ & + (C_{m,w} - C_s)\alpha D e^{k_s t_s} - C_{m,w}\alpha D e^{k_m\tau_m} \\ & + \left(h_{m,f} + S_{m,f}\right)\left(\frac{Q(2D + (P - D)n_2 - P)}{2P}\right) + \left(h_{m,f} - h_r\right)\tau_r D \\ & + \left(C_r - C_{m,f}\right)D e^{k_m t_m} + \frac{S_r Q}{2} - C_r D e^{(k_r t_m)}.\end{aligned} \tag{29}$$

The optimal order quantity that minimizes Eq. (29) is:

$$Q^{\text{III-III}} = \sqrt{\frac{A^{\text{III-III}}(n_1, n_2)}{h^{\text{III-II}}(n_1, n_2) + S^{\text{I-III}}(n_2)}}, \tag{30}$$

where

$$A^{\text{III-III}}\ (n_1,\ n_2) = \left[A_s + A_{m,f} + n_1 A_{m,w} + n_2 A_r\right]\frac{D}{n_2}.$$

3.2. Uncommon cycle length

The purpose of this section is to illustrate the adoption of an uncommon cycle length approach in constructing the mathematical models. In this case, the total cost model per cycle for each player has to be divided by the cycle length of the player. For illustrative purposes, the total cost model per cycle of the supplier under scenario I-I is as follows:

$$\Psi_s^{\text{I-I}} = A_s + C_s\alpha n_2 Q + \frac{n_1(n_1 - 1)}{2}(h_s + S_s)\left(\frac{\alpha n_2^2 Q^2}{P n_1^2}\right) + h_s\tau_m\alpha n_2 Q + (C_{m,w} - C_s)\alpha n_2 Q e^{k_s t_s} - C_{m,w}\alpha n_2 Q e^{k_s(\tau_m - t_s)}. \tag{31}$$

The total cost of the supplier per unit per year is determined by dividing Eq. (31) by the cycle length of the supplier ($T_{m,w} = \frac{n_2 Q}{P}$) that becomes as follows:

$$\psi_s^{\text{I-I}} = \frac{A_s P}{n_2 Q} + C_s\alpha P + \frac{n_1(n_1 - 1)}{2}(h_s + S_s)\left(\frac{\alpha n_2 Q}{n_1^2}\right) + h_s\tau_m\alpha P + (C_{m,w} - C_s)\alpha P e^{k_s t_s} - C_{m,w}\alpha P e^{k_s(\tau_m - t_s)}. \tag{32}$$

The mathematics for the other scenarios can be easily determined following the same procedure as in Eq. (32), and for keeping the paper concise they were left out. Fig. 2 shows the different cycle lengths for each player involved in the supply chain.

4. Numerical results

In this section, numerical examples are presented to differentiate between all scenarios that were developed in Section 3, when a delay in payments is considered as a coordination mechanism in a three-level (supplier–manufacturer–retailer) supply chain. The length of the permissible period and the time of the payments are considered as decision variables. The purpose of the numerical examples is to identify the best scenario that gives minimal total cost under permissible delay in payments. In order to set the ranges for the input parameters of the numerical examples that would be generated and solved, we surveyed many published papers that investigated coordination of a three-level in a supply chain, and looked closely into their numerical examples, to have an idea of the numerical example ranges. In addition, we considered the suggestions of Waters [33] with regard to the financial and physical holding costs. The number of simulation runs was decided to cover a wider range of possible numerical examples. Also, it was mentioned in Burton et al. [34] that the number of runs varied from 100 to 100,000 runs. Increasing the number of runs in a simulation reduces the error percentage. For example, in our numerical analysis, we have varied the annual demand for a thousand run, which gave an error range of 0.34–4.08% with a standard deviation 1.12%. Then, we ran the simulation for 5000 times and found that the error range was from 0.32% to 3.92% and a standard deviation of 1.17%. Finally, we ran it for a 10,000 times, and the error ranged from 0.35% to 1.61%

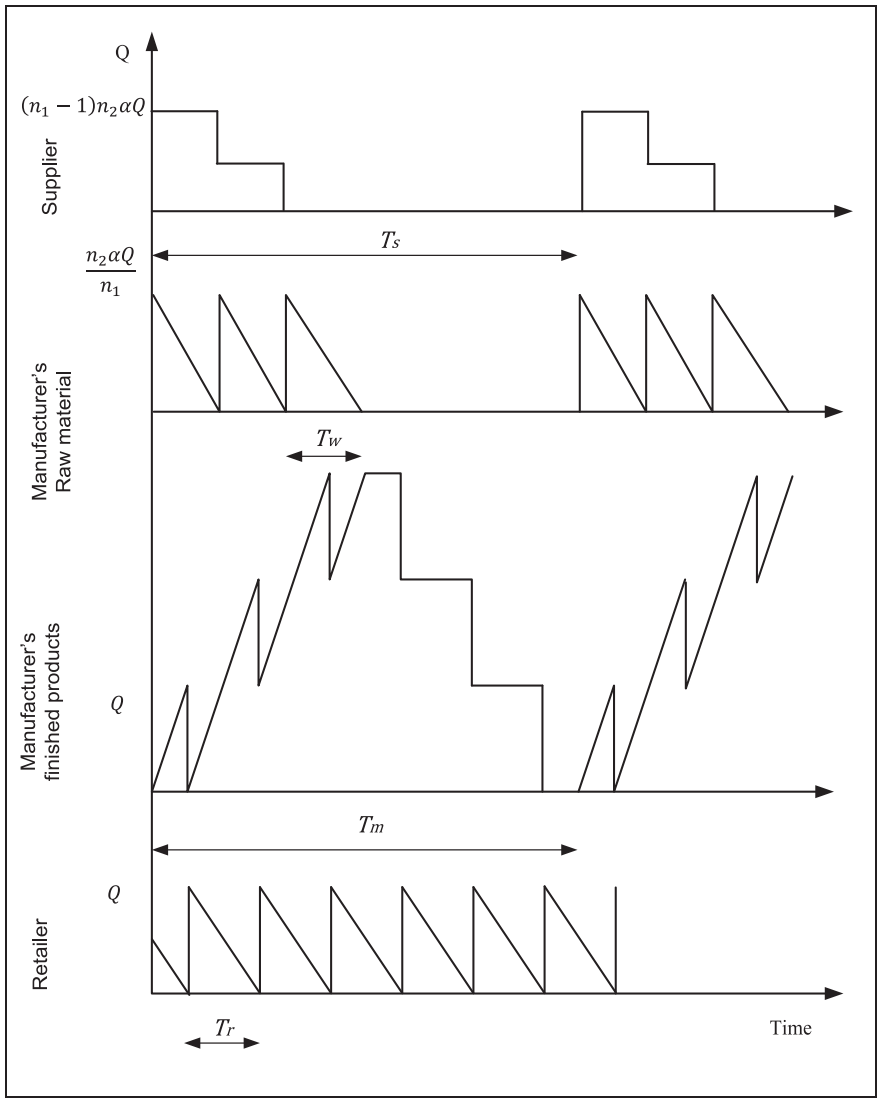

Fig. 2. Inventory behavior for the supplier, manufacturer and the retailer under uncommon cycle assumption.

Table 1
Ranges of the independent variables.

	D	P	A_s	$A_{m,w}$	$A_{m,f}$	A_r	h_s	s_s	$h_{m,w}$	$s_{m,w}$	$h_{m,f}$	$s_{m,f}$	h_r	s_r	k_s	k_m	k_r
Min	1000	1100	50	50	50	50	2	2	3	3	5	5	7	7	0%	0%	0%
Max	5000	10,000	500	500	500	500	6	6	9	9	15	15	20	20	10%	10%	10%
Average	3009.7	5485	247	243	274	276	4	4	6	6	9	9	13	13	5%	5%	5%

with standard deviation of 0.46%. Accordingly, we decided to consider the 10,000 runs in the numerical example to increase the accuracy of our analysis; the ranges of the independent variable parameters are shown in Table 1. The physical holding cost ranges between 9% and 20% of the unit cost and the financial holding cost ranges between 10% and 15% of the unit cost. One of the assumptions in this paper is to avoid shortages; to satisfy this assumption the production rate was always kept greater than the demand rate. In addition, the value of the product increases as it moves down the supply chain. This assumption is kept when solving the numerical examples by having the financial and physical holding costs of the supplier the lowest and those of the retailer the highest of the three levels.

Table 2
An illustration of assuming common cycle rather than uncommon cycle.

		I-I	I-II	I-III	II-I	II-II	II-III	III-I	III-II	III-III
Worst numerical examples	Common cycle	526,993.30	526,907.55	529,167.18	526,855.30	526,900.78	528,993.78	528,198.22	528,370.79	529,682.59
	Uncommon cycles	634,660.30	634,571.13	636,478.42	634,418.65	566,986.96	636,245.32	567,756.20	567,890.99	568,892.18
	Difference	16.96%	16.97%	16.86%	16.95%	7.07%	16.86%	6.97%	6.96%	6.89%
Best numerical examples	Common cycle	107,734.73	107,738.73	108,012.61	107,722.76	107,726.76	107,999.49	107,861.25	107,870.47	108,174.00
	Uncommon cycles	129,729.74	129,733.74	129,987.56	129,708.11	129,712.11	129,966.22	129,954.37	129,958.39	130,209.27
	Difference	16.95%	16.95%	16.91%	16.95%	16.95%	16.90%	17.00%	17.00%	16.92%

Table 3
Input parameters of the numerical example.

D	P	α	A_s	$A_{m,w}$	$A_{m,f}$	A_r	C_s	$C_{m,w}$	$C_{m,f}$	C_r	h_s	s_s	$h_{m,w}$	$s_{m,w}$	$h_{m,f}$	$s_{m,f}$	h_r	s_r	k_s	k_m	k_r
3069	4720	1	441	206	175	384	20	30	50	70	3	3	3	7.5	12	9	13.3	7.7	1%	8%	4%

Subscript: s = supplier, w = manufacturer's raw material, m = manufacturer, r = retailer, and u = unit.

Table 4
Results of the numerical example.

Scenario	n_1	n_2	Q	t_s	τ_m	t_m	τ_r	Supplier	Manufacturer	Retailer	Supply chain total cost
No delay	1	2	294	0	0	0	0	2268.37	99,184.59	221,926.17	77,859.13
I-I	1	2	296	0.1019	0.1019	0.0181	0.0181	3143.53	98,721.48	221,092.51	77,437.52
I-II	1	2	296	0.1020	0.1020	0.0081	0.0181	3143.71	98,501.52	221,349.11	77,474.34
I-III	1	2	296	0.1046	0.1046	0.0964	0.1064	2744.56	102,786.84	219,296.22	79,307.61
II-I	1	2	296	0	0.1068	0.0181	0.0181	3050.09	98,784.13	221,091.23	77,405.45*
II-II	1	2	297	0	0.1068	0.0080	0.0181	3050.09	98,563.07	221,349.10	77,442.27
II-III	1	2	290	0	0.1297	0.0944	0.1043	2861.43	102,481.08	219,372.60	79,195.12
III-I	1	2	290	0	0.1391	0.0182	0.0182	3373.42	98,482.75	221,113.57	77,449.75
III-II	1	2	301	0	0.1440	0.0082	0.0181	3332.48	98,335.59	221,336.03	77,484.11
III-III	1	2	304	0	0.1798	0.0990	0.1089	3178.34	102,546.02	219,254.9	79,403.90

Note: Asterisk (*) indicates the optimal values.

4.1. Comparison between uncommon and common cycle length

Using simulation, the worst and best numerical examples were used to compare between common and uncommon cycle in each scenario, it is illustrated in Table 2. The results show that adopting a common cycle length for the player in the supply chain will reduce the total cost of the supply chain. Accordingly, the numerical examples and the discussion from here and afterwards will be based on the common cycle length assumption.

4.2. Detailed examples for common cycle length

For illustrative purposes, a numerical example was selected randomly and is presented in this section, where Table 3 shows the input parameters for this numerical example. Solver is one of the analysis tools available in Microsoft Excel that is used for optimization problems. It was used to find the minimum total cost by setting n_1, n_2, t_s, t_m, τ_m and τ_r as decision variables. In the calculations of the numerical examples, the purchasing of raw materials and the production costs (some of the fixed costs in the system) at the manufacturer have been neglected, to clarify the difference between the total costs.

Table 4 illustrates the results of the numerical example for nine scenarios, and a scenario without the permissible delay in payments. The results show that the supplier achieves its minimum cost when no delay in payments is offered to downstream players. The reason is that the supplier's rate of return is very low. The manufacturer attains its minimum cost under scenario III-II because it does not incur a financial holding cost, generates profit from investment during the delay period, and receives interest from the retailer on a delay balance. The retailer achieves its minimum cost under scenario III-III, since the retailer does not incur a financial holding cost for items in its inventory and has more time to invest its money before paying the manufacturer. As this paper considers a centralized system, the decision of the best scenario is made based on the scenario that minimizes the total system cost (i.e. scenario II-I), where $n_1 = 1$, $n_2 = 2$, $t_s = 0$, $\tau_m = 0.1068$, $t_m = \tau_r = 0.0181$, $Q^* = 296$ and the total cost is 77,405.45$/year.

To verify the reliability of Solver's results, 1000 numerical examples were solved in MATLAB and Solver. When comparing the results obtained using both software, a small insignificant error that ranges between −1.036% and 0.309% was noted, thus we decided to use Excel Solver enhanced with Visual Basic for Applications (VBA) in our simulation. Ten thousand numerical examples were randomly generated and solved under the nine scenarios. The results are summarized in Table 5.

To form a conclusion about the most preferable scenario, we ran a regression analysis for the 10,000 numerical examples when the dependent variables being the permissible periods, the number of shipments and the total cost of the supply

Table 5
Summary of the 10,000 numerical example.

Optimal scenario	Best minimum cost	Average	Difference between the average total cost and the average of the optimal scenario									R
			No delay	I-I	I-II	I-III	II-I	II-II	II-III	III-I	III-II	III-III	
No delay	170	415,423.52	–	1239.18	1136.36	2513.68	1235.02	1033.33	2448.65	1900.52	1725.96	3541.86	6
I-I	2017	311,089.31	738.84	–	117.84	832.54	40.32	128.05	820.45	789.69	929.99	1800.08	2
I-II	108	327,435.63	569.11	117.46	–	1124.72	148.77	20.96	1107.97	910.24	794.70	2116.44	7
I-III	0	315,449.55	(123.32)	(923.91)	(852.72)	–	(1011.47)	(946.00)	(125.63)	(667.63)	(582.93)	402.11	10
II-I	6978	312,577.04	974.58	99.68	203.61	1000.64	–	106.46	871.95	318.26	442.40	1362.10	1
II-II	1032	318,466.73	806.04	227.02	120.41	1232.21	128.23	–	1074.57	389.56	293.87	1565.58	3
II-III	1	319,541.29	273.47	979.44	934.39	247.67	754.28	709.10	–	462.91	343.81	396.09	9
III-I	249	315,764.79	1392.96	423.05	516.69	1379.09	238.93	353.61	1145.41	–	108.36	1114.59	5
III-II	301	336,213.39	1954.85	633.69	561.10	1580.72	399.29	1097.42	1280.47	238.89	–	1065.44	4
III-III	2	177,975.00	4231.38	528.15	614.10	612.87	416.79	562.58	787.93	261.16	1.15	–	8

Note: R = ranking based on the number of achieving the best minimum cost from the simulation run.

Table 6
The strong effect of independent variables on each scenario.

	No delay	I-I	I-II	I-III	II-I	II-II	II-III	III-I	III-II	III-III
D	√	√	√	√	√	√	√	√	√	√
P	√	√	√	✗	√	√	✗	√	√	√
A_s	√	√	√	√	√	√	√	√	√	√
h_s	√	√	√	√	√	√	√	√	√	√
s_s	√	√	√	√	√	√	√	√	√	√
k_s	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
$A_{m,w}$	√	√	√	√	√	√	√	√	√	√
$h_{m,w}$	√	✗	✗	✗	✗	✗	✗	✗	✗	✗
$s_{m,w}$	√	√	√	√	√	√	√	√	√	√
$A_{m,f}$	√	√	√	√	√	√	√	√	√	√
$h_{m,f}$	√	√	√	√	√	√	√	√	√	√
$s_{m,f}$	√	√	√	√	√	√	√	√	√	√
k_m	✗	√	√	✗	√	√	√	√	√	√
A_r	√	√	√	√	√	√	√	√	√	√
h_r	√	√	√	√	√	√	√	√	√	√
s_r	√	√	√	√	√	√	√	√	√	√
k_r	✗	✗	✗	√	✗	✗	√	✗	✗	√

Note: √ = significantly affects the supply chain's cost, ✗ = insignificantly affects the supply chain's cost.

Table 7
A sample of the sensitivity when varying interest earnings and holding costs.

Interest rates	I-I	I-II	I-III	II-I	II-II	II-III	III-I	III-II	III-III
$k_r > k_m > k_s$	76,298.29	76,324.71	76,517.38	76,098.39	76,124.96	76,283.48	76,084.86	76,097.36	76,290.84
$k_m > k_r > k_s$	76,517.50	76,529.81	77,395.18	76,110.98	76,324.94	77,067.63	75,665.94	75,576.47	76,153.57
$k_s > k_m > k_r$	77,625.84	77,630.44	79,167.18	77,172.75	77,177.46	78,537.97	77,161.76	77,148.97	78,590.80
$k_s > k_r > k_m$	77,252.28	77,270.81	77,817.43	77,101.11	77,119.64	77,651.30	77,254.90	77,266.05	77,898.89
$k_s = k_m = k_r$	77,141.48	77,156.89	77,902.75	76,796.28	76,811.82	77,488.01	76,784.47	76,771.31	77,519.04

chain. The regression analyses show that the following independent variables, D, A_s, h_s, s_s, A_w, s_w, A_m, h_m, s_m, A_r, h_r and s_r, significantly affect the total cost in all scenarios. Moreover, the holding cost of the raw materials (h_w) has a significant effect on the total cost when delay in payments is not offered in a supply chain. In addition, the retailer's interest rate (k_r) has a significant effect in the following scenarios, I-III, II-III and III-III. However, the analysis shows that the supplier's interest rate (k_s) has an insignificant effect on the total cost in all scenarios. The production rate (P) is one of the independent variables having an insignificant effect in scenarios I-III and II-III. In addition, the manufacturer interest rate (k_m) has an insignificant effect when there is no delay in payments applied in coordinating the supply chain and in scenario I-III. Table 6 summarizes the effect of each independent variable in all scenarios.

In the simulation run, different cases have been considered. First, the ratio of demand over the production rate is varied between 0.1 and 0.99. Also, all scenarios are tested under five possible cases of interest as illustrated in Table 6. The first case, $k_r > k_m > k_s$, is when the interest rate of the manufacturer is between the supplier's and the retailers', with the retailer having the highest interest rate. The second case, $k_m > k_r > k_s$, is when the interest rate of the manufacturer is the highest, and so on. Moreover, different possible cases of comparing the holding cost between the players are considered in Tables 7 and 8.

Table 8
A sample of the sensitivity when varying interest earnings and holding costs.

Holding costs	I-I	I-II	I-III	II-I	II-II	II-III	III-I	III-II	III-III
$h_r > h_{m.f} > h_s$	76,327.63	76,331.32	77,156.77	76,327.33	76,297.70	77,061.31	76,323.99	76,324.91	77,218.08
$h_{m.f} > h_r > h_s$	76,691.05	76,703.44	79,019.63	76,658.24	76,670.63	78,907.46	76,687.58	76,684.90	79,058.00
$h_s > h_{m.f} > h_r$	75,585.55	75,600.94	77,510.96	75,605.84	75,621.23	77,531.25	78,888.69	78,934.90	81,013.90
$h_s > h_r > h_{m.f}$	74,735.01	74,743.14	74,835.82	74,755.30	74,763.42	74,856.10	78,114.02	78,162.35	78,385.97

Table 9
Summary of the performance for all scenarios when varying D/P.

D/P	Interest case	I-I	I-II	I-III	II-I	II-II	II-III	III-I	III-II	III-III
0.25	$k_r > k_m > k_s$	75,156	75,183	75,401	75,074	75,101	75,296	75,072	75,096	75,316
	$k_m > k_r > k_s$	75,802	75,814	76,899	75,647	75,730	76,742	75,468	75,457	76,461
	$k_s > k_m > k_r$	76,409	76,413	78,215	76,224	76,228	77,983	76,222	76,220	78,029
	$k_s > k_r > k_m$	75,903	75,921	76,547	75,839	75,858	76,476	75,907	75,909	76,569
	$k_s = k_m = k_r$	75,964	75,979	76,824	75,822	75,838	76,644	75,820	75,818	76,658
0.5	$k_r > k_m > k_s$	75,881	75,907	76,108	75,723	75,750	75,919	75,715	75,733	75,936
	$k_m > k_r > k_s$	76,254	76,266	77,258	75,943	76,105	76,993	75,613	75,576	76,441
	$k_s > k_m > k_r$	77,180	77,185	78,848	76,824	76,829	78,417	76,817	76,809	78,500
	$k_s > k_r > k_m$	76,760	76,778	77,351	76,640	76,658	77,218	76,763	76,770	77,413
	$k_s = k_m = k_r$	76,710	76,726	77,505	76,439	76,454	77,172	76,432	76,423	77,203
0.75	$k_r > k_m > k_s$	76,569	76,595	76,783	76,342	76,368	76,522	76,324	76,333	76,520
	$k_m > k_r > k_s$	76,596	76,608	77,319	75,987	76,319	76,955	75,449	75,330	75,839
	$k_s > k_m > k_r$	77,915	77,920	79,256	77,383	77,388	78,536	77,362	77,335	78,588
	$k_s > k_r > k_m$	77,970	77,989	78,430	77,400	77,419	77,934	78,783	78,844	79,273
	$k_s = k_m = k_r$	77,421	77,437	78,152	77,029	77,044	77,603	77,014	76,997	77,594
1	$k_r > k_m > k_s$	77,457	77,484	77,097	76,549	76,576	76,706	76,507	76,481	76,621
	$k_m > k_r > k_s$	76,281	76,294	76,875	76,173	75,834	76,407	74,221	73,576	73,668
	$k_s > k_m > k_r$	78,886	78,890	80,076	77,413	77,419	78,389	77,364	77,293	78,364
	$k_s > k_r > k_m$	79,845	79,863	80,478	78,630	78,599	78,753	79,753	79,898	80,368
	$k_s = k_m = k_r$	78,070	78,085	78,684	77,111	77,127	77,593	77,073	77,014	77,490

Table 9 shows that scenario II-I achieves the minimum total cost in around 70% of the 10,000 examples in all cases while the ratio of $\frac{D}{P}$ was varied between 0.1 and 0.99. When the ratio $\frac{D}{P}$ is 0.25 all scenarios achieved their best minimum total annual costs when the interest combination is $k_r > k_m > k_s$. When $\frac{D}{P} = 0.50$ all scenarios achieved their best minimum costs under the case of $k_r > k_m > k_s$, except scenarios III-I and III-II where they achieved their minimum when the interest rates combination is $k_m > k_r > k_s$. When the ratio of $\frac{D}{P} = 0.75$, four scenarios achieved their minimum total cost under case $k_r > k_m > k_s$ and the remaining five scenarios got their minimum cost when $k_m > k_r > k_s$. When setting the ratio $\frac{D}{P} = 1.0$, all scenarios achieved their minimum total cost under the case $k_m > k_r > k_s$. Although that scenario II-I has the best minimum total cost in most of the numerical examples, however, the difference between the best and worst scenario is just 1.44%.

On the other hand, all scenarios got their highest total cost when $\frac{D}{P}$ was 0.25 and 0.50 and when $k_s > k_m > k_r$.

5. Managerial insights

Permissible delay in payments has been widely implemented as a business practice as it is a coordination mechanism that helps companies in reducing their total cost. For instance, Wal-Mart has achieved a 1.2% reduction in its capital expenditures by having a one month delay in payment [4]. The permissible period varies across companies, from just a few weeks to months, as in the case of Wal-Mart. The scenarios and cases presented in this paper considered the delay in payments offered between adjacent players in a supply chain as a decision variable. One may argue if such an approach is practical. However, the suggested approach provides some guideline to decision makers in a supply chain when negotiating credit terms and payments schedules. For instance, it is best when the three players coordinate using a permissible delay in payment and operate under scenario II-I, i.e. when the manufacturer settles its balance beyond the permissible delay period and before receiving the new shipment from the supplier, while the retailer settles its balance before the end of the permissible delay period. This can be achieved by sharing the additional supply chain profit proportionally among the three players, so all of the players have an incentive to operate under the best scenario.

Our results are insensitive to the demand to production ratio, as the best scenario is mostly the same for different values of the ratio. An important outcome of the analysis is to provide decision makers with insights into which parameters significantly affect the total cost, so a closer look at these parameters is required, versus the ones that can be ignored. For instance, the raw material holding cost does not have a significant effect on the total cost, unlike manufacturer's and retailer's holding cost parameters. Thus, actions and best practices towards reducing the holding costs at the manufacturer and retailer are important to keep the supply chain cost low. Also, the supplier's and retailer's interest rates can be ignored as they are insignificant in affecting the supply chain's cost, unlike the manufacturer's interest rate. The production rate is

another parameter that needs to be closely monitored as it significantly affects the cost, in addition to the fixed setup costs for each player. A sensitivity analysis is also performed to study the effect of the relative comparison of the interest rates on the best scenario. Based on the relative ranking of the interest rates, decision makers can simply identify the best scenario and work together to operate under this specific scenario to minimize the total supply chain cost.

6. Summary and conclusions

This paper considers a three-level supply chain (supplier–manufacturer–retailer) with permissible delay in payments as a decision variable. The supplier offers a permissible delay in payments to the manufacturer who in turn, offers the retailer a permissible delay. The paper investigates nine different possible scenarios that occur in real life when permissible delay in payments is used as coordination mechanism.

Ten thousand numerical examples were considered to compare between all nine scenarios. In addition, we have considered the case when no permissible delay is assumed. A numerical example was selected randomly out of these 10,000 to illustrate the behavior of each scenario. For the supplier, it is better not to have a permissible delay between the players in order to minimize its total cost. The manufacturer reduces its total cost to the best minimal cost when the supply chain follows scenario III-II. The retailer gets its best minimal cost when the supply chain follows scenario I-III. However, the best scenario that generates the minimal total cost for the whole supply chain is scenario II-I.

Regression analyses were conducted to analyze the simulation run of 10,000 numerical examples, to illustrate which independent variables have a significant effect in each scenario. The independent variables that do not have a significant effect were D, A_s, h_s, s_s, A_{mr}, s_{mr}, A_m, h_m, s_m, A_r, h_r and s_r. Ranking the best scenarios, we observe that scenarios II-I, I-I and II-II are the best three. The worst three performing ones are I-III, II-III and III-III. A sensitivity analysis was implemented to check the limitations of the proposed model. All scenarios were tested under five different interest rates cases of interest rate comparison, besides varying the ratio of demand to production rate. The total cost of all scenarios increase by increasing the ratio of $\frac{D}{P}$. Under each interest case and for each ratio considered, the scenario that attains the best minimal total cost is scenario II-I.

In summary, we conclude that the best scenario to follow is II-I when coordinating a three-level of supply chain with permissible delay in payments. First, it has achieved the best minimal total cost in 70% of the cases of the simulation run. Second, this scenario helps the manufacturer in managing its financial flow, as the manufacturer would first receive the payment from the retailer and then pay the supplier. Third, although the total cost difference between all scenarios is small, it is better to execute a single scenario instead of alternating between multiple scenarios, which could cost more effort and time; a positive result could be achieved by getting all players in the chain to agree upon the scenario II-I.

The models that are presented in this paper can be extended in different possible directions. First, one option is to investigate the effect of the permissible delay in payments under different production polices rather than Hill's [25]. Second, assuming a stochastic demand and including a production policy at the supplier is another possible extension. One can also consider multiple un-identical retailers and suppliers.

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Appendix A

At the supplier

Total cost per cycle

A_s: Supplier's setup cost per cycle.

$C_s\alpha n_2 Q$: Supplier's production cost (supplier's cost of raw materials per cycle).

$\frac{n_1(n_1-1)}{2}(h_s+S_s)(\frac{\alpha n_2^2 Q^2}{Pn_1^2})$: Supplier's holding cost per cycle.

$h_s\tau_m\alpha n_2 Q$: Supplier's financial holding costs that are incurred until the balance is paid by the retailer.

$(C_{m,w}-C_s)\alpha n_2 Q e^{k_s t_s}$: Offering a delay in payments by the supplier for 't_s' period will cause him/her to incur a losing opportunity cost of investing the profit in a risk-free investment.

$C_{m,w}\alpha n_2 Q e^{k_s(\tau_m-t_s)}$: Interest charges that would be paid by the manufacturer if the payment is made beyond the permissible delay in payments.

$$\psi_s^{\mathrm{I-I}} = A_s + C_s\alpha n_2 Q + \frac{n_1(n_1-1)}{2}(h_s+S_s)\left(\frac{\alpha n_2^2 Q^2}{Pn_1^2}\right) + h_s\tau_m\alpha n_2 Q + (C_{m,w}-C_s)\alpha n_2 Q e^{k_s t_s} - C_{m,w}\alpha n_2 Q e^{k_s(\tau_m-t_s)}. \tag{A1}$$

Total annual cost: Divide Eq. (A1) by the common cycle length ($\frac{n_2Q}{D}$)

$$\psi_s^{\text{I–I}} = \frac{A_sD}{n_2Q} + C_s\alpha D + \frac{n_1(n_1-1)}{2}(h_s+S_s)\frac{\alpha n_2QD}{Pn_1^2} + h_s\tau_m\alpha D + (C_{m,w}-C_s)\alpha De^{k_st_s} - C_{m,w}\alpha De^{k_s(\tau_m-t_s)}. \tag{A2}$$

At the manufacturer's raw material

Total cost per cycle

n_1A_w: Manufacturer's order cost per cycle.

$C_{m,w}\alpha n_2Q$: Manufacturer's purchasing cost of the raw materials per cycle.

$h_{m,w}n_1\frac{(n_2\alpha Q-\alpha Pt_sn_1)^2}{2\alpha Pn_1^2} = h_{m,w}\frac{\alpha^2n_2^2Q^2}{2\alpha Pn_1} - h_{m,w}\alpha n_2Qt_s + h_{m,w}\frac{n_1\alpha Pt_s^2}{2}$: Manufacturer's holding cost per cycle that is incurred after the settlement of the balance payment is made by the manufacturer.

$S_{m,w}\frac{\alpha n_2^2Q^2}{2Pn_1}$: Manufacturer's physical holding cost per cycle.

$C_{m,w}\alpha n_2Q(e^{ks(\tau_m-t_s)} - e^{k_m\tau_m})$: Interest charges that would be paid by the manufacturer and the interest earnings generated from investing the balance in a risk-free investment during the permissible period.

$$\psi_{m,w}^{\text{I–I}} = n_1A_{m,w} + \frac{n_1C_{m,w}\alpha n_2Q}{n_1} + \frac{h_{m,w}n_1\alpha^2n_2^2Q^2}{2\alpha Pn_1^2} - \frac{h_{m,w}n_1\alpha n_2Qt_s}{n_1} + \frac{h_{m,w}n_1\alpha Pt_s^2}{2} + \frac{n_1\ S_{m,w}\alpha n_2^2Q^2}{2Pn_1^2} + \frac{n_1C_{m,w}\alpha n_2Q\left(e^{ks(\tau_m-t_s)} - e^{k_m\tau_m}\right)}{n_1}, \tag{A3}$$

Total annual cost: Divide Eq. (A3) by the cycle length ($\frac{n_2Q}{D}$)

$$\psi_{m,w}^{\text{I–I}} = \frac{n_1A_{m,w}D}{n_2Q} + \frac{C_{m,w}\alpha n_2QD}{n_2Q} + \frac{h_{m,w}\alpha^2n_2^2Q^2D}{2\alpha Pn_1n_2Q} - \frac{h_{m,w}\alpha n_2Qt_sD}{n_2Q} + \frac{h_{m,w}n_1\alpha Pt_s^2D}{2n_2Q} + \frac{S_{m,w}\alpha n_2^2Q^2D}{2Pn_1n_2Q} + \frac{C_{m,w}\alpha n_2QD}{n_2Q}\left(e^{ks(\tau_m-t_s)} - e^{k_m\tau_m}\right), \tag{A4}$$

At the manufacturer's finished product

The holding cost function has been adopted from [25] page 496.

Total cost per cycle

A_m: Manufacturer's setup cost for finished goods per cycle.

$C_{m,f}n_2Q$: Manufacturer's production cost per cycle.

$(h_{m,f}+S_{m,f})[\frac{n_2Q^2(2D+(P-D)n_2-P)}{2DP}]$: Manufacturer's holding costs for finished goods.

$h_{m,f}\tau_rn_2Q$: Manufacturer's financial holding cost resulting from offering the retailer a delay in payments.

$n_2Q(C_r-C_{m,f})e^{k_mt_m}$: Opportunity cost per cycle of investing the manufacturer's profit in a risk-free investment.

$C_rn_2Qe^{k_m(\tau_r-t_m)}$: Retailer's interest charges per cycle that would be paid for the manufacturer when the retailer settles its balance beyond the permitted period.

$$\psi_m^{\text{I–I}} = A_m + C_{m,f}n_2Q + (h_{m,f}+S_{m,f})\left[\frac{n_2Q^2(2D+(P-D)n_2-P)}{2DP}\right] + h_{m,f}\tau_rn_2Q + n_2Q\left(C_r-C_{m,f}\right)e^{k_mt_m} - C_rn_2Qe^{k_m(\tau_r-t_m)}. \tag{A5}$$

Total annual cost: Divide Eq. (A5) by the common cycle length ($\frac{n_2Q}{D}$)

$$\psi_m^{\text{I–I}} = \frac{A_{m,f}D}{n_2Q} + C_{m,f}D + (h_{m,f}+S_{m,f})\left[\frac{Q(2D_r+(P-D)n_2-P)}{2P}\right] + h_{m,f}\tau_rD + \left(C_r-C_{m,f}\right)De^{k_mt_m} - C_rDe^{k_m(\tau_r-t_m)}. \tag{A6}$$

At the retailer

Total cost per cycle

n_2A_r: Retailer's order cost per cycle.

C_rn_2Q: Retailer's purchasing cost per cycle.

$n_2h_r\frac{(Q-D_rt_m)^2}{2\ D}$: Retailer's holding costs incurred after settling the payments with the manufacturer.

$\frac{n_2s_r}{2D}Q^2$: Retailer's physical holding cost per cycle.

$n_2QC_r(1-e^{k_r t_m})$: Retailer's interest earnings of investing his/her balance during the permitted period.

$$\psi_r^{\mathrm{I-I}} = n_2A_r + C_r n_2 Q + n_2 h_r \frac{(Q - D_r t_m)^2}{2\ D} + \frac{n_2 s_r}{2D} Q^2 + n_2 Q C_r\left(1 - e^{k_r t_m}\right). \tag{A7}$$

Total annual cost: Divide Eq. (A7) by the cycle length ($\frac{n_2 Q}{D}$)

$$\psi_r^{\mathrm{I-I}} = \frac{A_r D}{Q} + C_r D + h_r \frac{(Q - Dt_m)^2}{2\ Q} + \frac{s_r}{2} Q + C_r D\left(1 - e^{k_r t_m}\right). \tag{A8}$$

Total annual cost of the system

Can be found by summing Eqs. (A2), (A4), (A6) and (A8)

$$\begin{aligned}
\psi_{sc}^{\mathrm{I-I}} = {} & \left(A_s + n_1 A_{m,w} + A_{m,f} + A_r n_2\right)\frac{D}{n_2 Q} + \left(C_s\alpha + C_{m,w}\alpha + C_{m,f} + C_r\right)D \\
& + (h_s\tau_m - h_{m,w}t_s)\alpha D \\
& + \frac{(n_1 - 1)(h_s + S_s)\ \alpha n_2 QD + h_{m,w}\alpha n_2 QD + S_{m,w}\alpha n_2 QD}{2Pn_1} \\
& + (C_{m,w} - C_s)\ \alpha D e^{k_s t_s} + \frac{h_{m,w}\ \alpha P n_1 D t_s^2 + h_r n_2 (Q - Dt_m)^2}{2n_2 Q} \\
& - C_{m,w}\alpha D e^{k_m \tau_m} + \frac{Q(2D + (P - D)n_2 - P)\left(h_{m,f} + S_{m,f}\right) + S_r PQ}{2P} \\
& + h_{m,f}\tau_r D + \left(C_r - C_{m,f}\right)D e^{k_m t_m}\ - C_r D e^{k_r t_m}, \\
\frac{\partial}{\partial Q}\psi_{sc}^{\mathrm{I-I}} = {} & -\left(A_s + n_1 A_{m,w} + A_{m,f} + A_r n_2\right)\frac{D}{n_2 Q^2} \\
& + \frac{(n_1 - 1)(h_s + S_s)\ \alpha n_2 D + h_{m,w}\alpha n_2 D + S_{m,w}\alpha n_2 D}{2Pn_1} - \frac{h_{m,w}\ \alpha P n_1 D t_s^2}{2n_2 Q^2} \\
& + \frac{h_r\ n_2 Q}{2n_2} - \frac{h_r n_2 D^2 t_m^2}{2n_2 Q}\ + \frac{(2D + (P - D)n_2 - P)\left(h_{m,f} + S_{m,f}\right) + S_r P}{2P}.
\end{aligned} \tag{5}$$

Hence, the second derivative with respect to Q is:

$$\frac{\partial^2}{\partial Q^2}\psi_{sc}^{\mathrm{I-I}} = \left(A_s + n_1 A_{m,w} + A_{m,f} + A_r n_2\right)\frac{D}{n_2 Q^4} + \frac{h_{m,w}\ \alpha P n_1 D t_s^2}{2n_2 Q^4} + \frac{h_r\ n_2}{2n_2} + \frac{h_r n_2 D^2 t_m^2}{2n_2 Q^2} > 0.$$

Therefore, there is an optimal Q that minimizes the total system cost and it is convex, which can found by setting $\frac{\partial}{\partial Q}\psi_{sc}^{\mathrm{I-I}} = 0$ and solving for Q:

$$Q^{\mathrm{I-I}} = \sqrt{\frac{A^{\mathrm{I-I}}\ (n_1,\ n_2,\ t_s,\ t_m)}{h(n_1, n_2) + S(n_2)}}, \tag{6}$$

where

$$A^{\mathrm{I-I}}\ (n_1,\ n_2,\ t_s,\ t_m) = \left[A_s + A_{m,f} + n_1 A_{m,w} + n_2 A_r + \frac{\alpha n_1 h_{m,w} P t_s^2}{2} + \frac{n_2 h_r D t_m^2}{2}\right]\frac{D}{n_2},$$

$$h(n_1, n_2) = [h_{m,w} + \ S_{m,w} + (n_1 - 1)(h_s + S_s)]\frac{\alpha n_2 D}{2n_1 P},$$

$$S(n_2) = \left[\frac{S_r P + h_r P + (h_{m,f} + S_{m,f})(2D + n_2(P - D) - P)}{2P}\right].$$

Appendix B

Joint optimization procedure

It was not possible to show that Eq. (5) is jointly convex in Q, n_1, and n_2. So we followed the procedure suggested by Grubbström and Erdem [35] and Zanoni and Grubbstrom [36] to prove that the system allows a joint optimization for Q, n_1 and n_2. The total cost for the system is equal to

$$\psi_{sc}^{\mathrm{I-I}}(Q,n_1,n_2) = \left(A_s + n_1A_{m,w} + A_{m,f} + A_r n_2\right)\frac{D}{n_2Q} + \left(C_s\alpha + C_{m,w}\alpha + C_{m,f} + C_r\right)D$$
$$+(h_s\tau_m - h_{m,w}t_s)\alpha D + \frac{(n_1-1)(h_s+S_s)\ \alpha n_2QD + h_{m,w}\alpha n_2QD + S_{m,w}\alpha n_2QD}{2Pn_1}$$
$$+(C_{m,w}-C_s)\ \alpha De^{k_st_s} + \frac{h_{m,w}\ \alpha Pn_1Dt_s^2 + h_rn_2(Q-Dt_m)^2}{2n_2Q} - C_{m,w}\alpha De^{k_m\tau_w}$$
$$+\frac{Q(2D+(P-D)n_2-P)\left(h_{m,f}+S_{m,f}\right)+S_rPQ}{2P} + h_{m,f}\tau_rD$$
$$+\left(C_r - C_{m,f}\right)De^{k_mt_m}\ - C_rDe^{k_rt_m}.$$

The optimum order quantity that minimize the system's total cost is written as follows:

$$Q^{\mathrm{I-I}}(n_1,n_2) = \sqrt{\frac{A^{\mathrm{I-I}}\ (n_1,\ n_2,\ t_s,\ t_m)}{h^{\mathrm{I-I}}(n_1,n_2)+S^{\mathrm{I-I}}(n_2)}},$$

where

$$A^{\mathrm{I-II}}(n_1,\ n_2,\ t_s,\ t_m) = \left[A_s + A_{m,f} + n_1A_{m,w} + n_2A_r + \frac{\alpha n_1h_{m,w}Pt_s^2}{2} + \frac{n_2h_rDt_m^2}{2}\right]\frac{D}{n_2},$$

$$h^{\mathrm{I-I}}(n_1,n_2) = [h_{m,w} + \ S_{m,w} + (n_1-1)(h_s+S_s)]\frac{\alpha n_2D}{2n_1P},$$

$$S^{\mathrm{I-I}}(n_2) = \left[\frac{S_rP + h_rP + (h_{m,f}+S_{m,f})(2D+n_2(P-D)-P)}{2P}\right].$$

Now, let

$$S(n_1,\ n_2) = \left[A_s + A_{m,f} + n_1A_{m,w} + n_2A_r + \frac{\alpha n_1h_{m,w}Pt_s^2}{2} + \frac{n_2h_rDt_m^2}{2}\right]\frac{D}{n_2},$$

and

$$H(n_1,\ n_2) = [h_{m,w} + \ S_{m,w} + (n_1-1)(h_s+S_s)]\frac{\alpha n_2D}{2n_1P} + \left[\frac{S_rP + h_rP + (h_{m,f}+S_{m,f})(2D+n_2(P-D)-P)}{2P}\right].$$

Then, the total annual cost of the system is written as:

$$\psi_{sc}^{\mathrm{I-I}}(n_1,n_2) = 2\sqrt{S(n_1,\ n_2)H(n_1,\ n_2)} + C,$$

where

$$C = \left(C_s\alpha + C_{m,w}\alpha + C_{m,f} + C_r\right)D + (h_s\tau_m - h_{m,w}t_s)\alpha D + (C_{m,w}-C_s)\ \alpha De^{k_st_s}$$
$$-C_{m,w}\alpha De^{k_m\tau_m} + h_{m,f}\tau_rD + \left(C_r - C_{m,f}\right)De^{k_mt_m}\ - C_rDe^{k_rt_m}.$$

The total variable costs ($TVC_{\mathrm{I-I}}$), joint optimization procedure for Q, n_1 and n_2

$$TVC_{\mathrm{I-I}}(Q^*(n_1,\ n_2),n_1,\ n_2) = 2\sqrt{S(n_1,\ n_2)H(n_1,\ n_2)},$$

$$(TVC_{\mathrm{I-I}}(Q^*(n_1,\ n_2),n_1,\ n_2))^2 = 4\left(\left(\left[A_s + A_{m,f} + n_1A_{m,w} + n_2A_r + \frac{\alpha n_1h_{m,w}Pt_s^2}{2} + \frac{n_2h_rDt_m^2}{2}\right]\frac{D}{n_2}\right)\right.$$
$$([h_{m,w} \ + \ S_{m,w} + (n_1-1)(h_s+S_s)]\frac{\alpha n_2D}{2n_1P}$$
$$\left.+\left[\frac{S_rP + h_rP + (h_{m,f}+S_{m,f})(2D+n_2(P-D)-P)}{2P}\right]\right)\Bigg).$$

- For $\boldsymbol{n}_1 = 1$ and $\boldsymbol{n}_2 = 1$

$$(TVC_{\mathrm{I-I}}(Q^*(1,\ 1),1,\ 1))^2 = 4\left(\left[A_s + A_{m,f} + A_{m,w} + A_r + \frac{\alpha h_{m,w}Pt_s^2}{2} + \frac{h_rDt_m^2}{2}\right]D\right)$$
$$\left([h_{m,w} + \ S_{m,w}]\frac{\alpha D}{2P} + \left[\frac{S_rP + h_rP + (h_{m,f}+S_{m,f})D}{2P}\right]\right).$$

Thus,

$$(TVC_{\text{I–I}}(Q^*(1,\ 1), 1,\ 1))^2 > 0.$$

- **For $n_1 \geq 1$ and $n_2 = 1$**

$$(TVC_{\text{I–I}}(Q^*(n_1,\ 1), n_1, 1))^2 = 4\left(\left(\left[A_s + A_{m,f} + n_1 A_{m,w} + A_r + \frac{\alpha n_1 h_{m,w} P t_s^2}{2} + \frac{h_r D t_m^2}{2}\right] D\right)\right.$$
$$\left.\left([h_{m,w} + S_{m,w} + (n_1 - 1)(h_s + S_s)]\frac{\alpha D}{2n_1 P} + \left[\frac{S_r P + h_r P + (h_m + S_m)D}{2P}\right]\right)\right),$$

$$(TVC_{\text{I–I}}(Q^*(n_1,\ 1), n_1, 1))^2 = 4\left(\left(A_s D + A_{m,f} D + n_1 A_{m,w} D + A_r D + \frac{\alpha n_1 h_{m,w} P t_s^2}{2} D + \frac{h_r D t_m^2}{2} D\right)\right.$$
$$\left.\left((h_{m,w} + S_{m,w})\frac{\alpha D}{2n_1 P} + (h_s + S_s)\frac{\alpha D}{2P} - (h_s + S_s)\frac{\alpha D}{2n_1 P} + \frac{S_r P + h_r P + (h_{m,f} + S_{m,f})D}{2P}\right)\right).$$

Since $(TVC_{\text{I–I}}(Q^*(n_1,\ 1), n_1, 1))^2 = a n_1 + \frac{b}{n_1} + c$, from the equation above

$$\boldsymbol{a} = 4D\left[A_{m,w} + \frac{\alpha h_w P t_s^2}{2}\right],$$

$$\boldsymbol{b} = 2\frac{\alpha D}{P}[h_{m,w} + S_{m,w} - h_s - S_s],$$

$$\boldsymbol{c} = 4\left(\frac{A_s D}{n_2} + \frac{A_{m,f} D}{n_2} + \frac{A_r D}{n_2} + \frac{h_r D t_m^2}{2} D\right)\left((h_s + S_s)\frac{\alpha D}{2P} + \frac{S_r P + h_r P + (h_{m,f} + S_{m,f})D}{2P}\right).$$

Therefore,

$n_1^* = \sqrt{\frac{b}{a}} = \sqrt{\frac{\frac{\alpha D}{P}[h_{m,w} + S_{m,w} - h_s - S_s]}{2D[A_{m,w} + \frac{\alpha h_w P t_s^2}{2}]}}$, where $h_{m,w} + S_{m,w} > h_s + S_s$, which is a valid constraint due to a basic assumption in the supply chain that the value of a product increases while moving down the chain. The minimum total variable costs equal to

$$TVC_{\text{I–I}}(Q^*(n_1,\ 1), n_1, 1) = \sqrt{2\sqrt{ab} + c}$$

$$= \sqrt{2\sqrt{\left(2D\left[A_{m,w} + \frac{\alpha h_w P t_s^2}{2}\right]\right)\left(\frac{\alpha D}{P}[h_{m,w} + S_{m,w} - h_s - S_s]\right)} + 4\left(\frac{A_s D}{n_2} + \frac{A_{m,f} D}{n_2} + \frac{A_r D}{n_2} + \frac{h_r D t_m^2}{2} D\right)}$$
$$\times \left((h_s + S_s)\frac{\alpha D}{2P} + \frac{S_r P + h_r P + (h_{m,f} + S_{m,f})D}{2P}\right).$$

For $n_1 = 1$ and $n_2 \geq 1$, the variable costs function is written as:

$$(TVC_{\text{I–I}}(Q^*(1,\ n_2), 1,\ n_2))^2 = 4\left(\left(\left[A_s + A_{m,f} + A_{m,w} + n_2 A_r + \frac{\alpha h_w P t_s^2}{2} + \frac{n_2 h_r D t_m^2}{2}\right]\frac{D}{n_2}\right)\right.$$
$$\left.\times\left((h_{m,w} + S_{m,w})\frac{\alpha n_2 D}{2P} + \left[\frac{S_r P + h_r P + (h_{m,f} + S_{m,f})(2D + n_2(P - D) - P)}{2P}\right]\right)\right),$$

since $(TVC_{\text{I–I}}(Q^*(1,\ n_2), 1,\ n_2))^2 = a n_2 + \frac{b}{n_2} + c$, therefore,

$$\boldsymbol{a} = 4\left((h_{m,w} + S_{m,w})\frac{\alpha D}{2P} + \left[\frac{(h_{m,f} + S_{m,f})(P - D)}{2P}\right]\right),$$

$$\boldsymbol{b} = 4\left(\left[A_s + A_{m,f} + A_{m,w} + \frac{\alpha h_w P t_s^2}{2}\right] D\right),$$

$$\boldsymbol{c} = 4\left(\left(\left[A_r + \frac{h_r D t_m^2}{2}\right] D\right)\left(\left[\frac{S_r P + h_r P + (h_{m,f} + S_{m,f})2D}{2P}\right] - \left[\frac{(h_{m,f} + S_{m,f})}{2}\right]\right)\right),$$

where $(h_r + S_r) > (h_m + S_m)$.

Accordingly,

$$n_2^* = \sqrt{\frac{b}{a}} = \sqrt{\frac{4D[A_s + A_{m,f} + A_{m,w} + \frac{\alpha h_{m,w} P t_s^2}{2}]}{4((h_{m,w} + S_{m,w})\frac{\alpha D}{2P} + [\frac{(h_{m,f} + S_{m,f})(P-D)}{2P}])}}$$

, where $P > D$ and the minimum variable costs equal to

$$\left(TVC_{I-I}\left(Q^*\left(1,\ n_2^*\right), 1,\ n_2^*\right)\right) = \sqrt{2\sqrt{ab} + c}$$

$$= \sqrt{\begin{array}{l} 2\sqrt{\left[4\left((h_{m,w} + S_{m,w})\frac{\alpha D}{2P} + \left[\frac{(h_m + S_m)(P-D)}{2P}\right]\right)\right]\left[4D\left[A_s + A_{m,f} + A_{m,w} + \frac{\alpha h_{m,w} P t_s^2}{2}\right]\right]} \\ +4\left(\left(\left[A_r + \frac{h_r D t_m^2}{2}\right]D\right)\left(\left[\frac{S_r P + h_r P + (h_{m,f} + S_{m,f})2D}{2P}\right] - \left[\frac{(h_{m,f} + S_{m,f})}{2}\right]\right)\right) \end{array}}.$$

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