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# Uncertainty quantification of the bond stress – displacement relationship of shoring anchors in different geologic units

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## ABSTRACT

The design of anchored wall systems is governed by the bond strength between the grout and the surrounding soil/rock in the grouted zone. Published work on the reliability of anchored retaining systems focuses on ultimate limit state considerations to ensure a target level of reliability against pullout of the anchor in the bonded zone. These studies do not shed light on the uncertainty in the bond stress – displacement relationship of the grout/soil interface. This uncertainty exists as a result of spatial variability in the soil/rock properties even within the same geologic unit. The primary objective of this paper is to quantify the uncertainty in the bond stress – displacement relationship to aid the serviceability limit state design of anchored wall systems. To achieve this objective, a database of shoring anchor tests from real projects that were executed in several sites around Beirut is assembled and analysed. The tests are categorised based on three different geologic units (Limestone, Marls, and Clays) that are common to many geologic settings in the world. The bond stress – displacement relationship is modelled with a hyperbolic model and the statistics of the model are derived from the assembled database and classified based on the geologic units analysed.

## ARTICLE HISTORY

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## KEYWORDS

Anchor; pull-out test; uncertainty; variability; deformation; bond strength; Clay; Limestone; Marl

## 1. Introduction and background

Lateral shoring systems which incorporate ground anchors provide practical solutions for controlling lateral deformations in open excavations and stabilising slopes and retaining systems (Sabatini et al. 1999; Liu et al. 2017). Ground anchors play a central/critical role in the overall stability and deformation control of the shoring system. Despite this critical role, the methodologies that are currently used for analysing and designing ground anchors are limited to ultimate limit state criteria which specify the ultimate load capacity of the anchor based on an assumed ultimate bond stress between the anchor and the surrounding rocks or soils. Once an ultimate bond stress is selected, the length of the anchor in the bonded zone is selected based on an assumed factor of safety against ultimate pullout of the anchor. Liu et al. (2017) states that the main limitation of this approach lies in the selection of the ultimate bond stress which generally spreads in a relatively wide range.

In order to address/minimize the impact of design uncertainties, field tests of anchors are mandated as part of the execution methodologies and specifications. These tests typically include pullout tests (failure tests) and proof tests (stressing to >125% of service design capacity). Several published studies have targeted the

design and performance of ground anchors. Examples include the work of Powers, Briaud, and Weatherby (1998), Kim (2003), Cherubini, Garrasi, and Petrolla (1992), Kwon et al. (2017), Basha and Babu (2008), Hegazy (2003), Basma (1991) and Ching, Liao, and Sue (2008).

The design approaches for ground anchors may benefit from an improved understanding of the load transfer between ground anchors and rocks/soils at different levels of anchor displacement in the bonded zone. With the advancement of reliability-based design methodologies in geotechnical engineering, better understanding of the mobilisation of bond stress with anchor displacement in the bonded zone is needed to allow for checking serviceability-based design considerations for retaining systems that are supported with ground anchors. A review of the literature indicates that there is lack of studies that target the bond stress–bond displacement relationship for ground anchors in different geologic units. There are very limited experimental data from full scale tests on the performance and behaviour of ground anchors, particularly in geologic settings that involve rocks (Liu et al. 2017) and other sedimentary units that involve stiff marls and clays.

The primary objective of this paper is to quantify the uncertainty in the bond stress – displacement relationship of ground anchors to aid the serviceability limit state design of anchored wall systems. To achieve this objective, a database of actual shoring anchor tests from real projects that were executed in several sites around Beirut is assembled and analysed. The tests are categorised based on three geologic units (Limestone, Marls, and Clays) that are common to many geologic settings in the world. The bond stress – displacement relationship is modelled with a hyperbolic model and the statistics of the model are derived from the assembled database and classified based on the geologic units analysed. The proposed work is intended to help move the field towards a greater understanding and appreciation of the importance of variability in anchor design and suggested means to incorporate it in actual practice.

## 2. Database

The database contains the results of 70 anchor tests in three geologic settings: 32 tests in Limestone, 26 in Marl, and 12 in Clay. The tests are executed in 28 different sites around Beirut. The baseline geological information available for Beirut indicates its simplest representation two main geological units that dominate the geologic setting: A relatively young Miocene (m2b) stratum of Marl and Marly Limestone and an older and more significant Limestone sequence, the Cretaceous Sannine limestone (c4). The depths to which the Miocene stratum is present are variable and depend to a large degree on the faulting/uplift and erosion along the fault which bisects the central parts of the city in a roughly N-S direction. In some areas of the Beirut, more recent quaternary deposits of alluvial nature are present and form the majority of the soil cover wherever they exist. In other locations residual soils resulting from the decalcification of the marls form the upper cover.

The database of anchor pullout tests is summarised in Table 1. In some sites, it is possible that anchors tests were conducted in two different geologic units depending on the elevation of the anchor that was tested. The main information that was available consisted of data of anchor head displacement versus applied pullout force. Information about each anchor included the bond length, free length, number of strands, and diameter of the drilled hole. In the majority of the cases analysed, the anchors were installed using a hydraulic rig mounted on crawlers. The holes were drilled by rotary methods with flushing and installation of temporary steel casing. Immediately after drilling, the hole was grouted and anchor strands were lowered inside the

hole. During the retrieval of the temporary casing, Type A grout, which consists of Portland cement type 2 with a cement to water ratio of about 2, was pumped to replace possible losses of grout. At least 24 h after its first installation, grout type A was injected inside the pressure sleeved pipes. After the stoppage of injection, the grouting tube is flushed with clear water and the operation is resumed the following day until reaching the pressure criterion. After verifying its geometric positioning, the anchor is wedged in its final position until the setting of the grout. After installation, each anchor is tested after the sealed grout has reached the sufficient hardness.

The testing of anchors was executed relying on the general principles in French Norm NF-P-94-153 and the interpretation was carried out per the guidelines established in the “Recommendations TA95”. The anchor test starts by positioning the anchor plate and connecting the multijacks to a single hydraulic power unit. During the loading stage, the load is applied in increments of 10% while measuring simultaneously the jack pressures and the elongation of the strands. The applied test load is maintained for 1 h and the corresponding displacements are measured.

## 3. Test results and analysis

The raw data from each anchor test consists of the applied load ( $T$ ) versus the measured total anchor displacement ( $\Delta$ ). The first step in the analysis consists of calculating the average bond stress in the grouted zone ( $f_b$ ) for each load increment using Equation (1):

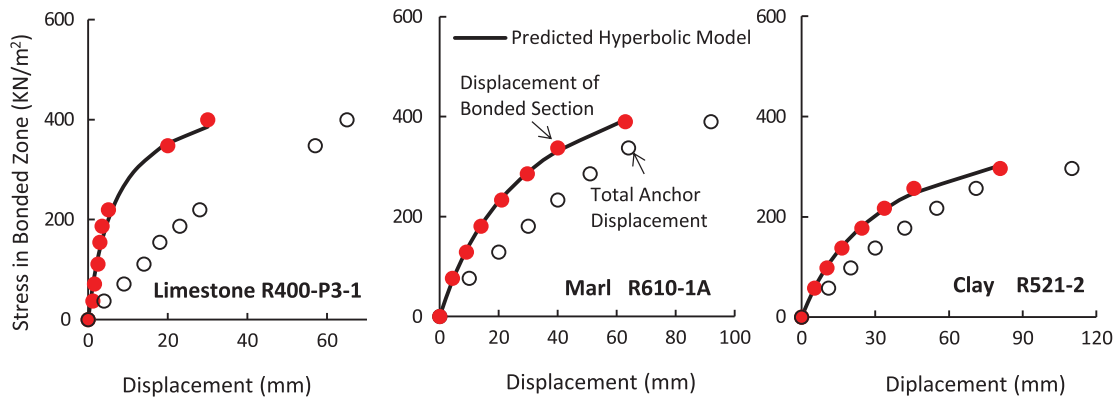
$$f_b = \frac{T}{\pi D L_b} \quad (1)$$

Where  $f_b$  = the average stress in the bonded length (kN/m<sup>2</sup>),  $T$  = the load applied on the anchor during the test (kN),  $D$  = drilled hole diameter (m), and  $L_b$  = bonded length (m). Data showing the average bond stress versus total anchor displacement is presented in Figure 1 using open circles for example cases involving anchors in limestone, marl, and clay, respectively. The results indicate total measured anchor displacements ranging from 70 mm (limestone) to 120 mm (clay) for the cases shown in Figure 1. Since part of the total measured displacement is attributed to elongation in the steel strands in the free length ( $\Delta_f$ ), this elastic elongation was subtracted from the total measured displacement to determine the average slip of the anchor in the bonded zone. The calculated slip in the bonded length ( $D_b$ ) is calculated from Equations (2) to (3) such that:

$$\Delta_b = \Delta - \Delta_f \quad (2)$$

**Table 1.** Data from anchor pullout tests.

Test Number	Total Length (m)	Bond Length (m)	Free Length (m)	Number of Steel Tendons	Area of Steel (mm <sup>2</sup> )	Inclination Angle (Degree)	Drilling Diameter (mm)	Hyperbolic Model Parameter a	Hyperbolic Model Parameter b	Type of Soil	Site
R400-P3-1	16	8	8	9	1350	30	140	5.87	0.84	Limestone	Site 1
R400-1	16	8	8	9	1350	30	140	17.75	0.85	Clay	Site 1
R400-2	16	8	8	9	1350	30	140	18.35	0.72	Marl	Site 1
R431-1	12	6	6	4	600	20	114	15.72	0.69	Marl	Site 2
R431-2	12	6	6	4	600	20	114	18.31	0.62	Marl	Site 2
R450-1	17.5	13.5	4	4	600	35	114	22.21	0.66	Clay	Site 3
R450-2	17.5	13.5	4	4	600	35	114	30.04	0.52	Clay	Site 3
R450-2G	17.5	13.5	4	4	600	35	114	30.45	0.51	Clay	Site 3
R482-1	10	5	5	4	600	20	114	3.07	0.93	Limestone	Site 4
R482-2	13	5	8	4	600	20	114	7.42	0.86	Limestone	Site 4
R482-3	13	5	8	4	600	20	114	15.25	0.74	Limestone	Site 4
R482-4	13	5	8	4	600	20	114	10.70	0.80	Limestone	Site 4
R482-5	13	5	8	4	600	20	114	11.65	0.82	Limestone	Site 4
R521-1	24	12	12	6	900	90	150	8.24	0.87	Marl	Site 1
R521-2	15	10	5	6	900	90	150	19.70	0.58	Clay	Site 1
R521-3	15	10	5	6	900	90	150	21.99	0.50	Clay	Site 1
R521-4	18	10	8	8	1200	90	150	7.76	0.92	Marl	Site 1
R521-5	18	10	8	8	1200	90	150	11.86	0.78	Marl	Site 1
R576-T42	16	8	8	4	600	35	114	18.26	0.63	Clay	Site 5
R595-1	10.5	5	5.5	3	450	20	114	29.31	0.51	Marl	Site 6
R595-2	10.5	5	5.5	3	450	30	114	23.96	0.66	Marl	Site 6
R595-3	11.5	6.5	5	4	600	20	114	17.74	0.67	Marl	Site 6
R598-1	11.5	5	6.5	5	750	20	130	15.65	0.74	Limestone	Site 7
R598-5	11	5	6	4	600	45	114	16.25	0.68	Marl	Site 7
R598-6	17	5	12	4	600	45	114	14.15	0.78	Marl	Site 7
R610-1A	10	5	5	4	600	20	114	26.57	0.85	Marl	Site 8
R610-1B	10	5	5	4	600	20	114	19.27	0.59	Marl	Site 8
R612-1	10	5	5	3	450	20	114	15.51	0.75	Marl	Site 9
R616-1	14	5	9	4	600	20	114	4.69	0.88	Limestone	Site 10
R656-1	10	5	5	5	750	20	114	11.26	0.76	Limestone	Site 11
R669-1	10	5	5	4	600	20	130	24.79	0.58	Limestone	Site 12
R669-2	10	5	5	4	600	20	130	6.57	0.89	Limestone	Site 12
R669-3	10	5	5	5	750	20	130	22.68	0.60	Marl	Site 12
R669-4	10	5	5	5	750	20	130	13.57	0.75	Limestone	Site 12
R675-1	10	5	5	4	600	20	130	25.34	0.56	Marl	Site 13
R675-2	10	5	5	4	600	20	130	17.38	0.71	Marl	Site 13
R675-3	10	5	5	4	600	20	130	18.56	0.71	Limestone	Site 13
R675-4	10	5	5	4	600	20	130	10.32	0.82	Limestone	Site 13
R682-1	10	5	5	5	750	20	130	7.46	0.87	Limestone	Site 14
R682-2	10	5	5	5	750	20	130	39.40	0.33	Limestone	Site 14
R682-3	10	5	5	5	750	20	130	12.60	0.77	Limestone	Site 14
R682-4	10	5	5	5	750	20	130	50.39	0.21	Marl	Site 14
R682-5	16.5	5	11.5	5	750	20	130	5.50	0.89	Limestone	Site 14
R705-1	10	5	5	5	750	20	130	30.21	0.51	Limestone	Site 15
R705-2	10	5	5	5	750	20	130	50.96	0.18	Marl	Site 15
R707-1	10	5	5	5	750	20	130	20.85	0.66	Limestone	Site 16
R707-2	10	5	5	5	750	20	130	49.89	0.12	Marl	Site 16
R707-3	10	5	5	5	750	20	130	20.58	0.31	Marl	Site 16
R721-1	10	5	5	5	750	33	130	13.69	0.72	Limestone	Site 17
R724-1	10	5	5	4	600	33	130	25.40	0.55	Marl	Site 18
R724-2	10	5	5	4	600	24	130	21.91	0.63	Clay	Site 18
R725-1	10	5	5	5	750	33	130	9.69	0.83	Limestone	Site 19
R746-1	10	5	5	4	600	30	130	25.99	0.58	Limestone	Site 20
R746-2	10	5	5	4	600	30	130	9.22	0.84	Limestone	Site 20
R749-2	13	8	5	3	450	30	130	19.71	0.67	Clay	Site 21
R751-1	15	10	5	5	750	20	130	31.19	0.51	Clay	Site 22
R751-2	13	8	5	3	450	20	130	14.26	0.76	Clay	Site 22
R761-1	11	5	6	5	750	20	130	20.68	0.67	Limestone	Site 23
R761-2	11	5	6	4	600	20	114	5.98	0.88	Limestone	Site 23
R761-4	10	5	5	4	600	20	114	28.44	0.52	Limestone	Site 23
R764-1	10	5	5	4	600	20	114	8.61	0.86	Marl	Site 24
R764-2	10	5	5	4	600	20	130	15.70	0.76	Limestone	Site 24
R768-1	10	5	5	4	600	20	130	13.70	0.77	Marl	Site 25
R768-4	10	5	5	4	600	20	130	19.44	0.87	Marl	Site 25
R768-5	15	10	5	5	750	20	130	28.42	0.51	Clay	Site 25
R769-1	12.5	5	7.5	4	600	20	130	9.33	0.85	Limestone	Site 26
R769-2	12.5	5	7.5	4	600	20	130	12.86	0.79	Limestone	Site 26
R787-1	16	6	10	4	600	20	114	18.53	0.74	Limestone	Site 27
R787-2	12	6	6	4	600	20	114	15.21	0.71	Marl	Site 27
R788-1	11.5	5	6.5	3	450	20	114	34.56	0.43	Limestone	Site 28



**Figure 1.** Example results from pullout tests in Limestone, Marl, and Clay.

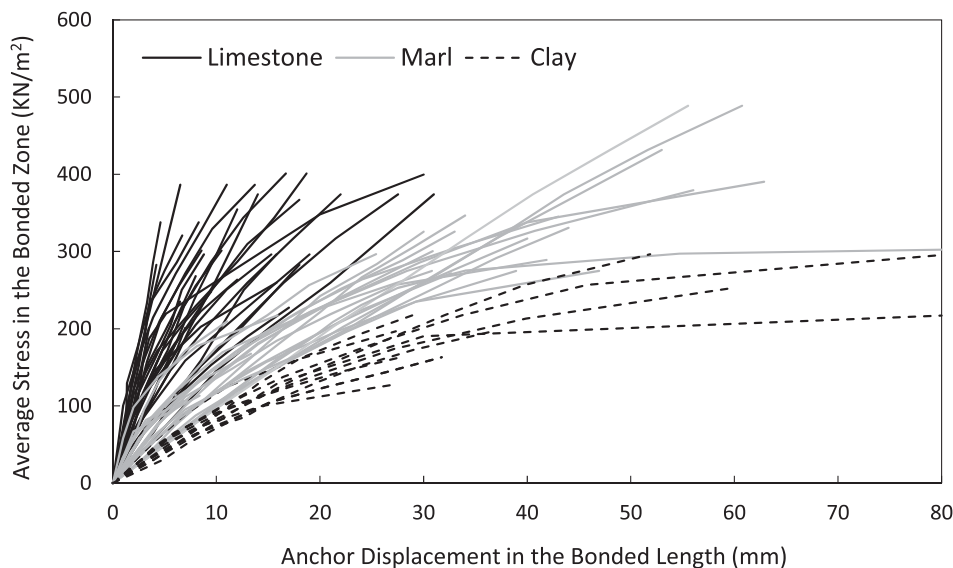
$$\Delta_f = \frac{T L_f}{E A} \quad (3)$$

Where  $\Delta_b$  = the slip in the bonded length (mm),  $\Delta$  = total anchor head displacement (mm),  $\Delta_f$  = the displacement in the free length (mm),  $L_f$  = free length (mm),  $E$  = modulus of elasticity of the steel strands ( $20 \times 10^8$  kPa), and  $A$  = area of the steel strands (m<sup>2</sup>). Figure 1 also shows the relationship between the average bond stress and the predicted slip/displacement in the bonded zone using solid circles. As indicated in the figure, a significant portion of the total anchor displacement is attributed to elongation of strands in the free length of the anchor.

Figure 2 shows the variation of the average stress in the bonded zone as a function of the predicted anchor slip/displacement in the bonded zone for all the tests in the three types of soil in the database. Results on Figure 2 indicate that the bond stress – displacement curves for anchors in a given geologic unit form groups

that can easily be distinguished based on the observed response. As expected, the curves representing anchors in limestone exhibit a relatively stiffer stress-displacement response, compared to anchors in marl, which in turn show a stiffer response compared to anchors in clay. It is also clear that even in the same geologic unit, the curves exhibit a significant degree of variability in the response due to variability in soil/rock properties, construction related factors, and properties of the grout.

To quantify the variability in the observed response, a hyperbolic model is used to fit the observed relationship between the average stress and anchor slip in the bonded zone. The hyperbolic model is a 2-parameter model that has been previously used to fit load-displacement relationships for problems involving piles (examples Phoon, Chen, and Kulhawy 2006, 2007; Dithinde et al. 2011; Stuedlein and Uzielli 2014; Reddy and Stuedlein 2017; Tang and Phoon 2018a, 2018b, 2018c) and shallow foundations (Uzielli and Mayne 2011; Huffman and



**Figure 2.** Variations of the average stress in the bonded zone as a function of displacement in the bonded zone for all the tests.

Stuedlein 2014; Najjar, Shammass, and Saad 2014, 2017). In the hyperbolic model, the response is captured by 2 parameters “a” and “b”, where “a” represents the initial slope and “b” represents the asymptotic value of the normalised load-settlement curve. In the hyperbolic model, the applied load/stress is usually normalised by the “ultimate/maximum” load/stress prior to fitting the curve to the observed response. In the problem at hand, the hyperbolic relationship is presented in Equation (4) and links the normalised stress in the bonded zone to the average predicted anchor slip in the bonded zone such that:

$$\frac{f_b}{f_{b,\max}} = \frac{\Delta_b}{a + b\Delta_b} \quad (4)$$

Where  $f_{b,\max}$  is the ultimate bond stress. Given that the ultimate bond stress was not reached in many of the anchor tests available in the database, ultimate bond stresses that were consistent with common design practice/experience for anchors in Beirut were adopted to normalise the data presented in Figure 2. Accordingly,  $f_{b,\max}$  was assumed to be equal to 600 kPa for anchors in limestone, 350 kPa for anchors in marl, and 250 kPa for anchors in clay. Exceptions to these assumptions included: (1) two tests in limestone where a reduced  $f_{b,\max}$  of 400 kPa was adopted due to the fact that the anchor tests approached failure at these values, and (2) three tests in marl, where a higher  $f_{b,\max}$  of 500 kPa was adopted given that the geology was classified as “marly limestone” rather than “marl”.

Following the normalisation of the data shown in Figure 2, two parameter hyperbolic models having the mathematical form presented in Equation (4) were fit to each test using linear regression. The resulting a and b parameters for each test were calculated and presented in Table 1. As expected, the uncertainty in the observed stress-displacement curves was translated clearly in the variability in the resulting values of a and b within a given geologic unit. The statistics representing the uncertainty in a and b for each geologic unit were calculated and presented in Table 2.

The results in Table 2 indicate that the parameter “a” seems to be more uncertain than parameter “b”, particularly for cases involving anchors in limestone and marl, where coefficients of variation as high as 0.61 are recorded for “a” compared to 0.33 for “b”. The parameters for anchors in clay show the least variability with coefficients of variation as low as 0.25 for “a” and 0.18 for “b”. The relatively small coefficients of variation for cases of anchors in clay may be attributed to the homogeneous nature of the clayey geologic units compared to the marly/rocky units which exhibit significant

**Table 2.** Statistical parameters of the hyperbolic bond stress – bond displacement relationship.

Statistics	Limestone		Marl		Clay	
	a	b	a	b	a	b
Mean	15	0.74	21.8	0.63	23	0.61
Standard Deviation	9.09	0.14	11.57	0.21	5.66	0.11
Coefficient of Variation	0.61	0.19	0.53	0.33	0.25	0.18
Beta Parameter, a	3	0	7	0	14	0.5
Beta Parameter, b	40	0.91	50	0.92	32	0.85
Beta Parameter, m	-3.46	5.63	-3.46	3.16	-0.88	-4.53
Beta Parameter, s	4.25	3.86	4.45	4.21	5.32	5.67
Correlation Coefficient	-0.99		-0.87		-0.79	

variations in strength and quality within the Lebanese geological context.

Interestingly, a plot showing the relationship between parameters a and b (Figure 3) for each geologic unit indicates that the parameters are highly correlated negatively with correlation coefficients as high as -0.99 for anchors in limestone and -0.79 for anchors in clay. Negative correlations between hyperbolic model parameters have been reported in many published studies for piles and shallow foundations.

To complete the statistical model for the hyperbolic parameters that model the relationship between the anchor bond stress and anchor slip in the bonded zone, candidate probability distributions were tested against the observed data for a and b. These include the conventional lognormal probability distribution in addition to a more general four-parameter beta distribution that is bounded in an interval [a b]. The beta distribution is based on a transformation (Equation 5) from a standard normally distributed random variable G, into a random variable X that is bounded on the interval a to b such that:

$$X = a + \frac{1}{2}(b - a) \left[ 1 + \tanh\left(\frac{m + sG}{2\pi}\right) \right] \quad (5)$$

Where  $m$ ,  $s$ ,  $a$  and  $b$  are the location, scale, lower bound and upper bound parameters, respectively. The probability density function of the bounded distribution of X is:

$$f_X(x) = \frac{\sqrt{\pi}(b - a)}{\sqrt{2s}(x - a)(b - x)} \cdot \exp\left\{-\frac{1}{2s^2} \left[ \pi \ln\left(\frac{x - a}{b - x}\right) - m \right]^2\right\} \quad (6)$$

The 4-parameters of the bounded distribution were calibrated to produce a cumulative distribution function (CDF) that best fits the observed CDF of the data. The calibration was conducted in a spreadsheet using a solver tool that allows for solving the optimisation problem in which the 4-parameters of the bounded distributions were obtained.

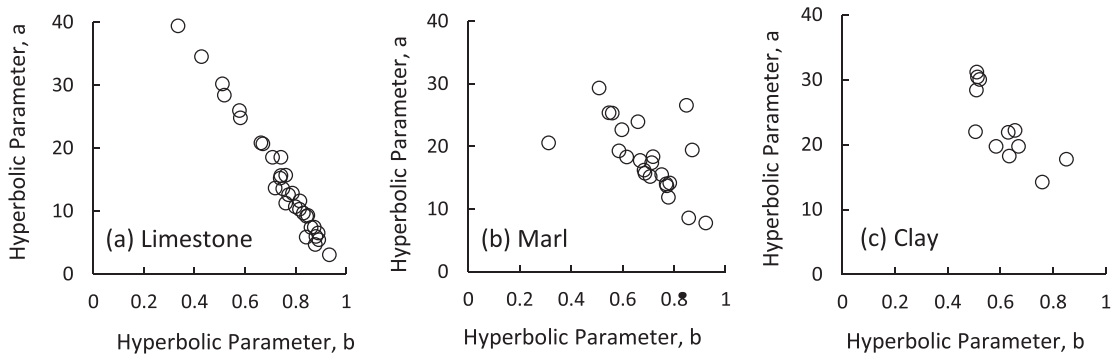


Figure 3. Relationship between the two hyperbolic model parameters from the anchor tests.

Figure 4 shows the observed CDFs for a and b in the different geologic units and the theoretical lognormal and beta distributions that were used to model the variability in the hyperbolic parameters. The four parameters that describe the beta distributions shown in Figure 4 are presented in Table 2 for the cases of anchors in

limestone, marl, and clay. Results on Figure 4 show that the four parameter beta distribution provides a more realistic representation of the uncertainty in a and b for almost all of the cases analysed. This distribution, along with the correlation coefficients shown in Table 2, can be used to model the joint probability

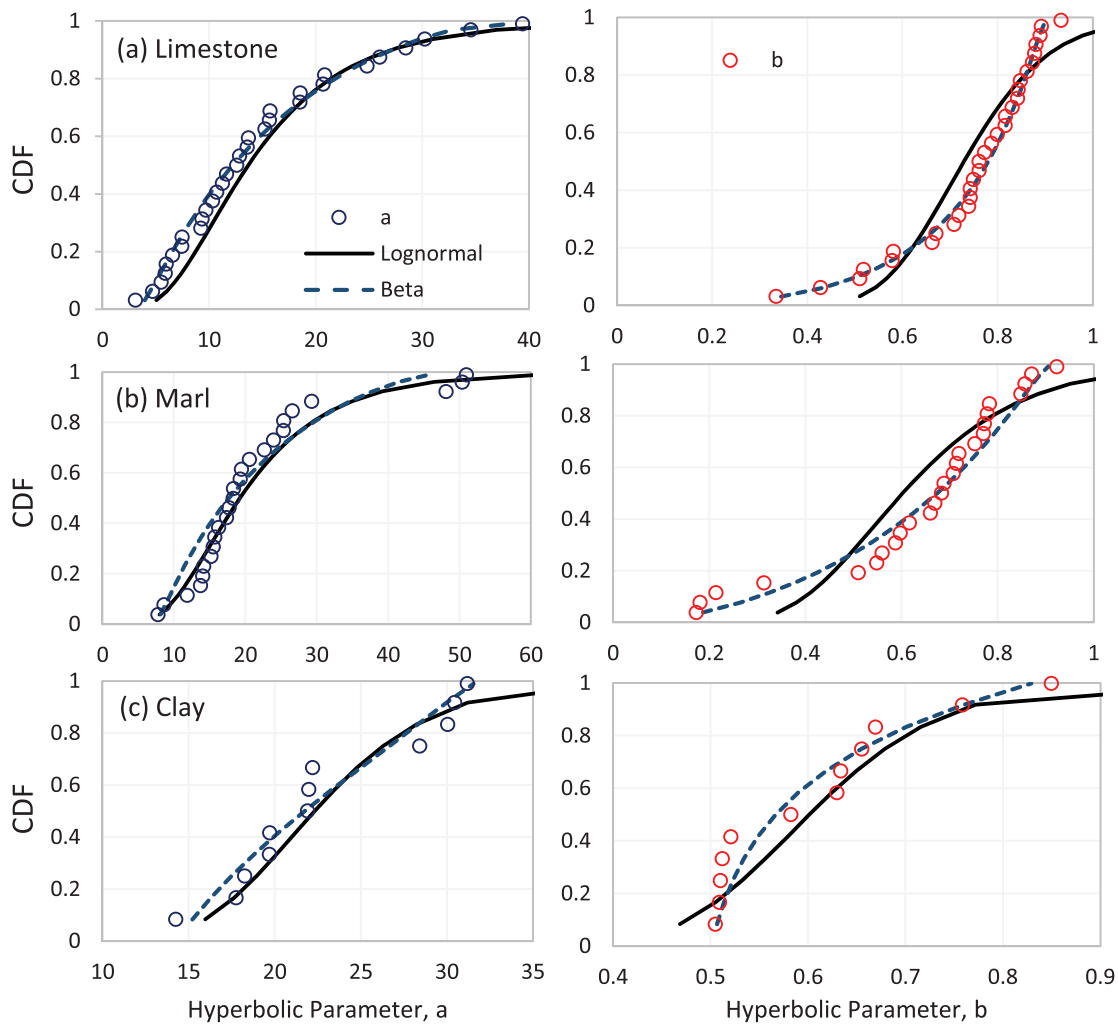


Figure 4. Actual and theoretical lognormal and beta distributions for modelling the uncertainty in the hyperbolic model parameters for anchors.

density functions representing the uncertainty in the hyperbolic models for the anchor tests that were analysed in this study.

#### 4. Conclusions and future work

In this paper, a database of 70 anchor pullout tests in geologic units involving limestone, marl, and clay was assembled and used to quantify the uncertainty in the bond stress – anchor slip relationship using a commonly used hyperbolic model. The statistics of the hyperbolic model parameters (a and b) can be used within a Monte Carlo Simulation framework to simulate bond stress – anchor slip curves that represent the expected variability in the anchor response in the bonded zone. Such curves are key to the serviceability limit state design of anchored retaining systems. Examples of studies where probabilistic load-displacement relationships were adopted to quantify the reliability of geotechnical systems include the work of Najjar, Shamma, and Saad (2014) and (2017) and Tang and Phoon (2018a, 2018b, 2018c), among others.

Current methodologies for the design of anchored retaining systems focus on the ultimate limit state and ignore the serviceability limit state design considerations. The lateral displacement of anchored retaining systems is a critical performance criterion that is the basis of performance-based design of geotechnical systems. Current design approaches quantify the design anchor capacity as a function of the ultimate pullout capacity divided by a factor of safety. The design anchor capacity is then included as part of a force equilibrium analysis for the retaining system to determine moments, shears, and displacements of the retained structure. There is currently no link that is established between the resulting wall displacements at the location of the anchors and the mobilised pullout anchor force that was used in design. Future work should focus on establishing this link through bond stress – anchor slip relationships that are similar to those established in this paper to couple the current ultimate limit state design methods with performance based criteria that are related to the anchor displacement.

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#### Disclosure statement

No potential conflict of interest was reported by the authors.

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