When Quantized Massive MIMO Meets Large MIMO With Higher Order Modulation

Asmaa Abdallah, *Student Member, IEEE*, Mohammad M. Mansour, *Senior Member, IEEE*, Louay M. A. Jalloul, *Senior Member, IEEE*, and Ali Chehab, *Senior Member, IEEE*

***Abstract*— In this letter, we compare the capacity of a massive multiple-input multiple-output (MIMO) system using a low-resolution analog-to-digital converter (ADC) and a linear detector against a conventional MIMO system with higher order modulation and near maximum likelihood (ML) detection. We show that in the low-signal-to-noise ratio (SNR) regime, the quantized massive MIMO system can outperform the conventional large MIMO system; however, for high SNR, the conventional MIMO system with a near ML detector can outperform the extreme 1-bit quantized massive MIMO system. An analytical framework that derives the achievable rate of a linear minimum mean-squared error (MMSE)-based detector in a massive MIMO configuration, with the assumptions that the front-end is limited to a low-resolution ADC and channel estimation is imperfect, is presented.**

***Index Terms*— Quantized massive MIMO, MMSE, detection.**

I. Introduction

MASSIVE MIMO is an emerging technology that scales up the number of antennas in a conventional MIMO system by orders of magnitude and improves both spectral and energy efficiency through simple linear signal processing techniques [1], [2].

Equipping the base station (BS) with a large number of antenna elements dramatically increases the associated hardware cost and resulting power consumption of the radio-frequency (RF) circuits and data converters. It is known that the power consumption of ADCs grows significantly with the number of quantization bits [3], [4] and with large sampling rates. One potential solution is the use low-resolution quantized massive MIMO (e.g., 1-bit ADCs) as a means of reducing costs and power consumption, and improving computational efficiency [4]–[10]. However the main drawback of reducing the ADC resolution is the need to compensate for the severe non-linearity introduced by coarse quantization, which might render traditional detection schemes using high-resolution ADCs highly sub-optimal.

Prior work has studied the case of uplink quantized massive MIMO [4]–[7], and analyzed the non-linearity effects of quantization. However, there has not been any comparison of achievable rates of the quantized massive MIMO with that of a conventional MIMO system employing higher-order modulation (e.g., 1024-QAM) schemes. In addition, linear detectors such as zero-forcing (ZF), and maximal ratio combining (MRC) have been previously analyzed in [4] for the uplink massive MIMO with 1-bit ADCs. In [11] and [12] the gradient projection algorithm is used to iteratively find a precoder that minimizes the MSE between the transmitted and the received signal for the downlink case, but [11] and [12] contain no mathematical analysis of the achievable rate.

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A. Abdallah, M. Mansour, and A. Chehab is with the Department of Electrical and Computer Engineering, American University of Beirut, 1107 2020 Beirut, Lebanon (e-mail: awa18@aub.edu.lb; mmansour@ieee.org; chehab@aub.edu.lb).

M. A. Jalloul is with Qualcomm Inc., San Jose, CA 95131 USA.

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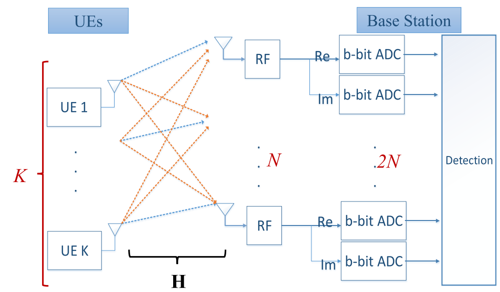

Fig. 1. Uplink quantized massive MIMO system model.

In this letter, we consider a quantized massive MIMO system. We propose an MMSE-based detector that incorporates the effects of coarse quantization in the ADC and channel estimation. We analytically derive the achievable rate, and compare it against the capacity of a large MIMO system with higher order modulation. By using low resolution ADCs with only few bits in massive MIMO, the same achievable rate obtained in the conventional MIMO system can be attained but with significantly less power consumption.

II. System Model and Proposed Detection Scheme

We consider a *single-cell* uplink system as depicted in Fig. 1, where K single-antenna users are served by a BS that is equipped with an array of $N \gg K$ antennas. The sub-channels between each transmit-receive antenna pair is modeled as a Rayleigh block-fading channel in which the channel stays constant over the channel coherence time.

The discrete-time complex baseband received signal over all antennas prior to quantization, is given as

$$\mathbf{y} = \sqrt{\rho}\,\mathbf{H}\mathbf{s} + \mathbf{w}, \tag{1}$$

where $\mathbf{y} \in \mathbb{C}^N$ is the received vector, ρ is the uplink SNR, $\mathbf{H} \in \mathbb{C}^{N\times K}$ is the channel matrix between the K users and the N BS antennas, and $\mathbf{s} \in \mathbb{C}^K$ denotes the channel input from all users. We assume that the channel gains $[\mathbf{H}]_{n,k} \sim \mathcal{CN}(0,1)$. Similarly, the entries $[\mathbf{w}]_n$ of the additive white gaussian noise vector $\mathbf{w} \in \mathbb{C}^N$ are $\mathcal{CN}(0,1)$ distributed. Moreover, $\mathbb{E}\left[\mathrm{tr}(\mathbf{s}\mathbf{s}^H) \leq K\rho\right]$ in which the average power constraint is satisfied, and $\mathrm{tr}\{\cdot\}$ represents the trace of a matrix.

A. Quantization in the ADC

The in-phase and quadrature components of the received signal at each antenna are quantized separately by an ADC of b-bit resolution. Following the notation of [4], we define a set of 2^b+1 quantization thresholds $\zeta_b = \{\upsilon_0, \cdots, \upsilon_{2^b}\}$ and a set of 2^b quantization labels $\mathcal{L}_b = \{\ell_0, \cdots, \ell_{2^b-1}\}$ where $\ell_i \in (\upsilon_i, \upsilon_{i+1}]$. Let $\mathcal{B}_b = \mathcal{L}_b \times \mathcal{L}_b$. The b-bit quantization is modeled by the function $Q_b(\cdot): \mathbb{C}^N \to \mathcal{B}_b^N$ that maps the received complex vector $\mathbf{y}$ with entries y_n to the quantized output $\mathbf{r}$ with entries $r_n = \ell_k + j\ell_l$ if and only if $\Re\{y_n\} \in [\upsilon_k, \upsilon_{k+1})$ and $\Im\{y_n\} \in [\upsilon_l, \upsilon_{l+1})$. $\Re\{\cdot\}$ and $\Im\{\cdot\}$ denote the real and imaginary parts of a complex scalar, respectively.

For simplicity, we consider the ADCs as symmetric uniform quantizers with step size Δ. First, note that when a signal is quantized, the average power in the received signal is not preserved. Therefore, we further assume that the output of the quantizer is scaled by a constant $\vartheta \in \mathbb{R}$ as in [4], to ensure that the variance of each entry of the quantized output $\mathbf{r}$ is $K\rho+1$. The entries ℓ_i of the quantization labels $\mathcal{L}$ are defined as

$$\ell_i = \vartheta\Delta\left(i - \frac{L-1}{2}\right), \quad i = 0, \ldots, L-1. \tag{2}$$

where $\vartheta = \sqrt{\frac{K\rho+1}{2\sum_{i=0}^{L-1}\ell_i^2\left(\Phi\left(\sqrt{\frac{2\upsilon_{i+1}^2}{K\rho+1}}\right)-\Phi\left(\sqrt{\frac{2\upsilon_i^2}{K\rho+1}}\right)\right)}}$ and $\Phi(x)$ is the CDF of a standard normal random variable.

Considering uniform quantizers, the quantization thresholds are given by $\upsilon_i = \Delta\left(i - \frac{L}{2}\right), \quad i = 1, \ldots, L-1$, where the step size Δ of the quantizers is chosen to minimize the distortion between the quantized and unquantized signal and can be found numerically (see [13] for details).

The b-bit quantized received signal can then be written as

$$\mathbf{r} \triangleq Q_b(\mathbf{y}) = Q_b(\sqrt{\rho}\mathbf{H}\mathbf{s} + \mathbf{w}). \tag{3}$$

In the 1-bit case (i.e., $b=1$), we can write the quantized received signal r_n at the nth antenna as follows:

$$Q_1(y_n) = \sqrt{\tfrac{K\rho+1}{2}}(\mathrm{sgn}(\Re\{y_n\}) + j\mathrm{sgn}(\Im\{y_n\})), \tag{4}$$

where $\mathrm{sgn}(\cdot)$ is the signum function defined as $\mathrm{sgn}(x) = -1$ if $x<0$ and $\mathrm{sgn} = 1$ if $x \geq 0$, and the quantized signal $Q_1(y_n)$ is scaled such that its variance is $K\rho+1$.

Bussgang decomposition for Gaussian inputs: The crosscorrelation of a Gaussian signal before and after applying a non-linear operation (quantization) are equal up to a constant [14]. When the input to the quantizer is Gaussian, Bussgang's theorem [14] can be used to decompose the quantized signal into a convenient form. Using Theorem 1 in [4] and assuming $\mathbf{y} \sim \mathcal{CN}(\mathbf{0}_N, \mathbf{C}_\mathbf{y})$ where $\mathbf{C}_\mathbf{y} \in \mathbb{C}^{N\times N}$, the quantized vector $\mathbf{r}$ is linearly related to $\mathbf{y}$ through some diagonal matrix $\mathbf{G}_b$

$$\mathbf{r} = \mathbf{G}_b\mathbf{y} + \mathbf{d}, \tag{5}$$

where the excess quantization distortion $\mathbf{d} \in \mathbb{C}^N$ and $\mathbf{y}$ are uncorrelated. The entries of the diagonal matrix $\mathbf{G}_b = G_b\mathbf{I}_N$ are real and given by $G_b = \sum_{i=0}^{2^b-1}\frac{\ell_i}{\sqrt{\pi\ (K\rho+1)}}\left(e^{-\frac{\upsilon_i^2}{K\rho+1}} - e^{-\frac{\upsilon_{i+1}^2}{K\rho+1}}\right)$, and the covariance matrix of $\mathbf{y}$ satisfies $\mathbf{C}_\mathbf{y} = (K\rho+1)\mathbf{I}_N$.

Both the Gaussian assumption and the diagonal structure of $\mathbf{C}_\mathbf{y} = (K\rho+1)\mathbf{I}_N$ are accurate at low SNR or when the number of UEs is large [4]. Moreover, due to the power normalization in (4), the covariance matrix $\mathbf{C}_\mathbf{r}$ of $\mathbf{r}$ becomes $\mathbf{C}_\mathbf{r} = (K\rho+1)\mathbf{I}_N$, and hence, the covariance matrix of the distortion $\mathbf{d}$ is $\mathbf{C}_\mathbf{d} = \mathbf{C}_\mathbf{r} - G_b^2\mathbf{C}_\mathbf{y} = \left(1-G_b^2\right)(K\rho+1)\mathbf{I}_N$.

For the infinite-resolution case ($b=\infty$), it can be deduced that $G_\infty = 1$, and that for the 1-bit-ADC case ($b=1$), we have $G_1 = \sqrt{2/\pi}$, which is a well-known result from [15] used to analyze the achievable rate with 1-bit ADCs.

B. Channel Estimation

In the channel estimation step, we assume that the coherence interval is divided into two parts: one dedicated for training and the other for data transmission. During the training phase, all K users simultaneously transmit their ($P \geq K$) sized pilot sequences to the BS. All pilot sequences used by different users are assumed to be pairwise orthogonal. Let $\mathbf{\Phi} \in \mathbb{C}^{K\times P}$ denote the pilot matrix transmitted from the K users such that $\mathbf{\Phi}\mathbf{\Phi}^H = P\rho\mathbf{I}_K$. Furthermore, let $\mathbf{Y}_p = \mathbf{H}\mathbf{\Phi} + \mathbf{W}_p$ and $\mathbf{R}_b = Q_b(\mathbf{Y_b})$, where $\mathbf{Y}_p, \mathbf{R}_p$ and $\mathbf{W}_p \in \mathbb{C}^{N\times P}$, denote the unquantized pilot sequences, quantized pilots sequences received from the K users at the BS during the training phase and the additive noise, respectively. The linear MMSE channel estimator [4] is given by

$$\hat{\mathbf{H}} = \frac{G_b\mathbf{R}_p\mathbf{\Phi}^H}{G_b^2P\rho + G_b^2 + (1-G_b^2)(K\rho+1)}. \tag{6}$$

Let $\mathbf{H} = \hat{\mathbf{H}} + \tilde{\mathbf{H}}$ where $\tilde{\mathbf{H}}$ represents the estimation error. The variance of the channel estimate and of the estimation error are given by: $\hat{\sigma}^2 = \frac{G_b^2P\rho}{G_b^2P\rho+G_b^2+(1-G_b^2)(K\rho+1)}$ and $\hat{\sigma}^2 = \frac{G_b^2P\rho}{G_b^2P\rho+G_b^2+(1-G_b^2)(K\rho+1)}$.

For the unquantized case ($b=\infty, G_\infty = 1$), linear MMSE channel estimation defaults back to its original form [16].

C. Data Detection

We consider the case when the BS employs an MMSE receiver. A soft estimate $\hat{s}_k$ of the transmitted symbol s_k from the kth user is obtained as $\hat{s}_k = \mathbf{a}_k^H\mathbf{r}$, where $\mathbf{a}_k \in \mathbb{C}^N$ denotes the linear (MMSE) receive filter for the kth user. Using (5) and assuming perfect CSI, we obtain

$$\hat{s}_k = \mathbf{a}_k^H(G_b\mathbf{y} + \mathbf{d}) = \sqrt{\rho}G_b\mathbf{a}_k^H\mathbf{H}\mathbf{s} + \mathbf{a}_k^H\mathbf{n}, \tag{7}$$

where we have defined $\mathbf{n} = G_b\mathbf{w} + \mathbf{d}$. Note that the noise $\mathbf{n}$ and the input vector $\mathbf{s}$ are uncorrelated such that $\mathbf{C}_\mathbf{y} = (K\rho+1)\mathbf{I}_N$ holds.

We employ an MMSE-based receiver while considering the b-quantization in which the detector matrix $\mathbf{A}$ is given by

$$\mathbf{A}^H = \left(\hat{\mathbf{H}}^H\hat{\mathbf{H}} + \tfrac{\tilde{\beta}_b}{G_b^2\rho}\mathbf{I}_K\right)^{-1}\hat{\mathbf{H}}^H, \tag{8}$$

where $\tilde{\beta}_b = G_b^2 + (1-G_b^2)(K\rho+1) + K\rho G_b^2\tilde{\sigma}^2$ represents the variance of $\mathbf{n}$ that accounts the additive noise, the quantization effect and the estimation error.

Therefore, the kth column of $\mathbf{A}$ can be written as

$$\mathbf{a}_k = \left(\hat{\mathbf{H}}\hat{\mathbf{H}}^H + \tfrac{\tilde{\beta}_b}{G_b^2\rho}\mathbf{I}_N\right)^{-1}\hat{\mathbf{h}}_k = \frac{\mathbf{\Upsilon}_k^{-1}\hat{\mathbf{h}}_k}{\hat{\mathbf{h}}_k^H\mathbf{\Upsilon}_k^{-1}\hat{\mathbf{h}}_k + 1}, \tag{9}$$

where, as shown in [17] but with low resolution quantization,

$$\mathbf{\Upsilon}_k \triangleq \sum_{i=1, i\neq k}^{K}\hat{\mathbf{h}}_i\hat{\mathbf{h}}_i^H + \tfrac{\tilde{\beta}_b}{G_b^2\rho}\mathbf{I}_N.$$

One can note that for the case of full resolution ($b = \infty$, $G_\infty = 1$), the detector matrix $\mathbf{A}$ for the linear MMSE defaults back to its original form $\mathbf{A}^H = \left(\hat{\mathbf{H}}^H\hat{\mathbf{H}} + \frac{1}{\rho}\mathbf{I}_K\right)^{-1}\hat{\mathbf{H}}^H$.

III. Analysis of Achievable Rate

We characterize the rate achievable in a low-resolution quantized massive MIMO uplink system for Gaussian inputs.

A. Sum-Rate Approximation for Gaussian Inputs

Using Bussgang's decomposition, the achievable rate can be approximated by treating the additive noise $\mathbf{a}_k^H\mathbf{n}$ in (7) and the channel input as Gaussian random variables. We then have the following general form for the achievable rate:

$$R_k(\rho) \approx \mathbb{E}_{\hat{\mathbf{H}}}\left[\log_2\left(1 + \frac{\rho|\mathbf{a}_k^H\hat{\mathbf{h}}_k|^2}{\rho\sum\limits_{j\neq k}^{K}|\mathbf{a}_k^H\hat{\mathbf{h}}_j|^2 + \tilde{\beta}_b\|\mathbf{a}_k\|^2}\right)\right] \tag{10}$$

where the terms in the denominator correspond to the interference, the estimation error and the quantization distortion.

1) Achievable Rate of the Proposed MMSE-Based Detector: Using (9), the signal-to-interference-plus-noise ratio can be written $\text{SINR}_{\text{MMSE}} = \hat{\mathbf{h}}_k^H\boldsymbol{\Upsilon}_k^{-1}\hat{\mathbf{h}}_k$, and by applying some straight-forward linear algebraic calculations, we obtain the following approximation:

$$R_k^{\text{MMSE}}(\rho) = -\mathbb{E}_{\hat{\mathbf{H}}}\left[\log_2\left[\left(\mathbf{I}_K + \frac{G_b^2\rho}{\tilde{\beta}_b}\hat{\mathbf{H}}^H\hat{\mathbf{H}}\right)^{-1}\right]_{k,k}\right] \tag{11}$$

Using Jensen's inequality, we obtain the following lower bound on the approximate achievable uplink rate in (10):

$$R_k^{\text{MMSE}}(\rho) \geq \tilde{R}_k^{\text{MMSE}}(\rho) = \log_2\left(1 + \frac{1}{\mathbb{E}\left[1/\gamma_k\right]}\right) \tag{12}$$

where $\gamma_k = \frac{1}{\left[\left(\mathbf{I}_K + \frac{G_b^2\rho}{\tilde{\beta}_b}\hat{\mathbf{H}}^H\hat{\mathbf{H}}\right)^{-1}\right]_{k,k}} - 1$.

Using a similar methodology as in [18] while considering the b-bit quantization, we approximate the exact distribution of γ_k with a Gamma distribution which has an analytically tractable form. Hence, the PDF of γ_k is given by [19]:

$$p_{\gamma_k}(\gamma) = \frac{\gamma^{\hat{\alpha_b}-1}e^{-\gamma/\hat{\theta}_b}}{\Gamma(\hat{\alpha_b})\hat{\theta}_b^{\hat{\alpha_b}}} \tag{13}$$

where $\hat{\alpha_b} = \frac{(N-K+1+(K-1)\mu)^2}{N-K+1+(K-1)\kappa}$, $\hat{\theta}_b = \frac{N-K+1+(K-1)\kappa}{N-K+1+(K-1)\mu}\frac{G_b^2\rho\hat{\sigma}^2}{\tilde{\beta}_b}$, and $\Gamma(\cdot)$ is the Gamma function. Moreover, μ and κ are obtained by solving following equations:

$$\mu = \frac{1}{N\frac{G_b^2\rho\hat{\sigma}^2}{\tilde{\beta}_b}\left(1-\frac{K-1}{N}+\frac{K-1}{N}\mu\right)+1}$$

$$\kappa\left(1+\frac{(K-1)G_b^2\rho\hat{\sigma}^2}{\tilde{\beta}_b}\mu^2\right) = \frac{(K-1)G_b^2\rho\hat{\sigma}^2}{\tilde{\beta}_b}\mu^3 + (K-1)\mu^2$$

Proposition 1: Using the approximate PDF of γ_k given by (13), and the proposed linear MMSE-based detector, the lower bound on the achievable rate for the kth user is

$$\tilde{R}_k^{\text{MMSE}}(\rho) = \log_2(1 + (\hat{\alpha_b} - 1)\hat{\theta}_b). \tag{14}$$

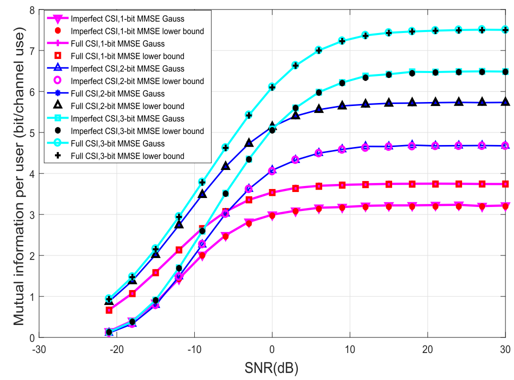

Fig. 2. Mutual information per user versus SNR for a $K \times N$ massive MIMO system where $K = 16$ users and $N = 128$ BS antennas and $b = \{1, 2, 3\}$ quantization bits; analytical versus simulated for the proposed MMSE-based detection with imperfect channel estimation.

Proof: Substituting (13) into (12) and using the identity $\Gamma(\hat{\alpha_b}) = (\hat{\alpha_b} - 1)\Gamma(\hat{\alpha_b} - 1)$, the result follows. $\square$

Remark: From (10), the achievable rate $R_k(\rho)$ can be rewritten as

$$\begin{aligned} R_k(\rho) &= \mathbb{E}_{\hat{\mathbf{H}}}\left[\log_2\left(1 + \frac{|\mathbf{a}_k^H\hat{\mathbf{h}}_k|^2}{\mathbf{a}_k^H\boldsymbol{\Upsilon}_k\mathbf{a}_k}\right)\right] \\ &\leq \mathbb{E}_{\hat{\mathbf{H}}}\left[\log_2\left(1 + \frac{\|\mathbf{a}_k^H\boldsymbol{\Upsilon}_k^{1/2}\|^2\|\boldsymbol{\Upsilon}_k^{-1/2}\hat{\mathbf{h}}_k\|^2}{\mathbf{a}_k^H\boldsymbol{\Upsilon}_k\mathbf{a}_k}\right)\right] \\ &= \mathbb{E}_{\hat{\mathbf{H}}}\left[\log_2(1 + \hat{\mathbf{h}}_k^H\boldsymbol{\Upsilon}_k^{-1}\hat{\mathbf{h}}_k)\right] = R_k^{\text{MMSE}}(\rho). \end{aligned} \tag{15}$$

The inequality is obtained by using Cauchy-Schwarz' inequality, which holds with equality when using the proposed detector in (11) with $\mathbf{a}_k = \alpha\boldsymbol{\Upsilon}_k^{-1}\hat{\mathbf{h}}_k$, for any $\alpha \in \mathbb{C}$. Therefore, the proposed MMSE-based linear detector is optimal in the sense that it maximizes the achievable rate given by (10).

IV. Experimental Simulation Results

For our simulation, we consider a single-cell b-bit massive MIMO uplink with $K = \{16, 32\}$ users and $N = \{128, 256\}$ BS antennas for QPSK and Gaussian inputs.

1) To compute the achievable rate for the QPSK inputs, the achievable rate is $R_k(\rho) = I(s_k; \hat{s}_k \,|\, \mathbf{H})$ as shown in [20]. We expand the mutual information $I(s_k; \hat{s}_k \,|\, \mathbf{H})$ as follows:

$$I(s_k; \hat{s}_k \,|\, \mathbf{H}) = \mathbb{E}_{s_k,\hat{s}_k,\mathbf{H}}\left[\log_2\frac{P_{\hat{s}_k|s_k,\mathbf{H}}(\hat{s}_k|s_k,\mathbf{H})}{P_{\hat{s}_k|\hat{\mathbf{H}}}(\hat{s}_k|\mathbf{H})}\right]. \tag{16}$$

Here, the conditional probability mass functions $P_{\hat{s}_k|s_k,\mathbf{H}}(\hat{s}_k|s_k,\mathbf{H})$ and $P_{\hat{s}_k|\mathbf{H}}(\hat{s}_k|\mathbf{H}) = E_{s_k}[P_{\hat{s}_k|s_k,\mathbf{H}}(\hat{s}_k|s_k,\mathbf{H})]$ are needed to compute (16). We use Monte-Carlo simulations to estimate them since no closed-form expressions are available for these quantities. In particular, we simulate many noise and interference realizations, and map the resulting $\hat{s}_k$ to points over a rectangular grid in the complex plane.

2) A conventional MIMO system with higher-order modulation can be developed that uses near ML detection and achieves capacity that is upper bounded by [20]:

$$R(\rho) = \mathbb{E}_{\mathbf{H}}\left[\log_2\{\det\{\mathbf{I}_N + \tfrac{\rho}{K}\mathbf{H}\mathbf{H}^H\}\}\right], \tag{17}$$

where $\det\{\cdot\}$ represents the determinant of a matrix.

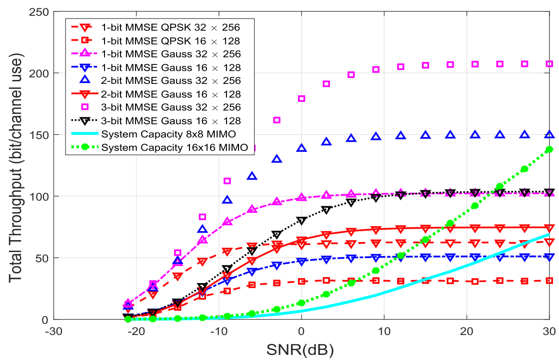

Fig. 3. Comparison of total system throughput for a $K \times N$ quantized b-bit massive MIMO with $K = \{16, 32\}$ users and $N = \{128, 256\}$ with imperfect channel estimation, vs. two conventional 8×8 and 16×16 MIMO systems.

The following simulations will address the attainability of an equivalent performance in the uplink to that of conventional MIMO using another less-complex system by adding more low-cost antennas with low resolution ADCs in the RF front-end at the receiver side, and employ linear processing for MIMO detection in the baseband.

We evaluate the validity of our closed-form expression for the achievable rate for the MMSE-based linear detector given in (14) for Gaussian inputs with perfect and imperfect CSI. In Fig. 2, we show the accuracy of the proposed closed form of the MMSE-based detector (14) with the simulated form (10) for low bit resolution ADCs ($b = \{1, 2, 3\}$). One can notice that the closed form achievable rate perfectly matches with Monte Carlo simulated rates for perfect and imperfect CSI. This indicates that our derived expression (14) is a valid predictor for the performance of b-bit massive MIMO system.

Figure 3 compares the sum rate between the b-bit quantized massive MIMO with linear detection against the conventional MIMO systems with near ML detection, assuming imperfect channel estimation. For the $b = 1$ bit quantization case, the SNR regimes are bifurcated into two regions: for the low SNR regime, the massive MIMO system can outperform the conventional MIMO even with imperfect channel estimation; however, for high SNR, the conventional MIMO system with a near-ML detector can outperform the massive MIMO system. For instance, to achieve a sum rate of 62 bits/channel use for the one bit massive MIMO case with $N = 256$ antennas at BS and $K = 32$ users the needed SNR is -6 dB with just QPSK modulation; however, for a large MIMO (e.g., 16×16) system with near-ML detection the needed SNR is 15 dB, and for an 8×8 MIMO system the needed SNR is around 25 dB. Furthermore, it can be seen that for few bits (i.e. 2 or 3 bits) of quantization, the proposed MMSE-based detector with massive MIMO deployment outperforms the conventional MIMO.

V. Conclusion

In this letter, an uplink massive MIMO system with low resolution quantized ADCs is considered. Using the Bussgang decomposition, a new MMSE-based linear detection scheme that incorporates the non-linear effects of quantization has been derived. A closed form expression for the uplink achievable rate has been derived, and used to analyze and compare the performance of a quantized massive MIMO system against both a large MIMO system that employs higher-order modulation. In particular, it has been shown that for a few bits of quantization (e.g., 2 or 3 bits), the quantized massive MIMO can outperform the conventional MIMO.

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